PHYS 3313 – Section 001
Lecture #12

Monday, Feb. 25, 2019

Dr. Jaehoon Yu

- Rutherford Scattering Experiment and Rutherford Atomic Model
- The Classic Atomic Model
- Bohr Radius
- Bohr's Hydrogen Model and Its Limitations

Announcements

- Reminder: Midterm Exam
  - In class Wednesday, March. 6
  - Covers from CH1.1 through what we learn March 4 plus the math refresher in the appendices
  - Mid-term exam constitutes 20% of the total
  - **Please do NOT miss the exam! You will get an F if you miss it.**
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of handwritten formulae and values of constants for the exam
  - No derivations, word definitions or setups or solutions of any problems!
    - No Lorentz velocity addition formula!
    - No Maxwell's equations!
  - No additional formulae or values of constants will be provided!
- Quiz 2 results
  - Class average: 33.7/60
    - Equivalent to 56.2/100
    - Previous quiz: 37.2/100
  - Top score: 58/60

Reminder: Special Project #4

- A total of N_i incident projectile particle of atomic number Z_1 kinetic energy KE scatter on a target of thickness t and atomic number Z_2 and has n atoms per volume. What is the total number of scattered projectile particles at an angle θ? (20 points)
- Please be sure to clearly define all the variables used in your derivation! Points will be deducted for missing variable definitions.
- This derivation must be done on your own. Please do not copy the book, internet or your friends'.
- Due this Wednesday, Feb. 27

Prefixes, expressions and their meanings

- deca (da): 10^{1}
- hecto (h): 10^{2}
- kilo (k): 10^{3}
- mega (M): 10^{6}
- giga (G): 10^{9}
- tera (T): 10^{12}
- peta (P): 10^{15}
- exa (E): 10^{18}

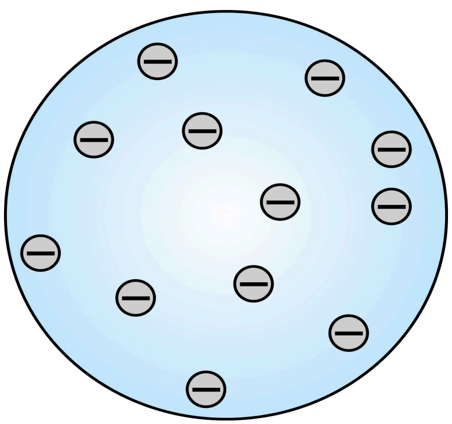
- deci (d): 10^{-1}
- centi (c): 10^{-2}
- milli (m): 10^{-3}
- micro (μ): 10^{-6}
- nano (n): 10^{-9}
- pico (p): 10^{-12}
- femto (f): 10^{-15}
- atto (a): 10^{-18}

The Atomic Models of Thomson and Rutherford

- Without seeing it, 19^{th} century scientists believed atoms have structure.
- Pieces of evidence that scientists had in 1900 to indicate that the atom was not a fundamental unit
- There are simply too many kinds of atoms (~70 known at that time), belonging to a distinct chemical element
  - Too many to be fundamental!!
- Atoms and electromagnetic phenomena seem to be intimately related
- The issue of **valence** ➔ Why certain elements combine with some elements but not with others?
  - Is there a characteristic internal atomic structure?
- The discoveries of radioactivity, x rays, and the electron

