PHYS 1444 – Section 002
Lecture #9

Monday, Feb. 24, 2020

Dr. Jaehoon Yu

- CH 23
  - Electric Potential due to Point Charges
  - Shape of the Electric Potential
  - Electric Potential by Charge Distributions
  - Equi-potential Lines and Surfaces
  - Electric Potential Due to Electric Dipole
  - Electrostatic Potential Energy

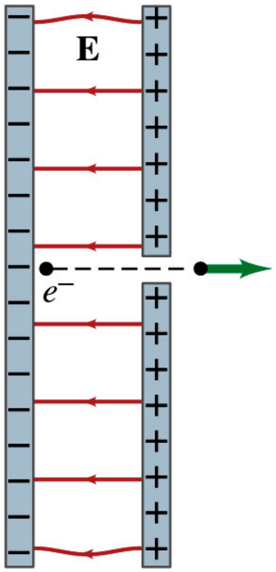
Today's homework is homework #6, due 11pm, Monday, Mar. 2!!

Announcements

- Bring out special project #2
- Reading assignments: CH23.9
- Mark your calendars for the triple extra credit colloquium
  - 4pm, Mar. 18: Dr. Pedro Machado, Fermilab

Reminder: Special Project #3

- **Particle Accelerator**. A charged particle of mass **M** with charge **-Q** is accelerated in the uniform field **E** between two parallel charged plates whose separation is **D** as shown in the figure on the right. The charged particle is accelerated from an initial speed $\mathbf{v_0}$ near the negative plate and passes through a tiny hole in the positive plate.
  - Derive the formula for the electric field E to accelerate the charged particle to a fraction *f* of the speed of light *c*. Express E in terms of **M, Q, D,** ***f*****, c** and $\mathbf{v_0}$.
  - (a) Using the Coulomb force and the kinematic equations. (8 points)
  - (b) Using the work-kinetic energy theorem. (8 points)
  - (c) Using the formula above, evaluate the strength of the electric field E to accelerate an electron from 0.1% of the speed of light to 90% of the speed of light. You need to look up and write down the relevant constants, such as mass of the electron, charge of the electron and the speed of light. (5 points)
- Must be handwritten and not copied from anyone else! **All of those involved in copying will get 0 credit!**
- Due beginning of the class next Monday, March 2



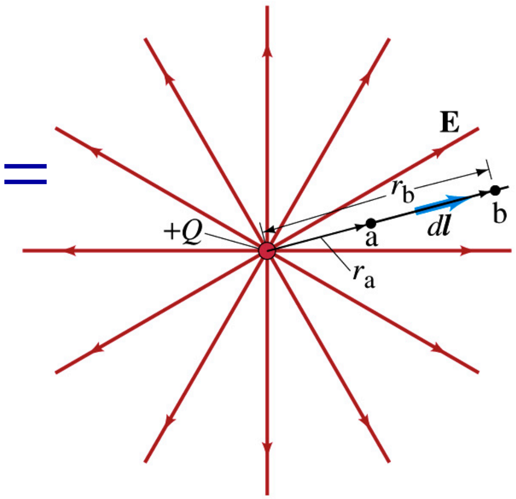
Electric Potential due to Point Charges

- What is the electric field by a single point charge Q at a distance r?

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = k\frac{Q}{r^2}$$

- Electric potential due to the field E for moving from point r_a to r_b in the radial direction away from the charge Q is

$$V_b - V_a = -\int_{r_a}^{r_b} \vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{\hat{r}}{r^2}\cdot\hat{r}\,dr =$$

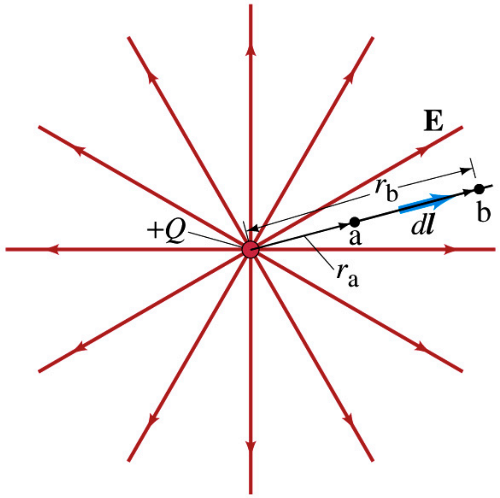


$$= -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{1}{r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b}-\frac{1}{r_a}\right)$$

