PHYS 3313 – Section 001
Lecture #16

Monday, March 25, 2019

Dr. Jaehoon Yu

- X-ray scattering & Bragg's Law
- de Broglie Waves
- Bohr's Quantization Conditions
- Electron Scattering
- Wave Motion and Properties
- Wave Packets and Packet Envelops
- The Uncertainty Principle

Announcements

- Homework #4: CH5 end of the chapter problems
  - 8, 10, 16, 24, 26, 37 and 47
  - Due: Monday Apr. 1, 2019
- Reading Assignments: CH5.1 and CH5.3
- The workshop volunteers needed
  - Two workshops in April
    - April 12 and 13, Friday and Saturday: New Opportunities at the Next Generation Neutrino Experiments
      - 8:30am – noon, Friday, Apr. 12 in front of CPB303: four for registration desk help through 9am then two at the registration desk and two in the room
      - Noon, Friday, Apr. 12 – 4pm Saturday, Apr. 13: Need help of one person in the room at all times
    - April 18 and 19, Thursday and Friday: US DUNE Near Detector Workshop
      - 8:30am – noon, Thursday, Apr. 18 in front of CPB303: four for registration desk help through 9am then two at the registration desk and two in the room
      - 2pm, Thursday, Apr. 18 – 11:30am Friday, Apr. 19: Need help of two persons in each of the four breakout rooms\
      - 11:30am – 1:30pm, Friday, Apr. 19: One person in CPB303
  - Eman Alasadi will coordinate this with Dr. Jonathan Asaadi

Special Project #6

- Derive the following using trigonometric identities

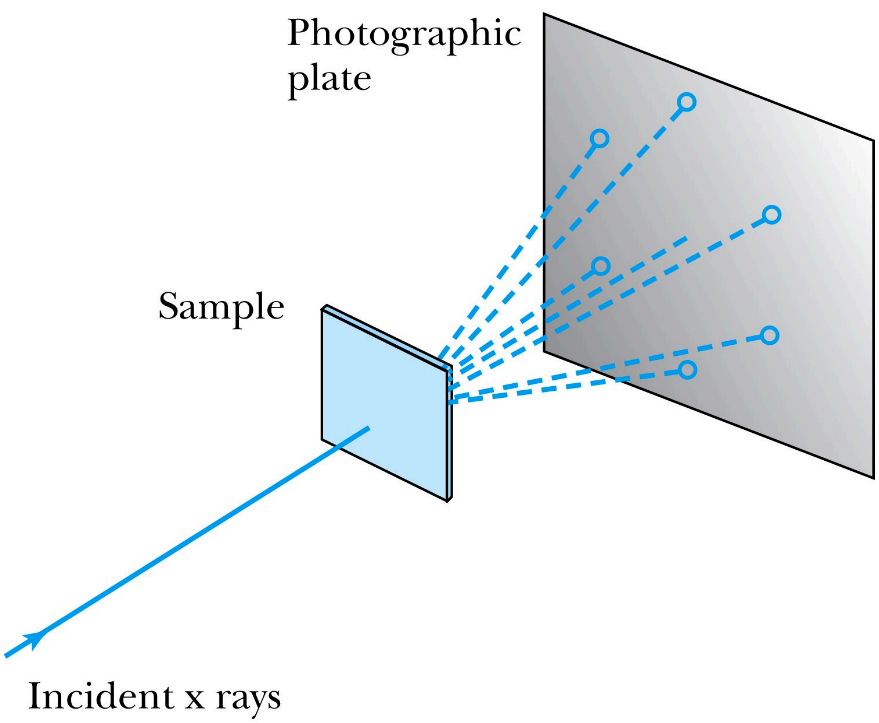
$$A\cos\left(k_1x-\omega_1t\right)+A\cos\left(k_2x-\omega_2t\right)=$$

$$2A\cos\left(\frac{k_1-k_2}{2}x-\frac{\omega_1-\omega_2}{2}t\right)\cos\left(\frac{k_1+k_2}{2}x-\frac{\omega_1+\omega_2}{2}t\right)=$$

- 10 points total for this derivation
- Due for this special project is Wednesday, Apr. 5.
- You MUST have your own answers!

X-Ray Scattering

- Max von Laue (1914 Nobel) suggested that if X rays were a form of electromagnetic radiation, interference effects should be observed. (Wave property!!)
- Crystals act as three-dimensional gratings, scattering the waves and producing observable interference effects.



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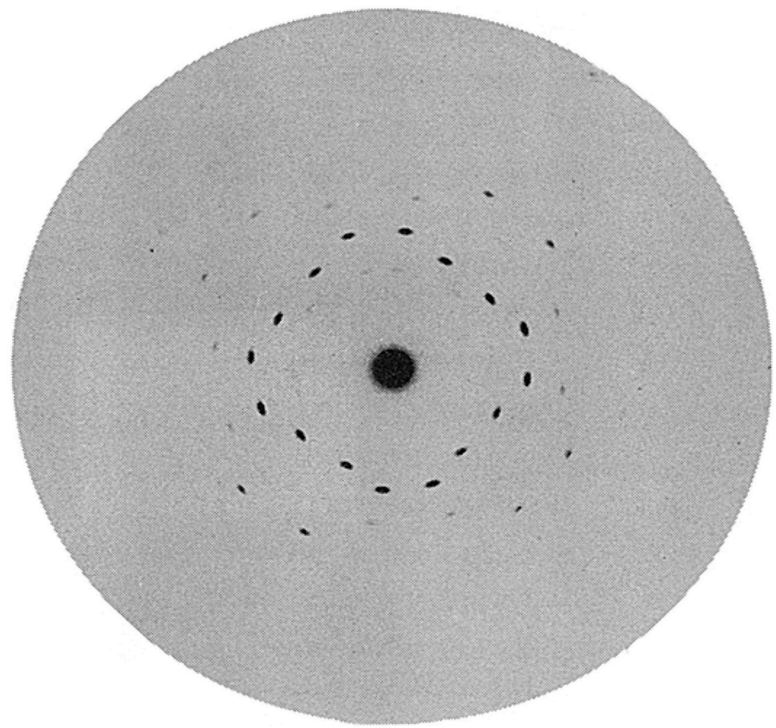

