PHYS 3313 – Section 001
Lecture #13

Wednesday, Feb. 27, 2019
Dr. Jaehoon Yu

- Bohr Radius
- Bohr's Hydrogen Model
- The Correspondence Principle
- Importance of Bohr's Hydrogen Model
- Success and Failure of Bohr's Model
- Characteristic X-ray Spectra

Announcements

- Homework #3
  - End of chapter problems on CH4: 5, 14, 17, 21, 23 and 45
  - Due: Monday, March 18
- Reminder: Midterm Exam
  - In class next Wednesday, March. 6
  - Covers from CH1.1 through what we learn March 4 plus the math refresher in the appendices
  - Mid-term exam constitutes 20% of the total
  - **Please do NOT miss the exam! You will get an F if you miss it.**
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of handwritten formulae and values of constants for the exam
  - No derivations, word definitions or setups or solutions of any problems!
    - No Lorentz velocity addition formula!
    - No Maxwell's equations!
  - No additional formulae or values of constants will be provided!
- Special Colloquium this Friday
  - 4pm, Friday, Mar. 1, SH101
  - Mr. Steve Battel, Battel Engineering, member of National Academy of Engineering

The UTA Department of Physics presents a special colloquium

Steve Battel

Member, National Academy of Engineering

Come prepared to help Steve teach you what he doesn't know!

Steve Battel is the president of Battel Engineering and is also an adjunct clinical professor of engineering at the University of Michigan. He has 42 years of experience working as an engineer on over 100 space projects from Hubble, Cassini, Mars Rovers, Parker Solar Probe, GOES and DirectTV down to small commercial satellites and CubeSats.

He will share his personal experiences with you on how to work creatively in partnership with others to translate engineering knowledge and ideas into working space systems at all scales of design. He will discuss how to ask questions when thinking about a problem and then how to successfully systematize, build and test a product that is the outcome of your ideas.

After his talk, Steve will also spend time with you in a brainstorming dialogue related to your specific interests and questions.

4 p.m. Friday, March 1

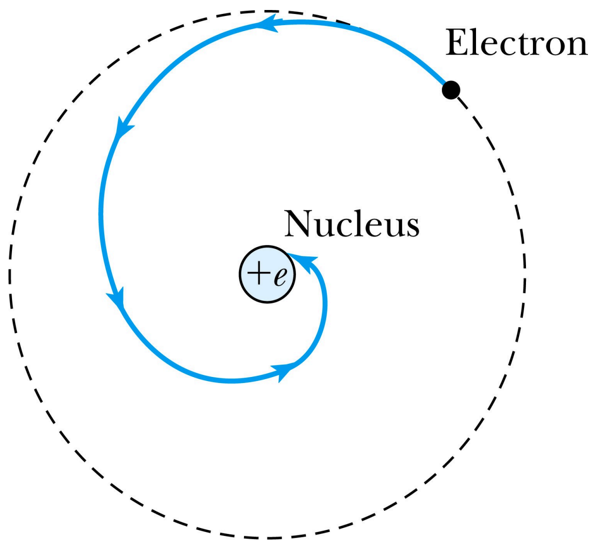
Science Hall Room 101

Reception starts at 3:30 p.m. in SH 108

In addition to being an NAE member, Steve is a Fellow of the American Institute of Aeronautics and Astronautics, a Fellow of the American Association for the Advancement of Science, a senior member of IEEE and a member of Sigma Xi. Steve's areas of specialization include low-noise instrumentation, avionics and power systems for space applications. He is internationally recognized for his expertise in the design and development of space high voltage systems especially for systems intended for operation in challenging planetary environments.

The Planetary Model is Doomed

- From the classical E&M theory, an accelerated electric charge radiates energy (electromagnetic radiation) which means total energy must decrease. ➔*Radius r must decrease!!*



Electron crashes into the nucleus!?

- Physics had reached a turning point in 1900 with Planck's hypothesis of the quantum behavior of radiation.