Thomson's Atomic Model

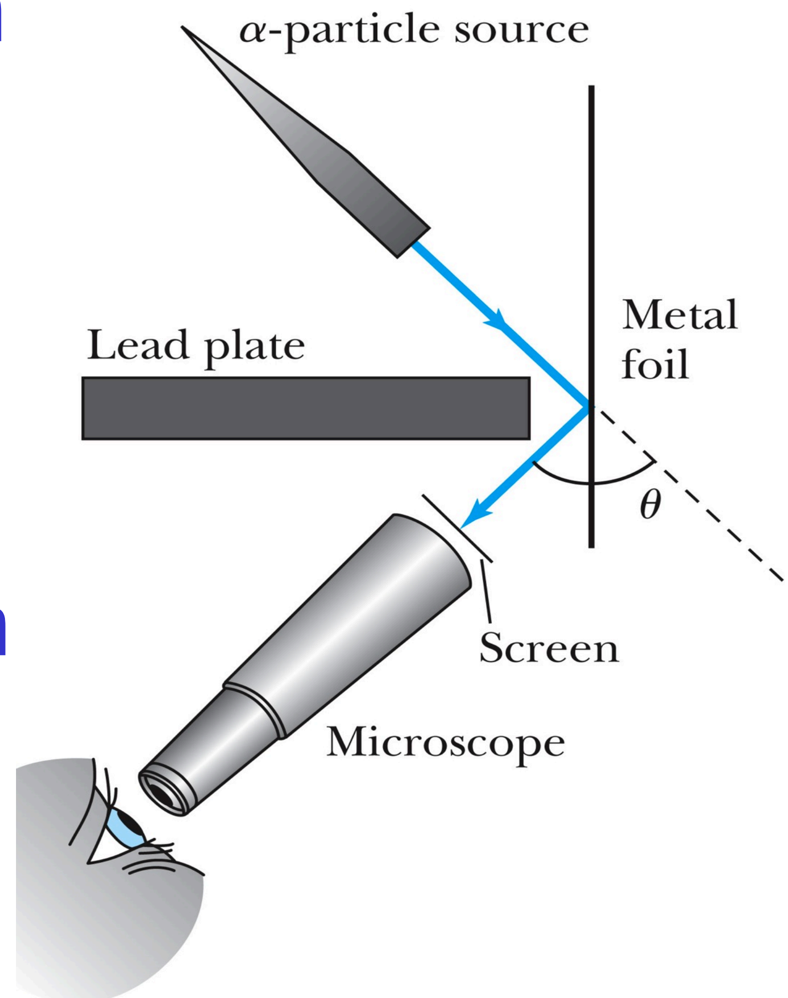
- Thomson's "plum-pudding" model
  - Atoms are electrically neutral and have electrons in them
  - Atoms must have an equal amount of positive charges in it to balance electron negative charges
  - So how about positive charges spread uniformly throughout a sphere the size of the atom with the newly discovered "negative" electrons embedded in a uniform background.



- Thomson thought when the atom was heated the electrons could vibrate about their equilibrium positions and thus produce electromagnetic radiation.

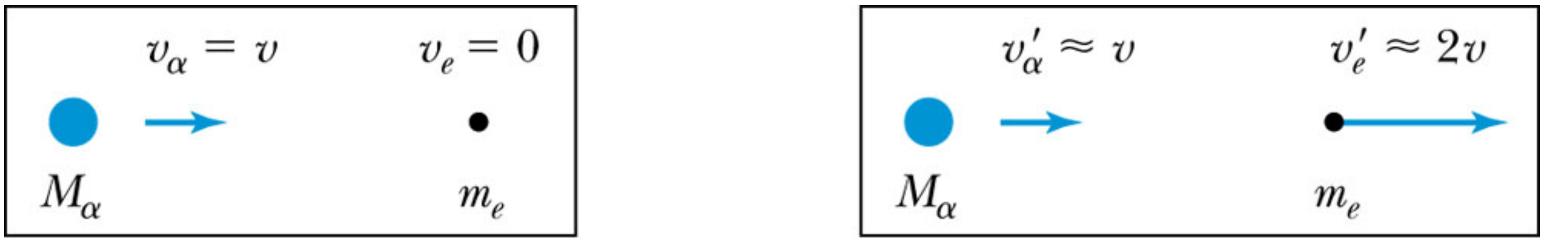
Experiments of Geiger and Marsden

- Rutherford, Geiger, and Marsden conceived a new technique for investigating the structure of matter by scattering a particle off atoms.
- Geiger showed that many particles were scattered from thin gold-leaf targets at backward angles greater than 90°.
- Time to do some calculations!



Ex 4.1: Maximum Scattering Angle

Geiger and Marsden (1909) observed backward-scattered (θ>=90°) α particles when a beam of energetic α particles was directed at a piece of gold foil as thin as $6.0\text{x}10^{-7}$m. Assuming an α particle scatters from an electron in the foil, what is the maximum scattering angle?