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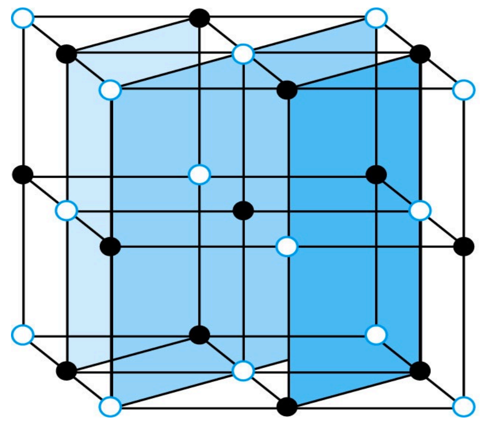
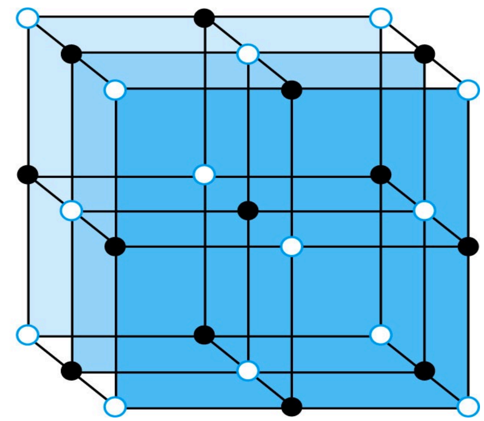
Bragg's Law

- William Lawrence Bragg (1915 Nobel) interpreted the x-ray scattering as the reflection of the incident X-ray beam from a unique set of planes of atoms within the crystal.
- There are two conditions for **constructive interference** of the scattered X rays:

1) The angle of incidence must equal the angle of reflection of the outgoing wave. (total reflection)
2) The difference in path lengths between two rays must be an integral number of wavelengths.

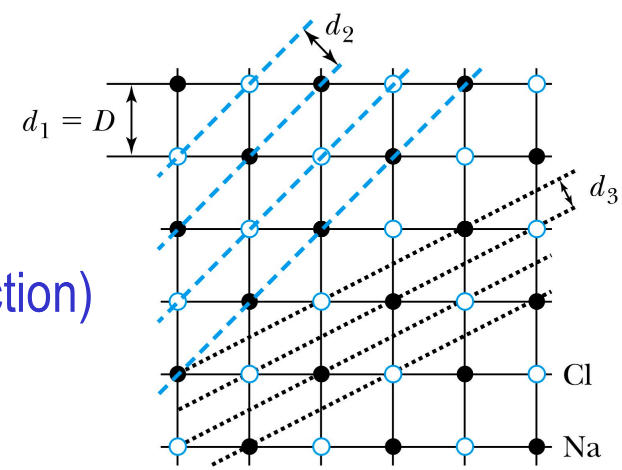


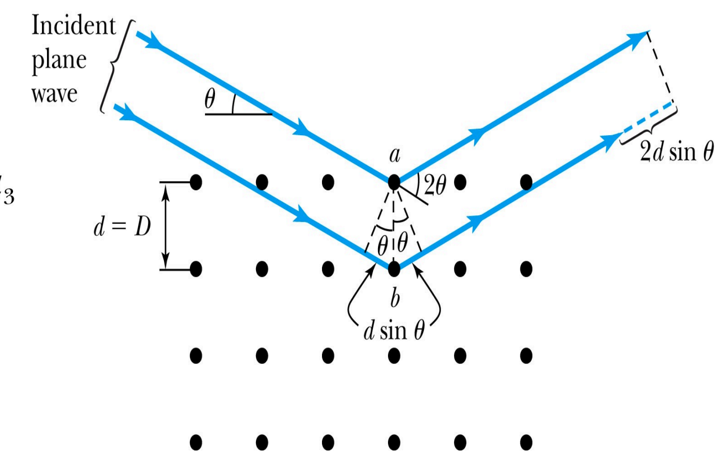


- **Bragg's Law:**

  $n\lambda = 2d\sin\,\theta$

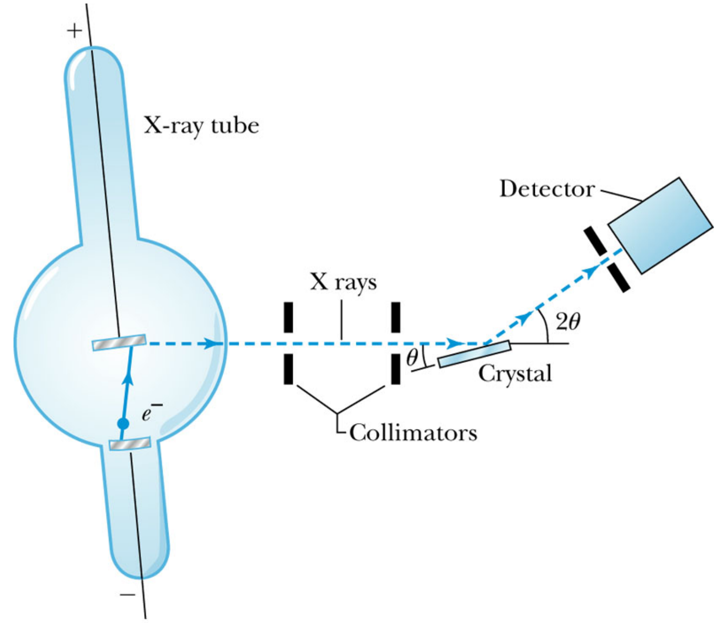
  (n = integer, order of reflection)