The Bohr Model of the Hydrogen Atom – **The assumptions**

- "Stationary" states or orbits must exist in atoms, i.e., orbiting electrons ***do not radiate*** energy in these orbits. These orbits or stationary states are of a fixed definite energy E. (Niels Bohr was awarded Nobel in 1922 for this!)
- The emission or absorption of electromagnetic radiation can occur only in conjunction with a transition between two stationary states. The frequency, f, of this radiation is proportional to the *difference* in the energies of the two stationary states:
- $E = E_1 - E_2 = hf$
- *where h is Planck's Constant*
  - *Bohr thought this has to do with the fundamental length of order ~10^{-10}m*
- Classical laws of physics govern the dynamic equilibrium of the stationary state but they do not apply to the transitions between stationary states.
- The mean kinetic energy of the electron-nucleus system is quantized as ***$K = nhf_{orb}/2$***, where f_{orb} is the frequency of rotation. This is equivalent to the angular momentum of a stationary state to be an integral multiple of $h/2\pi = \hbar$

How did Bohr Arrived at the angular momentum quantization?

- The mean kinetic energy of the electron-nucleus system is quantized as $K = nhf_{orb}/2$, where f_{orb} is the frequency of rotation in the given orbit. This is equivalent to the angular momentum of a stationary state to be an integral multiple of h/2π.
- Kinetic energy can be written $K = \frac{nhf}{2} = \frac{1}{2}mv^2$
- Angular momentum is defined as $\left|\vec{L}\right| = \left|\vec{r} \times \vec{p}\right| = mvr$
- The relationship between linear and angular quantifies $v = r\omega;\ \omega = 2\pi f$
- Thus, we can rewrite $K = \frac{1}{2}mvr\omega = \frac{1}{2}L\omega = \frac{1}{2}2\pi Lf = \frac{nhf}{2}$

$$2\pi L = nh \Rightarrow \quad L = n\frac{h}{2\pi} = n\hbar, \text{where } \hbar = \frac{h}{2\pi}$$

Bohr's Quantized Radius of Hydrogen

- The angular momentum is $\left|\vec{L}\right| = \left|\vec{r} \times \vec{p}\right| = mvr = n\hbar$
- So the speed of an orbiting e can be written $v_e = \frac{n\hbar}{m_e r}$
- From the Newton's law for a circular motion

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{m_e v_e^2}{r} \Rightarrow v_e = \frac{e}{\sqrt{4\pi\varepsilon_0 m_e r}}$$

- So from the above two equations, we can get

$$v_e = \frac{n\hbar}{m_e r} = \frac{e}{\sqrt{4\pi\varepsilon_0 m_e r}} \Rightarrow r = \frac{4\pi\varepsilon_0 n^2\hbar^2}{m_e e^2}$$

Bohr Radius

- The radius of the hydrogen atom for the n[th] stationary state is

$$r_n = \frac{4\pi\varepsilon_0\hbar^2 n^2}{m_e e^2} = a_0 n^2$$

Where the **Bohr radius** for a given stationary state is:

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = \frac{\left(1.055\times10^{-34}\,J\cdot s\right)^2}{\left(8.99\times10^{9}\,N\cdot m^2/C^2\right)\cdot\left(9.11\times10^{-31}\,kg\right)\cdot\left(1.6\times10^{-19}\,C\right)^2} = 0.53\times10^{-10}\,m$$

- The smallest diameter of the hydrogen atom is

$$d = 2r_1 = 2a_0 \approx 10^{-10}\,m \approx 1\,\mathring{A}$$

  - OMG!! The fundamental length!!
- $n = 1$ gives its lowest energy state (called the **"ground" state**)

Ex. 4.6 Justification for non-relativistic treatment of orbital e

- Are we justified for the non-relativistic treatment of the orbital electrons?
  - When do we apply relativistic treatment?
    - When v/c>0.1
- Orbital speed: $v_e = \frac{e}{\sqrt{4\pi\varepsilon_0 m_e r}}$
- Thus

$$v_e = \frac{\left(1.6\times10^{-16}\right)\cdot\left(9\times10^{9}\right)}{\sqrt{\left(9.1\times10^{-31}\right)\cdot\left(0.5\times10^{-10}\right)}} \approx 2.2\times10^{6}\left(m/s\right) < 0.01c$$