Before After

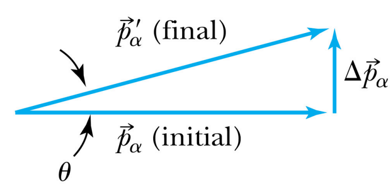
- The maximum scattering angle corresponds to the maximum momentum change
- Using the momentum conservation and the KE conservation for an elastic collision, the maximum momentum change of the α particle is in a head-on collision

$$M_\alpha \vec{v}_\alpha = M_\alpha \vec{v}'_\alpha + m_e \vec{v}'_e$$

$$\frac{1}{2} M_\alpha v_\alpha^2 = \frac{1}{2} M_\alpha v'^2_\alpha + \frac{1}{2} m_e v'^2_e$$

$$\Rightarrow \Delta \vec{p}_\alpha = M_\alpha \vec{v}_\alpha - M_\alpha \vec{v}'_\alpha = m_e \vec{v}'_e \Rightarrow \Delta p_{\alpha-\max} = 2 m_e v_\alpha$$

- Determine θ by letting Δp_{max} be perpendicular to the direction of motion.



$$\theta_{\max} = \frac{\Delta p_{\alpha-\max}}{p_\alpha} = \frac{2 m_e v_\alpha}{M_\alpha v_\alpha} = \frac{2 m_e}{M_\alpha} = 2.7 \times 10^{-4}\, rad = 0.016^\circ$$

Multiple Scattering from Electrons

- If an α particle were scattered by many electrons, then *N* electrons results in $\langle\theta\rangle_{total} \sim \sqrt{N} \cdot \theta$
- The number of atoms across a thin gold layer of 6×10^{-7} m:

$$\frac{N_{Molecules}}{cm^3} = N_{Avogadro}\left(molecules/mol\right) \times \left[\frac{1}{g-molecular-weight}\left(\frac{mol}{g}\right)\right] \cdot \left[\rho\left(\frac{g}{cm^3}\right)\right]$$

$$= 6.02 \times 10^{23}\left(\frac{molecules}{mol}\right) \cdot \left(\frac{1mol}{197g}\right) \cdot \left(19.3\frac{g}{cm^3}\right)$$

$$= 5.9 \times 10^{22}\frac{molecules}{cm^3} = 5.9 \times 10^{28}\frac{atoms}{m^3}$$

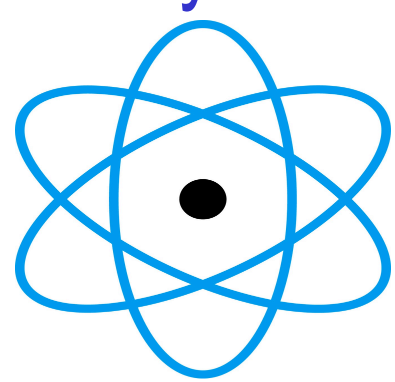
- Assume the distance between atoms is $d = \left(5.9 \times 10^{28}\right)^{-1/3} = 2.6 \times 10^{-10}\left(m\right)$ and there are $N = \dfrac{6 \times 10^{-7}m}{2.6 \times 10^{-10}m} = 2300\left(atoms\right)$

  This gives $\langle\theta\rangle_{total} = \sqrt{2300}\left(0.016^{\circ}\right) = 0.8^{\circ}$

Rutherford's Atomic Model

- $\langle\theta\rangle_{total} \sim 0.8*\sqrt{79} = 7.1^{o}$ even if the α particle scattered from all 79 electrons in each atom of gold

- The experimental results were inconsistent with Thomson's atomic model.
- Rutherford proposed that an atom has a positively charged core (nucleus) surrounded by the negative electrons.

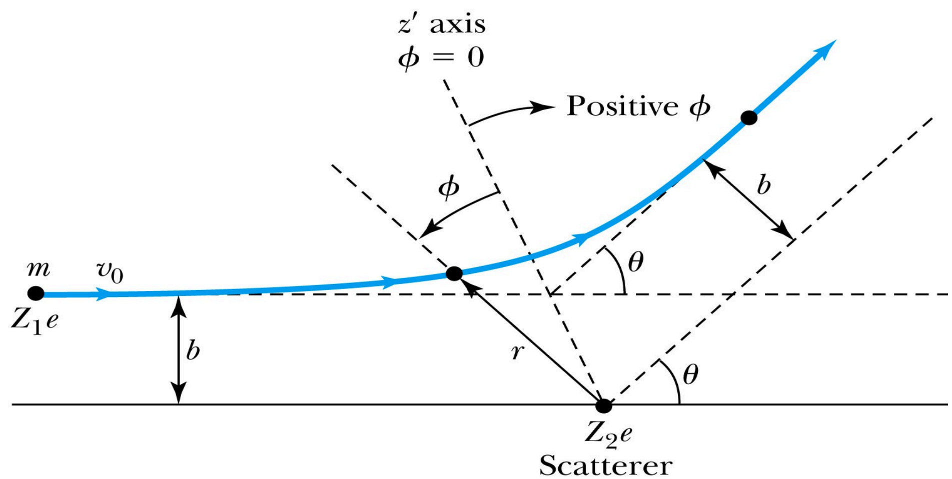


Assumptions of Rutherford Scattering

1. The scatterer is so massive that it does not recoil significantly; therefore the initial and final KE of the α particle are practically equal.
2. The target is so thin that only a single scattering occurs.
3. The bombarding particle and target scatterer are so small that they may be treated as point masses with electrical charges.
4. Only the Coulomb force is effective.