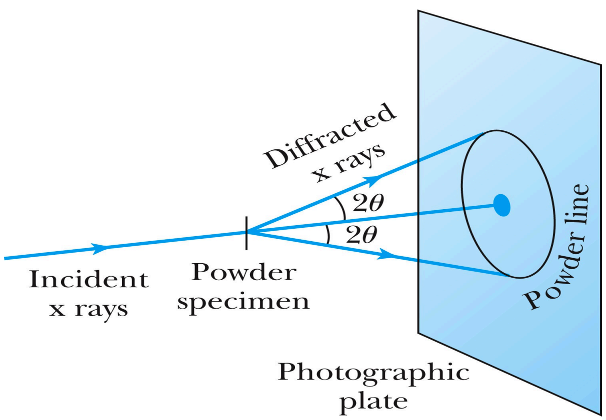




The Bragg Spectrometer

- A Bragg spectrometer scatters X rays from several crystals. The intensity of the diffracted beam is determined as a function of scattering angle by rotating the crystal and the detector.
- When a beam of X rays passes through the powdered crystal, the dots become a series of rings due to random arrangement.





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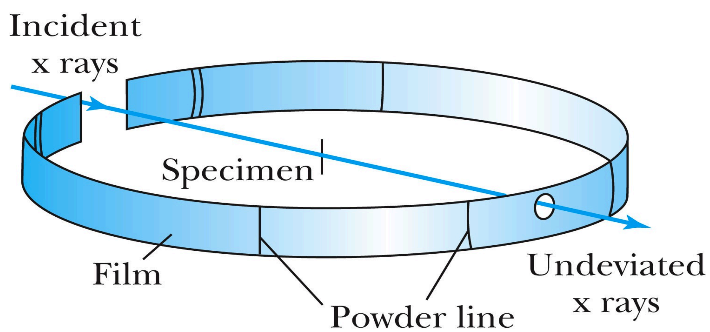

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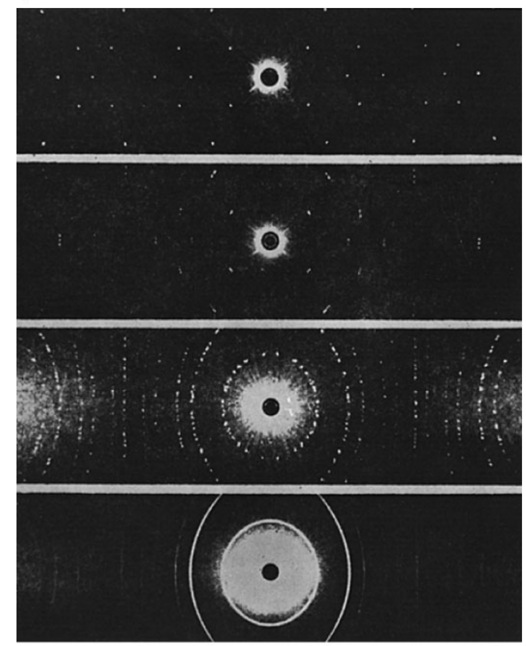

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Ex 5.1: Bragg's Law

X rays scattered from rock salt (NaCl) are observed to have an intense maximum at an angle of 20° from the incident direction. Assuming n=1 (from the intensity), what must be the wavelength of the incident radiation? NaCl density is $2.16 g/cm^3$ and the atomic mass is 58.5g/mol.

- Bragg's law: $n\lambda = 2d\sin\theta$
- *What do we need to know to use this? The lattice spacing d!*
- *We know n=1 and 2θ=20°.*
- *We use the density of NaCl to find out what d is.*

$$\frac{N_{molecules}}{V} = \frac{N_A \rho_{NaCl}}{M} = \frac{\left(6.02\times10^{23}\, molecules/mol\right)\cdot\left(2.16\, g/cm^3\right)}{58.5\, g/mol} =$$

$$= 2.22\times10^{22}\,\frac{molecules}{cm^3} = 4.45\times10^{22}\,\frac{atoms}{cm^3} = 4.45\times10^{28}\,\frac{atoms}{m^3}$$

$$\frac{1}{d^3} = 4.45\times10^{28}\,\frac{atoms}{m^3} \Rightarrow d = \frac{1}{\sqrt[3]{4.45\times10^{28}}} = 0.282nm$$

$$\lambda = 2d\sin\theta = 2\cdot0.282\cdot\sin10^o = 0.098nm$$

De Broglie Waves

- Louis V. de Broglie suggested that mass particles should have wave properties similar to electromagnetic radiation, **matter waves** (1929 Nobel) ➔ many experiments supported this!
- Thus the wavelength of a matter wave is called the **de Broglie wavelength**:

$$\lambda = \frac{h}{p}$$

- This can be considered as the probing beam length scale
- Since for a photon, $E = pc$ and $E = hf$, the energy can be written as

$$E = hf = pc = p\lambda f$$

Ex 5.2: De Broglie Wavelength

Calculate the De Broglie wavelength of (a) a tennis ball of mass 57g traveling 25m/s (about 56mph) and (b) an electron with kinetic energy 50eV.

- What is the formula for De Broglie Wavelength? $\lambda = \frac{h}{p}$
- (a) for a tennis ball, m=0.057kg.

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{0.057 \cdot 25} = 4.7 \times 10^{-34} m$$

This is why we don't observe a wave-like properties for a macroscopic objects.