Uncertainties

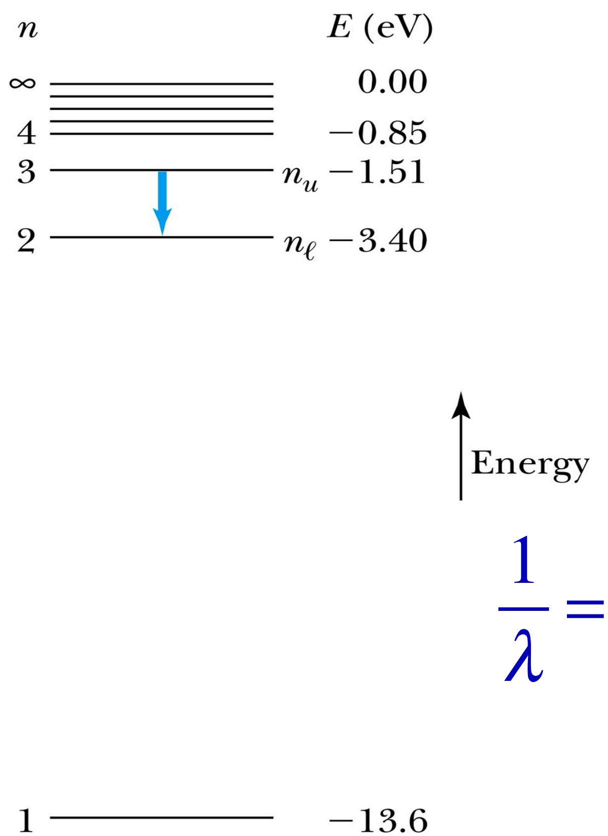
- Statistical Uncertainty: A naturally occurring uncertainty due to the number of measurements
  - Usually estimated by taking the square root of the number of measurements or samples, $\sqrt{N}$
- Systematic Uncertainty: Uncertainty due to unintended biases or unknown sources
  - Biases made by personal measurement habits
  - Some sources that could impact the measurements
- In any measurement, the uncertainties provide the significance to the measurement

The Hydrogen Atom – Rydberg Equation

- Recalling the total E of an e in an atom, the n[th] stationary states, E_n

$$E_n = -\frac{e^2}{8\pi\varepsilon_0 r_n} = -\frac{e^2}{8\pi\varepsilon_0 a_0 n^2} = -\frac{E_0}{n^2} \qquad E_0 = \frac{e^2}{8\pi\varepsilon_0 a_0} = \frac{\left(1.6\times10^{-19}\right)^2}{8\pi\cdot\left(8.85\times10^{-12}\right)\left(0.53\times10^{-10}\right)} = 13.6(eV)$$

where E_1 is the lowest energy or ground state energy



- Emission of light occurs when the atom is in an excited state and decays to a lower energy state ($n_u \longrightarrow n_\ell$).