Electric Potential due to Point Charges

- Since only the difference in potential have physical meaning, we can choose $V_b = 0$ at $r_b = \infty$.
- Thus, the electrical potential V at a distance r from a single point charge Q is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$



- So the absolute potential by a single point charge can be thought of **the potential difference by a single point charge between r and infinity**

Properties of the Electric Potential

- What are the differences between the electric potential and the electric field?
  - Electric potential
    - Electric potential energy per unit charge
    - Inversely proportional to the distance
    - Simply add the potential by each of the source charges to obtain the total potential due to multiple charges, since potential is a **scalar quantity**

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$$

  - Electric field
    - Electric force per unit charge
    - Inversely proportional to the **square** of the distance
    - Need **vector** sums to obtain the total field due to multiple source charges

$$\boxed{\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

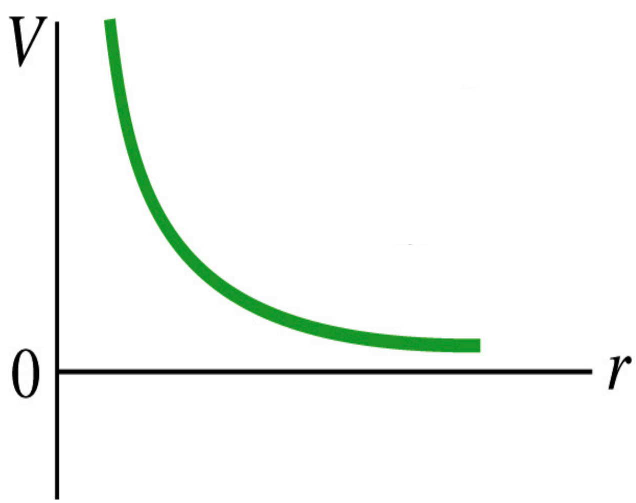
- Potential due to a positive charge is a large positive near the charge and decreases towards 0 at large distance.
- Potential due to a negative charge is a large negative near the charge and increases towards 0 at large distance.

Shape of the Electric Potential

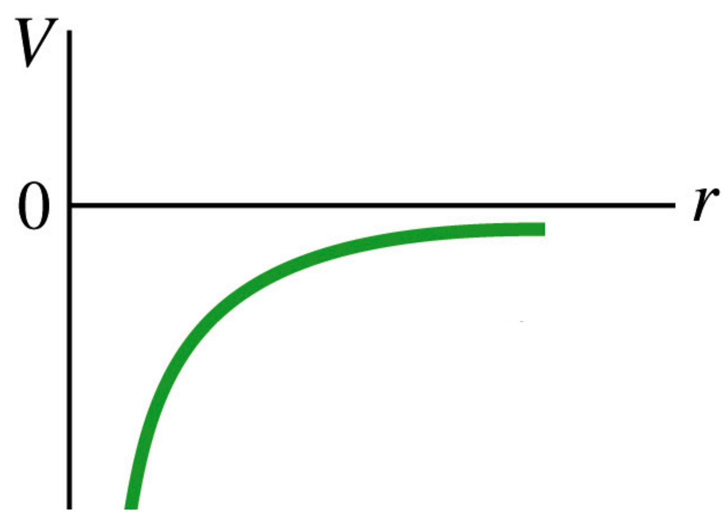
- So, what does the electric potential look like as a function of distance?
  - What is the formula for the potential by a single charge again?

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Positive Charge



Negative Charge



Uniformly charged sphere would have the potential the same as a single point charge.

What does this mean? **Uniformly charged sphere behaves like all the charge is on the single point in the center.**

Example 23 – 6

Work to bring two positive charges close together: What is minimum work required for an external force to bring the charge q=3.00μC from a great distance away (r=∞) to a point 0.500m from a charge Q=20.0 μC?