- (b) for electron with 50eV KE, since KE is small, we can use non-relativistic expression of electron momentum!

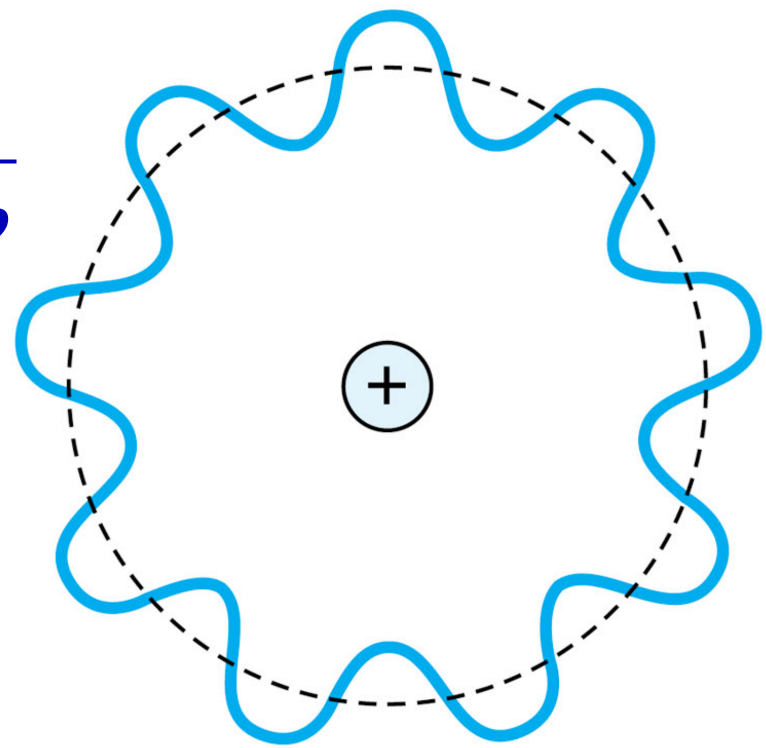
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e K}} = \frac{hc}{\sqrt{2m_e c^2 K}} = \frac{1240 eV \cdot nm}{\sqrt{2 \cdot 0.511 MeV \cdot 50 eV}} = 0.17 nm$$

- What are the wavelengths of you running at the speed of 2m/s? What about your car of 2 metric tons at 100mph? How about the proton with 14TeV kinetic energy?
- What is the momentum of the photon from a green laser (λ=532nm)?

Bohr's Quantization Condition

- One of Bohr's assumptions concerning his hydrogen atom model was that the angular momentum of the electron-nucleus system in a stationary state is an integral multiple of $h/2\pi$.
- The electron is a standing wave in an orbit around the proton. This standing wave will have nodes, and an integral number of wavelengths fit into the orbit!

$$2\pi r = n\lambda = n\frac{h}{p} \Rightarrow r = \frac{nh}{2\pi p}$$

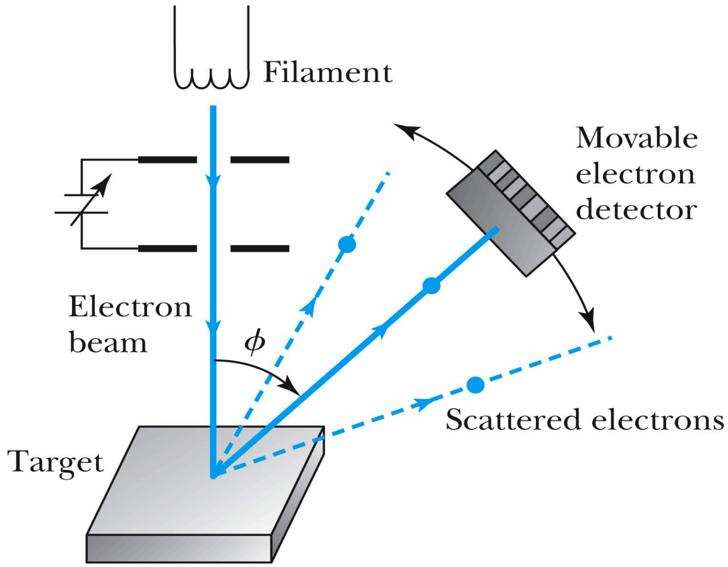


- The angular momentum becomes:

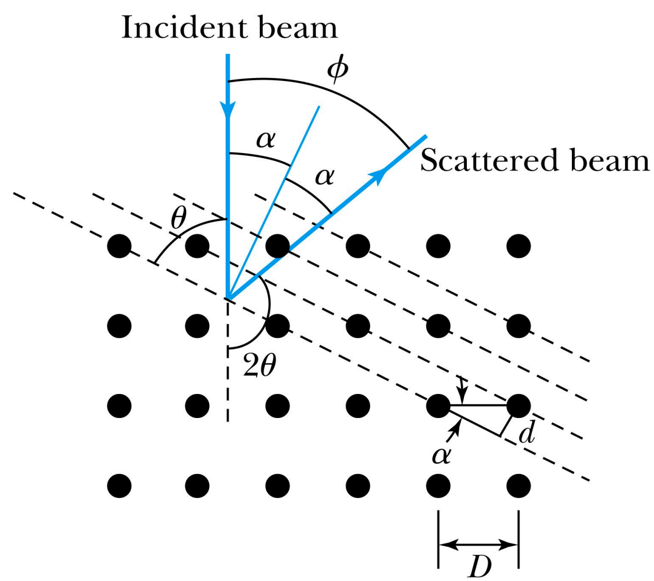
$$L = rp = \frac{nh}{2\pi p}p = n\frac{h}{2\pi} = n\hbar$$

Electron Scattering

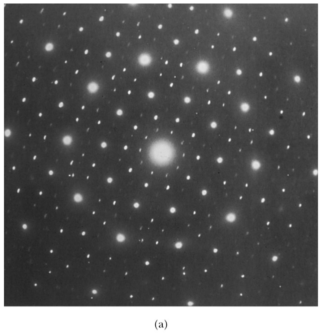
- Davisson* and Germer experimentally observed that electrons were diffracted much like X rays in nickel crystals. ➔ direct proof of the de Broglie wave!