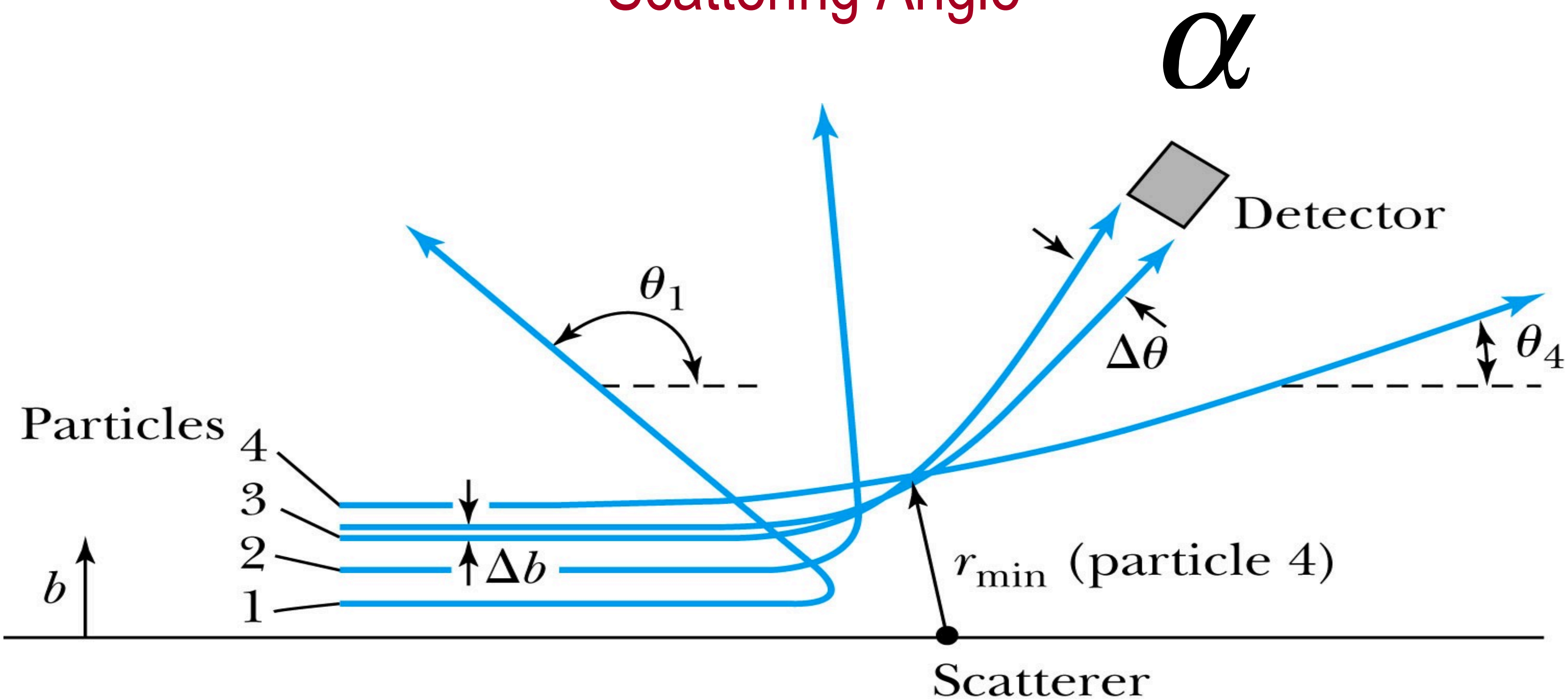
Rutherford Scattering

- Scattering experiments help us study matter too small to be observed directly by measuring the angular distributions of the scattered particles
  - What is the force acting in this scattering?
- There is a relationship between the impact parameter *b* and the scattering angle θ.



- When *b* is small,
- *r* gets small.
- Coulomb force gets large.
- θ can be large and the particle can be repelled backward.