What is the work done by the electric field in terms of potential energy and potential?

$$W = -qV_{ba} = -\frac{q}{4\pi\varepsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right)$$

Since $r_b = 0.500m, r_a = \infty$ we obtain

$$W = -\frac{q}{4\pi\varepsilon_0}\left(\frac{Q}{r_b} - 0\right) = -\frac{q}{4\pi\varepsilon_0}\frac{Q}{r_b} = -\frac{\left(8.99\times10^9 N\cdot m^2/C^2\right)\cdot\left(3.00\times10^{-6}C\right)\left(20.00\times10^{-6}C\right)}{0.500m} = -1.08J$$

Electric force does negative work. In other words, an external force must do +1.08J of work to bring the 3.00μC charge from infinity to 0.500m to the charge 20.0μC.

Electric Potential by Charge Distributions

- Let's consider a case of n individual point charges in a given space, and let V=0 at r=∞.
- Then the potential V_{ia} due to the charge Q_i at point a, distance r_{ia} from Q_i is

$$V_{ia} = \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}}$$

- Thus the total potential V_a by all n point charges is

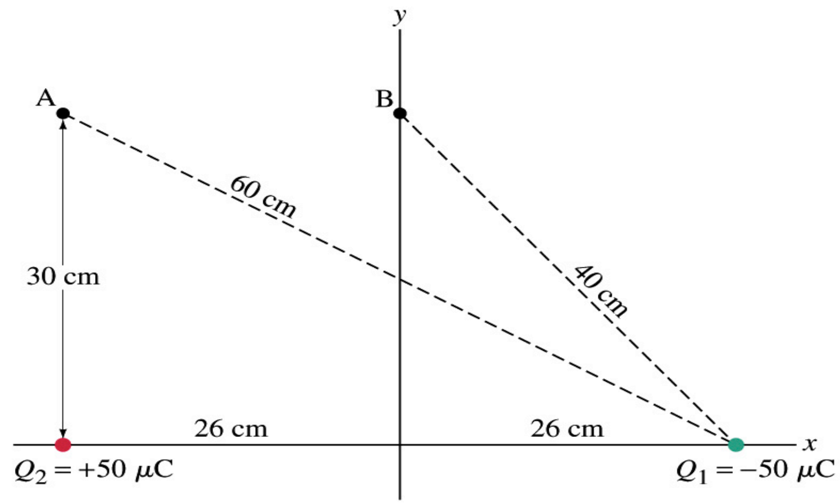
$$V_a = \sum_{i=1}^{n} V_{ia} = \sum_{i=1}^{n} \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}}$$

- For a continuous charge distribution, we obtain

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}}$$

Example

- **Potential due to two charges**: Calculate the electric potential (a) at point A in the figure due to the two charges shown, and (b) at point B.



- Electric potential is a scalar quantity, so one adds the potential by each of the source charge, as if they are numbers.

(a) potential at A is $V_A = V_{1A} + V_{2A} = \sum \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{iA}} =$

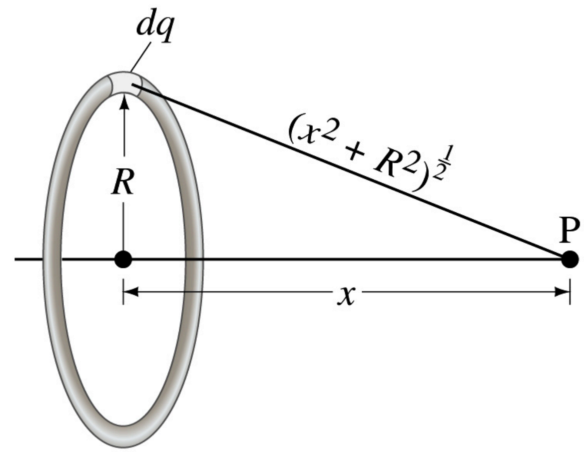
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_{1A}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_{2A}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{r_{1A}} + \frac{Q_2}{r_{2A}}\right)$$

$$= 9.0\times10^9\left(\frac{-50\times10^{-6}}{0.60} + \frac{50\times10^{-6}}{0.30}\right) = 7.5\times10^5 V$$

(b) How about potential at B?

Example 23 – 8

- **Potential due to a ring of charge**: A thin circular ring of radius R carries a uniformly distributed charge Q. Determine the electric potential at a point P on the axis of the ring at distance x from its center.