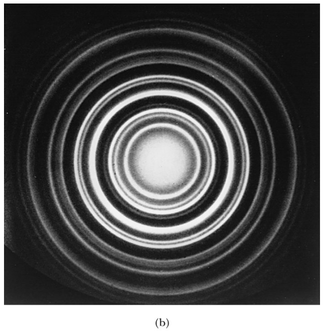
$$\lambda = \frac{D\sin\phi}{n}$$



- George P. Thomson* (1892–1975, 1937 Nobel), the son of J. J. Thomson (1906 Nobel), reported seeing the effects of electron diffraction in transmission experiments. The first target was celluloid, and soon after that gold, aluminum, and platinum were used.
- The randomly oriented polycrystalline sample of SnO_2 produces rings as shown in the figure right.



Crystal



Powder

- Photons, which we thought were waves, act particle like (eg Photoelectric effect or Compton Scattering)
- Electrons, which we thought were particles, act wave like (e.g. electron scattering)
- de Broglie: All matter has intrinsic wavelength.
  - Wavelength is inversely proportional to momentum
  - The more massive… the smaller the wavelength… the harder to observe the wavelike properties
  - So while photons appear mostly wavelike, electrons (next lightest particle!) appear mostly particle like.
- How can we reconcile the wave/particle views?

Wave Motion

- de Broglie matter waves suggest a further description of particles in wave. The displacement of a typical traveling wave is

$$\Psi(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

- This is a solution to the wave equation

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$$

- Define the wave number *k* and the angular frequency ω as:

$$k \equiv \frac{2\pi}{\lambda} \quad \text{and} \quad \omega = \frac{2\pi}{T} \qquad \lambda = vT$$

- The wave function can be rewritten: $\Psi(x,t) = A\sin[kx - \omega t]$

Wave Properties

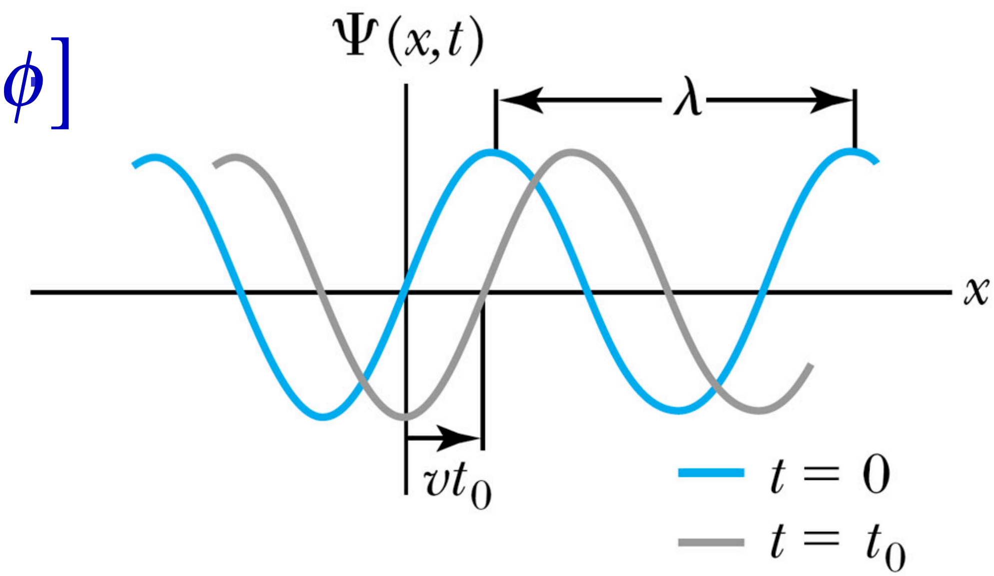
- The phase velocity is the velocity of a point on the wave that has a given phase (for example, the crest) and is given by

$$v_{ph} = \frac{\lambda}{T} = \frac{\lambda}{2\pi}\frac{2\pi}{T} = \frac{\omega}{k}$$

- The phase constant φ shifts the wave:

$$\Psi(x,t) = A\sin[kx - \omega t + \phi]$$
$$= A\cos[kx - \omega t]$$

(When φ=π/2)



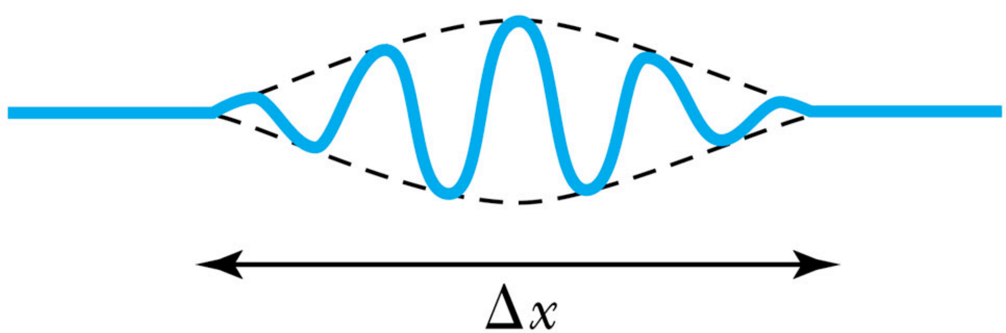
Principle of Superposition

- When two or more waves traverse in the same region, they act independently of each other.
- Combining two waves of the same amplitude yields:

$$\Psi(x,t) = \Psi_1(x,t) + \Psi_2(x,t) = A\cos(k_1 x - \omega_1 t) + A\cos(k_2 x - \omega_2 t) =$$

$$2A\cos\left(\frac{k_1 - k_2}{2}x - \frac{\omega_1 - \omega_2}{2}t\right)\cos\left(\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t\right) = 2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\cos(k_{av}x - \omega_{av}t)$$

