PHYS 1444 – Section 002
Lecture #11

Monday, Mar. 2, 2020

Dr. Jaehoon Yu

- CH 24 Capacitance etc..
  - Single Conductor Capacitance
  - Capacitors in Series or Parallel
  - Electric Energy Storage
  - Effect of Dielectric

CH 25

- Electric Current and Resistance

Today's homework is homework #7, due 11pm, Monday, Mar. 16!!

Announcements

- Bring out the special project #3
- Reading Assignment: CH24.5
- Quiz #3
  - This Wednesday, Mar. 4 at the beginning of the class
  - Covers CH23.2 through end of CH24
  - Bring your calculator but DO NOT input formula into it!
    - Cell phones or any types of computers cannot replace a calculator!
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the quiz
  - No derivations, word definitions or solutions of any problems!
  - No additional formulae or values of constants will be provided!
- Exam 1 results
  - Class average: 51.3/86
    - Equivalent to: 59.7/100
  - Top score: 82/86

Single Conductor Capacitance

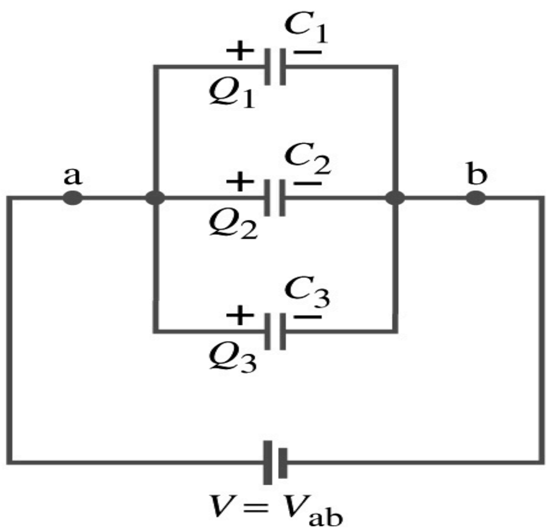
- A single isolated conductor can be said to have capacitance, C.
- C can still be defined as the ratio of the charge to the absolute potential V on the conductor.
  - So Q=CV.
- The potential of a single conducting sphere of radius r_b can be obtained as

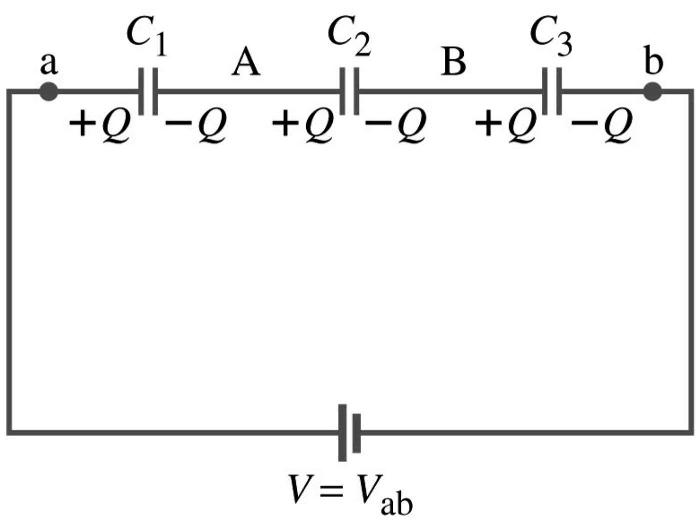
$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = \frac{Q}{4\pi\varepsilon_0 r_b} \qquad \text{where} \quad r_a \to \infty$$

- So its capacitance is $\boxed{C = \frac{Q}{V} = 4\pi\varepsilon_0 r_b}$

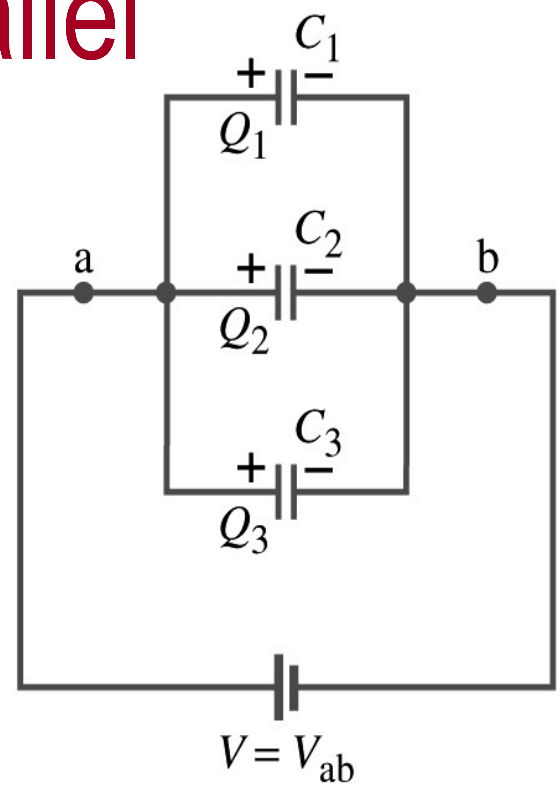
Capacitors in Series or Parallel

- Capacitors may be used in an electric circuit
- What is an electric circuit?
  - A closed path of conductors, usually wires connecting capacitors and other electrical devices, in which
    - charges can flow
    - And includes a voltage source such as a battery
- Capacitors can be connected in various ways.
  - In parallel, in series or in combination