- Each point on the ring is at the same distance from the point P. What is the distance?

$$r = \sqrt{R^2 + x^2}$$

- So the potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0 r}\int dq =$$

What's this?

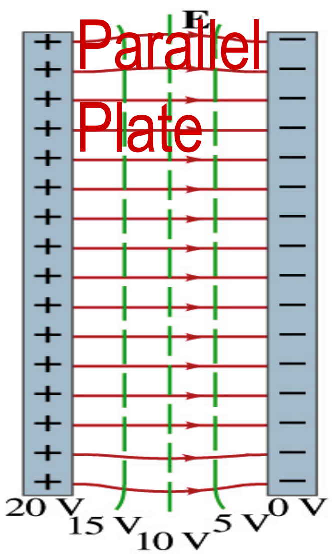
$$\frac{1}{4\pi\varepsilon_0\sqrt{x^2+R^2}}\int dq = \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$$

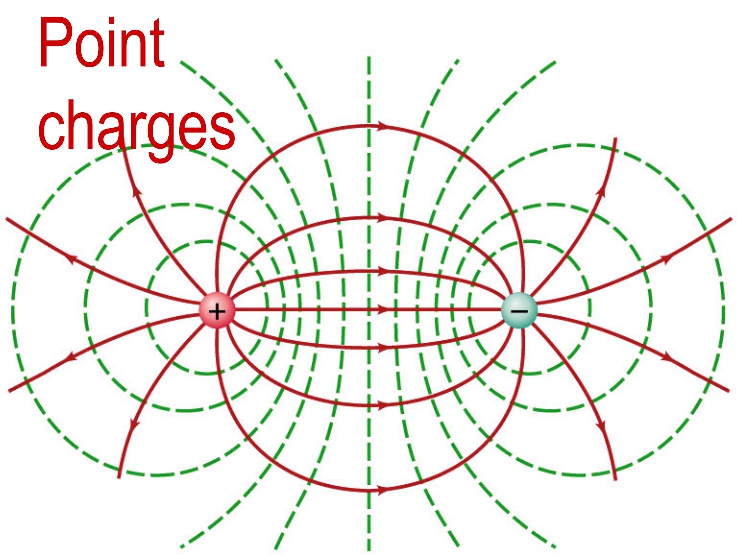
Equi-potential Surfaces

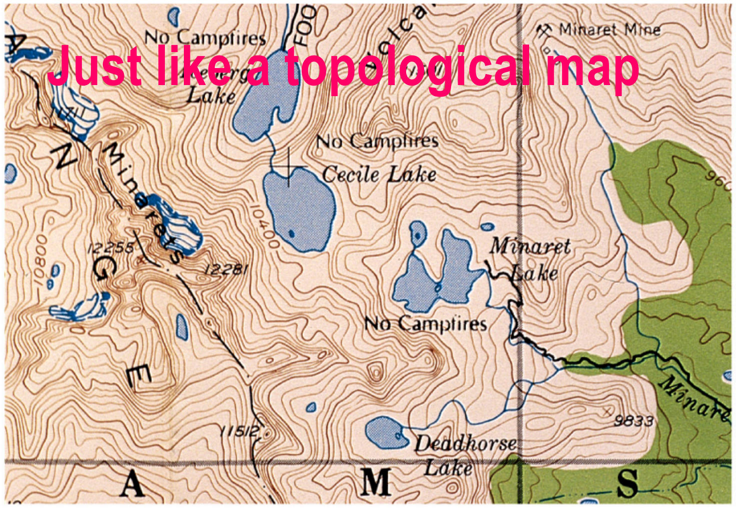
- Electric potential can be graphically shown using the equipotential lines in 2-D or the equipotential surfaces in 3-D
- Any two points on the equipotential surfaces (lines) are at the same potential
- What does this mean in terms of the potential difference?
  - The potential difference between any two points on an equipotential surface is 0.
- How about the potential energy difference?
  - Also 0.
- What does this mean in terms of the work to move a charge along the surface between these two points?
  - No work is necessary to move a charge between these two points.