- The combined wave oscillates within an envelope that denotes the maximum displacement of the combined waves.
- When combining many waves with different amplitudes and frequencies, a pulse, or **wave packet w/** localized amplitude, can be formed and can move with a **group velocity**:



$$u_{gr} = \frac{\Delta\omega}{\Delta k}$$

Fourier Series

- The sum of many waves that form a wave packet is called a **Fourier series**:

$$\Psi(x,t)=\sum_i A_i \cos\left[k_i x-\omega_i t\right]$$

- Summing an infinite number of waves yields the Fourier integral:

$$\Psi(x,t)=\int \tilde{A}(k)\cos\left[kx-\omega t\right]dk$$

Wave Packet Envelope

- The superposition of two waves yields the wave number and angular frequency of the wave packet envelope.

$$\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)$$

- The range of wave numbers and angular frequencies that produce the wave packet have the following relations:

$$\Delta k \Delta x = 2\pi \qquad \Delta\omega\Delta t = 2\pi$$

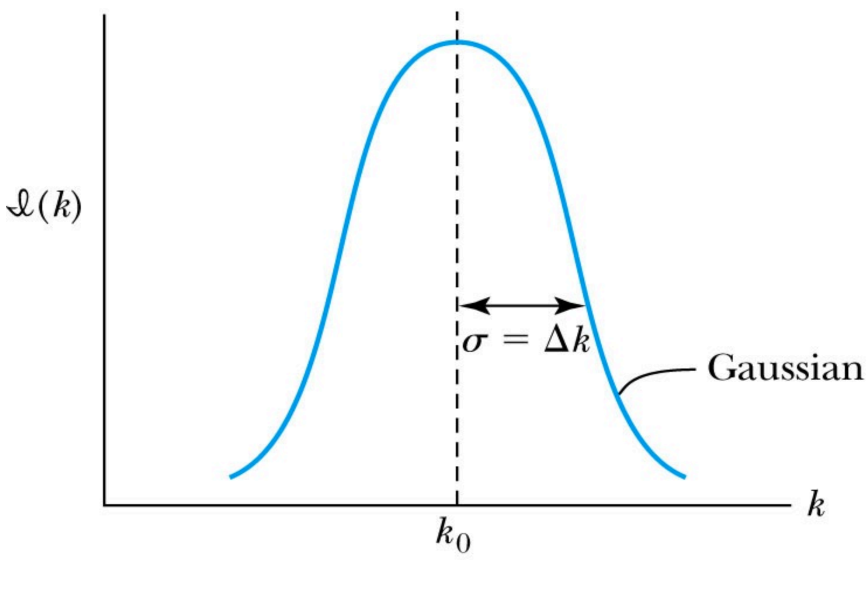
- A **Gaussian wave packet** has similar relations:

$$\Delta k \Delta x = \frac{1}{2} \qquad \Delta\omega\Delta t = \frac{1}{2}$$

- The localization of the wave packet over a small region to describe a particle position precisely requires a large range of wave numbers. Conversely, a small range of wave numbers cannot produce a wave packet localized within a small distance.

Gaussian Function

- A Gaussian wave packet describes the envelope of a pulse wave. $$\Psi(x,0)=\Psi(x)=Ae^{-\Delta k^2 x^2}\cos(k_0 x)$$