$$hf = E_u - E_l$$

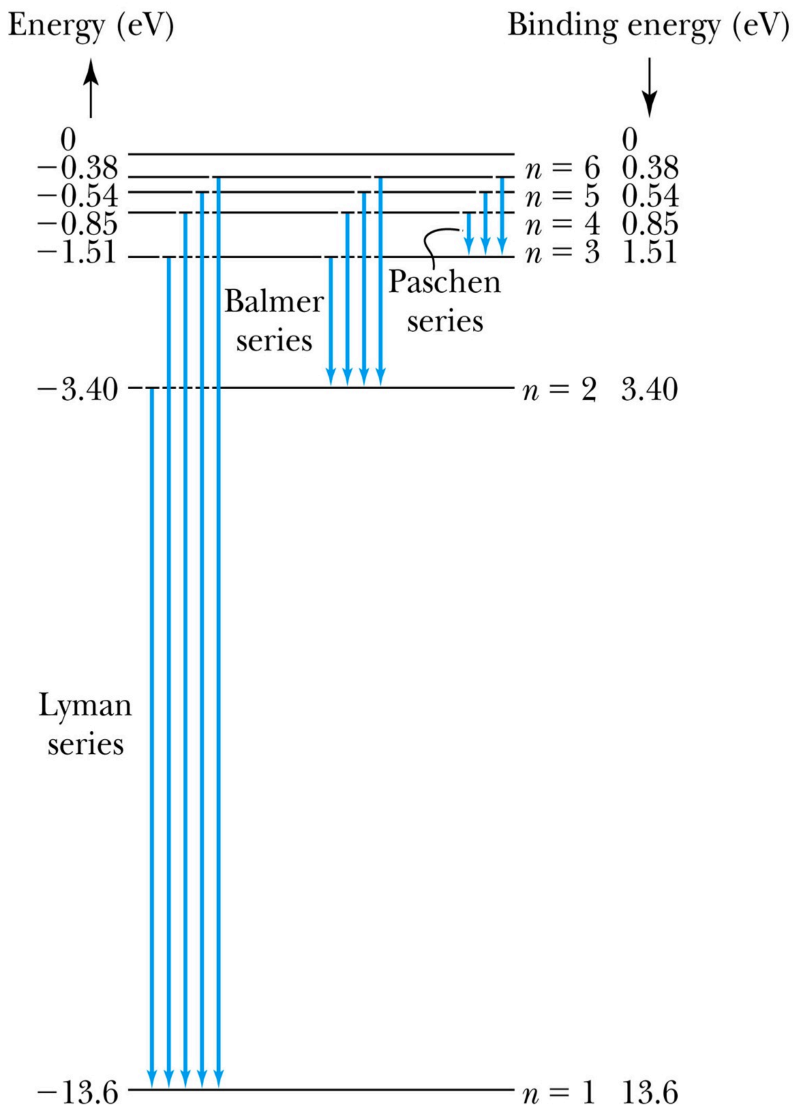
where *f* is the frequency of a photon.

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{E_u - E_l}{hc} = \frac{E_0}{hc}\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right) = R_\infty\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right)$$

R_∞ is the **Rydberg constant**. $R_\infty = E_0/hc$

$$R_H = 1.096776\times10^7 m^{-1}$$

Transitions in the Hydrogen Atom



- **Lyman series (n=1):** The atom will remain in the excited state for a short time before emitting a photon and returning to a lower stationary state. All hydrogen atoms exist in $n = 1$ (invisible).
- **Balmer series (n=2):** When sunlight passes through the atmosphere, hydrogen atoms in water vapor absorb the wavelengths (visible).

Fine Structure Constant

- The electron's speed on an orbit in the Bohr model:

$$v_e = \frac{n\hbar}{m_e r_n} = \frac{n\hbar}{m_e \dfrac{4\pi\varepsilon_0 n^2\hbar^2}{m_e e^2}} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_0\hbar}$$

- On the ground state, $v_1 = 2.2 \times 10^6$ m/s ~ less than 1% of the speed of light
- The ratio of v_1 to c is the **fine structure constant, α**.

$$\alpha \equiv \frac{v_1}{c} = \frac{\hbar}{m a_0 c} = \frac{e^2}{4\pi\varepsilon_0\hbar c} =$$

$$\frac{\left(8.99\times10^{9}\,N\cdot m^2/C^2\right)\cdot\left(1.6\times10^{-19}\,C\right)^2}{\left(1.055\times10^{-34}\,J\cdot s\right)\cdot\left(3\times10^{8}\,m/s\right)} \approx \frac{1}{137}$$

Ex. 4.7: Determine the longest and shortest wavelengths observed in Paschen Series (n=3)

- We use the equation

$$\frac{1}{\lambda} = R_\infty \left(\frac{1}{n_l^2} - \frac{1}{n_u^2} \right)$$

$$\frac{1}{\lambda_{Max}} = 1.0974 \times 10^7 \cdot \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = 5.335 \times 10^5 \left(m^{-1} \right)$$

$$\Rightarrow \lambda_{Max} = 1875 (nm)$$

$$\frac{1}{\lambda_{Min}} = 1.0974 \times 10^7 \cdot \left(\frac{1}{3^2} - \frac{1}{\infty^2} \right) = 1.219 \times 10^6 \left(m^{-1} \right)$$

$$\Rightarrow \lambda_{Min} = 820 (nm)$$