The Relationship Between the Impact Parameter b and the Scattering Angle



The relationship between the impact parameter b and scattering angle θ: Particles with small impact parameters approach the nucleus most closely (r_{min}) and scatter to the largest angles. Particles within a certain range of impact parameters b will be scattered within the window Δθ.

Rutherford Scattering

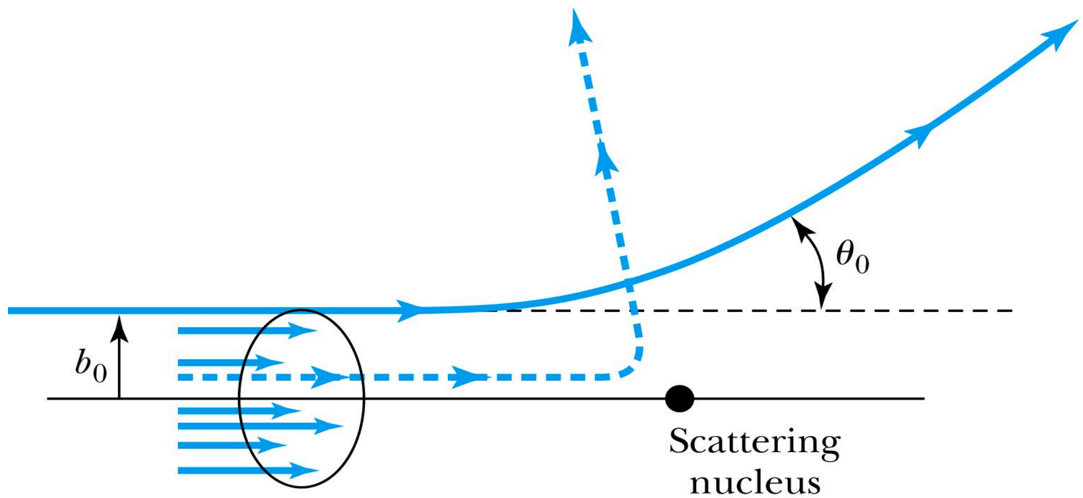
- What are the quantities that can affect the scattering?
  - What was the force again?
    - The Coulomb force

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{r^2}\hat{r}_e$$

  - The charge of the incoming particle ($Z_1 e$)
  - The charge of the target particle ($Z_2 e$)
  - The minimum distance the projectile approaches the target (r)
- Using the fact that this is a totally elastic scattering under a central force, we know that
  - Linear momentum is conserved $\vec{p}_i^{\,\alpha} = \vec{p}_f^{\,\alpha} + \vec{p}^{\,N}$
  - KE is conserved $\frac{1}{2}mv_{\alpha i}^2 = \frac{1}{2}mv_{\alpha f}^2 + \frac{1}{2}mv_n^2$
  - Angular momentum is conserved $mr^2\varpi = mv_{\alpha i}b$
- From this, impact parameter $b = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 m v_{\alpha i}^2}\cot\frac{\theta}{2} = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 KE_i}\cot\frac{\theta}{2}$

Rutherford Scattering - probability

- Any particle inside the circle of area πb_0^2 will be similarly scattered.



- The **cross section** $\sigma = \pi b^2$ is related to the **probability** for a particle being scattered by a nucleus.

$$nt\pi b^2 = \pi nt\left(\frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 KE_i}\cot\frac{\theta}{2}\right)^2$$