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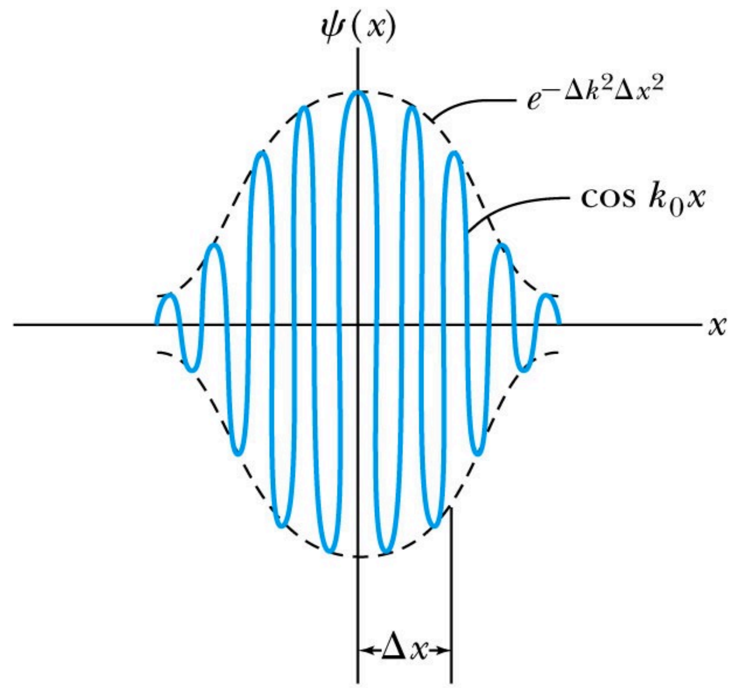
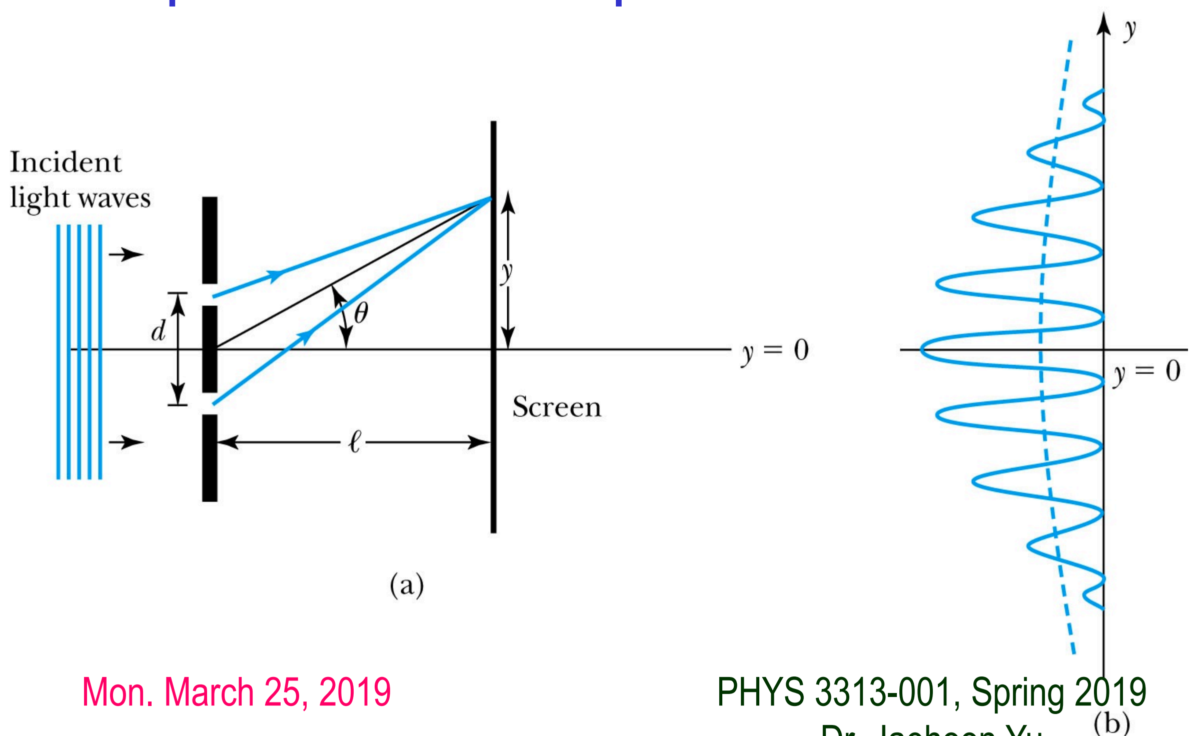


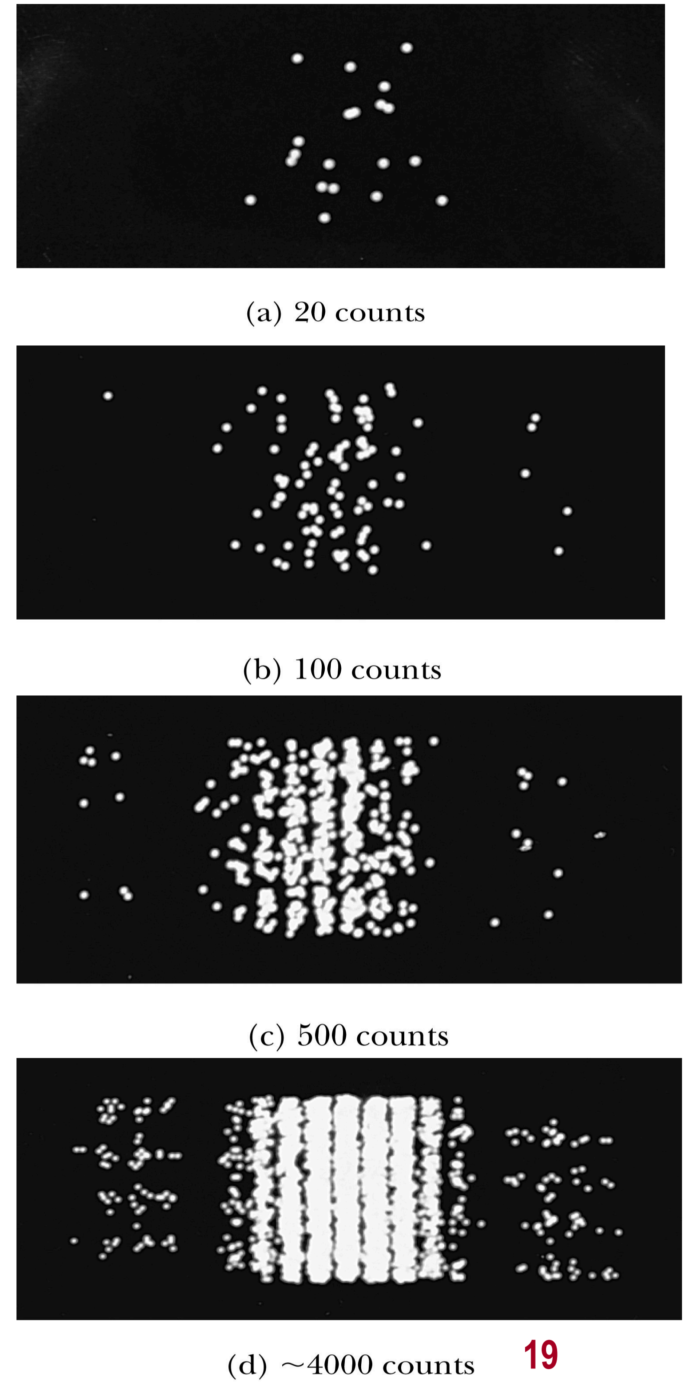
(b)

- The group velocity is $u_{gr}=\dfrac{d\omega}{dk}$

Waves or Particles?