Equi-potential Surfaces

- An equipotential surface (line) must be perpendicular to the electric field. Why?
  - If there are any parallel components to the electric field, it would require work to move a charge along the surface.
- Since the equipotential surface (line) is perpendicular to the electric field, we can draw these surfaces or lines easily.
- Since there can be no electric field within a conductor in a static case, the entire volume of a solid conductor must be at the same potential.
- So the electric field must be perpendicular to the conductor surface.

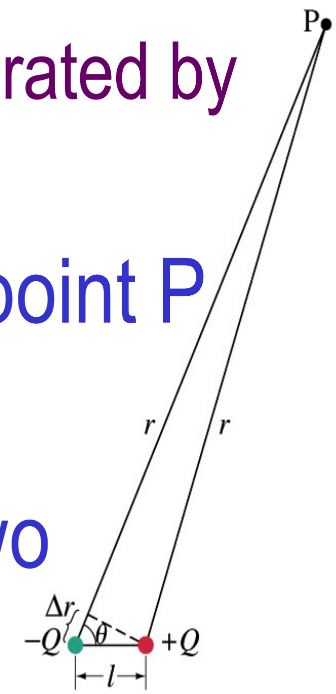






Electric Potential due to Electric Dipoles

- What is an electric dipole?
  - Two equal point charge Q of opposite signs separated by a distance ℓ and behaves like one entity: $p=Q\ell$
- For the electric potential due to a dipole at a point P
  - We take V=0 at r=∞
- The simple sum of the potential at P by the two charges is



$$V = \sum \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r}+\frac{(-Q)}{r+\Delta r}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{r+\Delta r}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{\Delta r}{r(r+\Delta r)}$$

- Since $\Delta r = \ell\cos\theta$ and if $r \gg \ell$, $r \gg \Delta r$, thus $r+\Delta r \sim r$ and

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{l\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

V by a dipole at a distance r from the dipole

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}}$$

E Determined from V

- Potential difference between two points under an electric field is $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$
- So in a differential form, we can write

$$dV = -\vec{E} \cdot d\vec{l} = -E_l dl$$

  - What are dV and E_ℓ?
    - dV is the infinitesimal potential difference between the two points separated by a distance $d\ell$
    - E_ℓ is the field component along the direction of $d\ell$.
- Thus we can write the field component E_ℓ as

$$E_l = -\frac{dV}{dl}$$

Physical Meaning?

The component of the electric field in any direction is equal to the negative rate of change of the electric potential as a function of distance in that direction!!

E Determined from V

- The quantity $dV/d\ell$ is called the **gradient of V** in a particular direction
  - If no direction is specified, the term gradient refers to the direction in which **V changes most rapidly** and this would be the direction of the field vector **E** at that point.
  - So if **E** and $d\ell$ are parallel to each other, $E = -\dfrac{dV}{dl}$
- If E is written as a function x, y and z, the ℓ refers to x, y and z $\quad E_x = -\dfrac{\partial V}{\partial x} \quad E_y = -\dfrac{\partial V}{\partial y} \quad E_z = -\dfrac{\partial V}{\partial z}$
- $\dfrac{\partial V}{\partial x}$ is the "partial derivative" of V with respect to x, while y and z are held constant
- In vector form, $\vec{E} = -gradV = -\vec{\nabla} V = -\left(\vec{i}\dfrac{\partial}{\partial x} + \vec{j}\dfrac{\partial}{\partial y} + \vec{k}\dfrac{\partial}{\partial z} \right) V$

$\vec{\nabla} = -\left(\vec{i}\dfrac{\partial}{\partial x} + \vec{j}\dfrac{\partial}{\partial y} + \vec{k}\dfrac{\partial}{\partial z} \right)$ is called *del* or the *gradient operator* and is a *vector operator*.

Electrostatic Potential Energy

- Consider a case in which a point charge q is moved between points *a* and *b* where the electrostatic potential due to other charges in the system is V_a and V_b
- The change in electrostatic potential energy of q in the field by other charges is

$$\Delta U = U_b - U_a = q\left(V_b - V_a\right) = qV_{ba}$$

- Now what is the electrostatic potential energy of a system of charges?
  - Let's choose V=0 at r=∞
  - If there are no other charges around, single point charge Q_1 in isolation has no potential energy and is under no electric force