t: target thickness
n: atomic number density

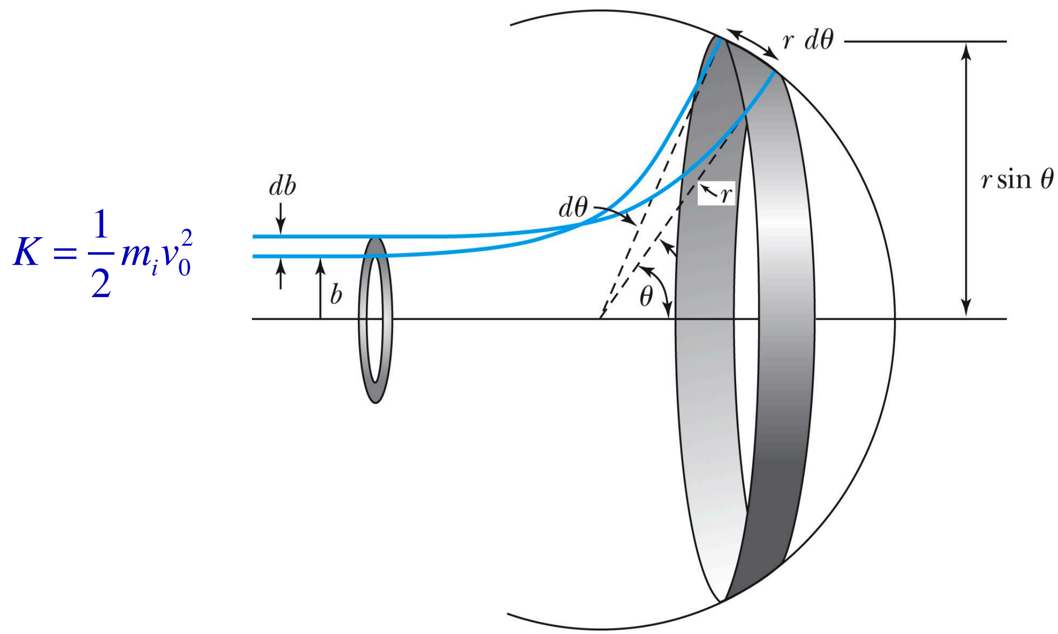
- The fraction of the incident particles scattered is

$$f = \frac{\text{target area exposed by scatterers}}{\text{total target area}}$$

- The number of scattering nuclei per unit area $nt = \frac{\rho N_A N_M t}{M_g} \cdot \frac{atoms}{cm^2}$

Rutherford Scattering Equation

- In an actual experiment, a detector is positioned from θ to $\theta + d\theta$ that corresponds to incident particles between b and $b + db$.



- The number of particles scattered into the the angular coverage per unit area is

$$N(\theta) = \frac{N_i n t}{16} \left(\frac{e^2}{4\pi\varepsilon_0} \right)^2 \frac{Z_1^2 Z_2^2}{r^2 K^2 \sin^4(\theta/2)}$$

The Important Points

1. The scattering is proportional to the **square of the atomic numbers** of *both* the incident particle (Z_1) and the target scatterer (Z_2).
2. The number of scattered particles is **inversely proportional to the square of the kinetic energy** of the incident particle.
3. For a scattering angle θ, the scattering is **proportional to the 4th power of sin(θ/2)**.
4. The Scattering is **proportional to the target thickness** for thin targets.

The Classical Atomic Model

As suggested by the Rutherford Model, an atom consists of a small, massive, positively charged nucleus surrounded by moving electrons. This then suggested consideration of a planetary model of the atom.

Let's consider atoms in a planetary model.

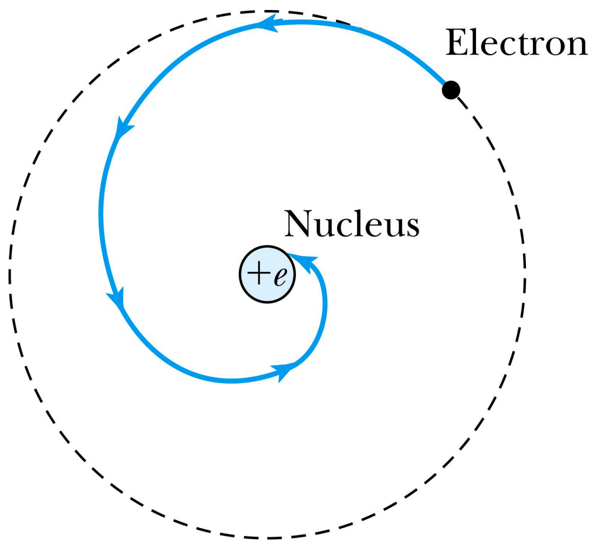
- The force of attraction on the electron by the nucleus and Newton's 2nd law give $$\vec{F}_e = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\hat{e}_r = -\frac{mv^2}{r}\hat{e}_r$$

  where v is the tangential speed of an electron.

- The total energy is $E = K + V = \dfrac{e^2}{8\pi\varepsilon_0 r} - \dfrac{e^2}{4\pi\varepsilon_0 r} = -\dfrac{e^2}{8\pi\varepsilon_0 r}$

The Planetary Model is Doomed

- From the classical E&M theory, an accelerated electric charge radiates energy (electromagnetic radiation) which means total energy must decrease. ➔*Radius r must decrease!!*



Electron crashes into the nucleus!?

- Physics had reached a turning point in 1900 with Planck's hypothesis of the quantum behavior of radiation.