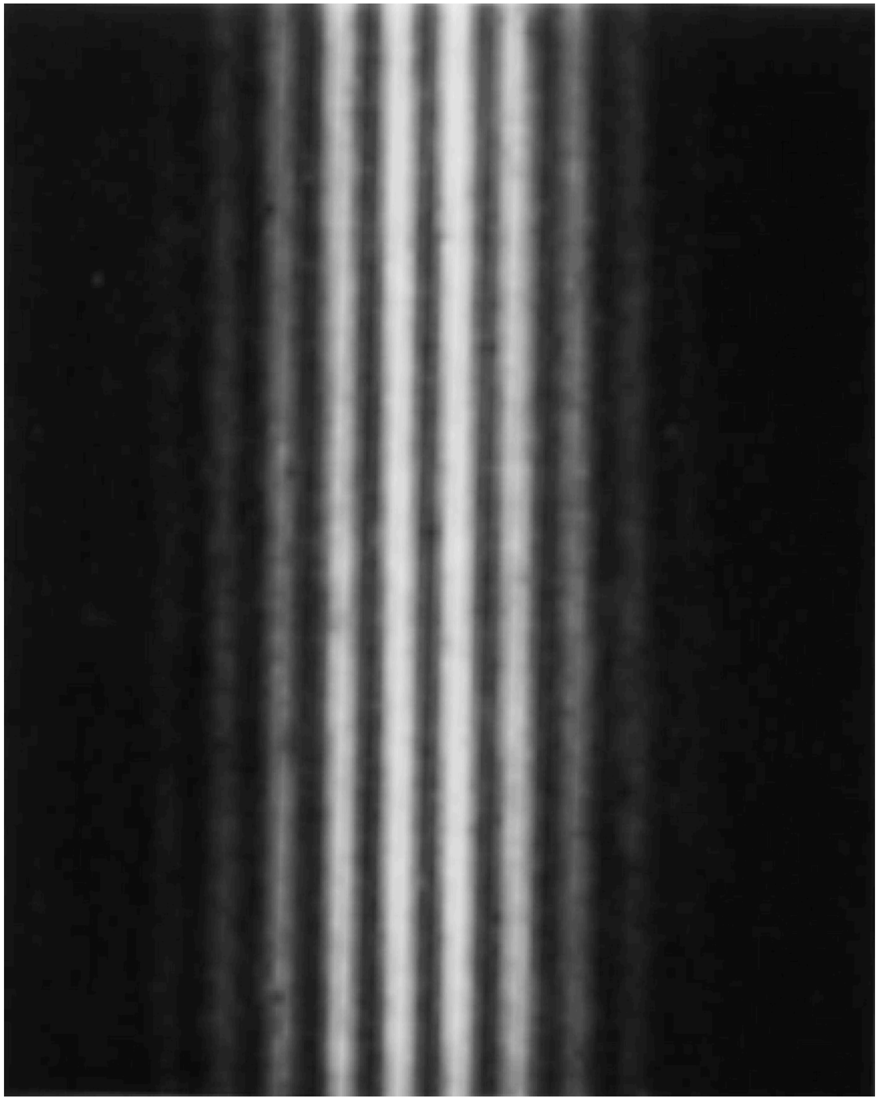
- Young's double-slit diffraction experiment demonstrates the wave property of the light.
- However, dimming the light results in single flashes on the screen representative of particles.




(a) 20 counts
(b) 100 counts
(c) 500 counts
(d) ~4000 counts

Electron Double-Slit Experiment

- C. Jönsson of Tübingen, Germany, succeeded in 1961 in showing double-slit interference effects for **electrons** by constructing very narrow slits and using relatively large distances between the slits and the observation screen.
- This experiment demonstrated that precisely the same behavior occurs for both light (waves) and electrons (particles).



Wave particle duality solution

- The solution to the wave particle duality of an event is given by the following principle.
- **Bohr's principle of complementarity**: It is not possible to describe physical observables simultaneously in terms of both particles and waves.
- **Physical observables** are the quantities such as position, velocity, momentum, and energy that can be experimentally measured. In any given instance we must use either the particle description or the wave description.

Uncertainty Principles

- It is impossible to measure simultaneously, with no uncertainty, the precise values of *k* and *x* for the same particle. The wave number *k* may be rewritten as

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/p} = p\frac{2\pi}{h} = \frac{p}{\hbar}$$

- For the case of a Gaussian wave packet we can write

$$\Delta k \Delta x = \frac{\Delta p}{\hbar}\Delta x = \frac{1}{2} \quad \Rightarrow \Delta p_x \Delta x = \frac{\hbar}{2}$$

Thus for a single particle, the Heisenberg's **uncertainty principle**:

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$ Momentum-position

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$ Energy-time

Probability, Wave Functions, and the Copenhagen Interpretation

- The wave function determines the likelihood (or probability) of finding a particle at a particular position in space at a given time.

$$P(y)dy = \left|\Psi(y,t)^2\right| dy$$

- The total probability of finding the particle is 1. Forcing this condition on the wave function is called the **normalization**.

$$\int_{-\infty}^{+\infty} P(y)dy = \int_{-\infty}^{+\infty} \left|\Psi(y,t)^2\right| dy = 1$$

The Copenhagen Interpretation

- Bohr's interpretation of the wave function consisted of 3 principles:
  1) The uncertainty principle of Heisenberg
  2) The complementarity principle of Bohr
  3) The statistical interpretation of Born, based on probabilities determined by the wave function
- Together, these three concepts form a logical interpretation of the physical meaning of quantum theory. According to the Copenhagen interpretation, physics depends on the outcomes of measurement.