Capacitors in Parallel



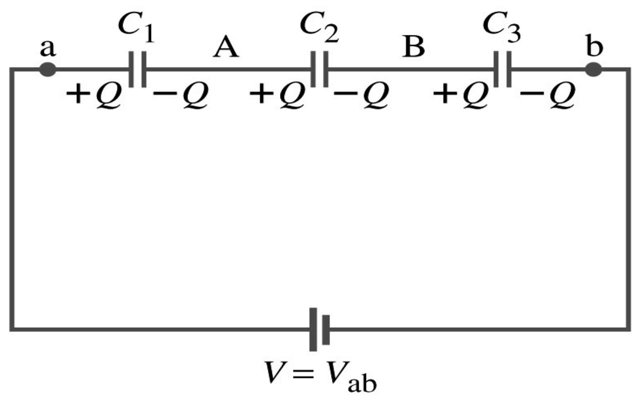
- Parallel arrangement provides the **same voltage** across all the capacitors.
  - Lefthand plates are at V_a, while the righthand plates are at V_b
  - So each capacitor plate acquires charges given by the formula
    - $Q_1=C_1V$, $Q_2=C_2V$, and $Q_3=C_3V$
- The total charge Q that must leave the battery is then
  - $Q=Q_1+Q_2+Q_3=V(C_1+C_2+C_3)$
- Consider that the three capacitors behave like an equivalent one
  - $Q=C_{eq}V= V(C_1+C_2+C_3)$
- Thus the equivalent capacitance in parallel is $\boxed{C_{eq} = C_1 + C_2 + C_3}$

What is the net effect?

The capacitance increases!!!

Capacitors in Series

- Series arrangement is more interesting
  - When battery is connected, +Q flows to the left plate of C_1 and –Q flows to the right plate of C_3.
  - Since capacitors in between were originally neutral, charges get induced to neutralize the ones in the middle.
  - So the charge on each capacitor plate is the same value, Q. (**Same charge**)



- Consider that the three capacitors behave like an equivalent one
  - $Q=C_{eq}V$
- The total voltage V across the three capacitors in series must be equal to the sum of the voltage across each capacitor.
  - $V=V_1+V_2+V_3=Q/C_1+Q/C_2+Q/C_3$
- Putting all these together, we obtain:
- $V=Q/C_{eq}=Q(1/C_1+1/C_2+1/C_3)$
- Thus the equivalent capacitance is

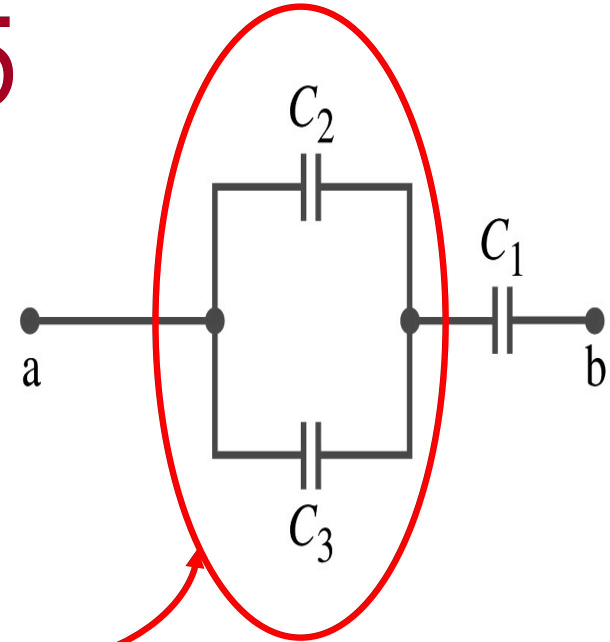
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

What is the net effect?

The capacitance smaller than the smallest C!!!

Example 24 – 5

Equivalent Capacitor: Determine the capacitance of a single capacitor that will have the same effect as the combination shown in the figure. Take $C_1=C_2=C_3=C$.



We should do these first!!

How? These are in parallel so the equivalent capacitance is:

$$C_{eq1} = C_1 + C_2 = 2C$$

Now the equivalent capacitor is in series with C1.

$$\frac{1}{C_{eq}} = \frac{1}{C_{eq1}} + \frac{1}{C_2} = \frac{1}{2C} + \frac{1}{C} = \frac{3}{2C}$$

Solve for C_{eq}

$$C_{eq} = \frac{2C}{3}$$

Electric Energy Storage

- A charged capacitor stores energy.
  - The stored energy is the same as the amount of the work done to charge it.
- The net effect of charging a capacitor is removing one type of charge from one plate and put them on to the other.
  - Battery does this when it is connected to a capacitor.
- Capacitors do not get charged immediately.
  - Initially when the capacitor is uncharged, no work is necessary to move the first bit of charge. Why?
    - Since there is no charge, there is no field that the external force needs to work to overcome.
  - When some charge is on each plate, it requires work to add more charges of the same sign due to the electric repulsion.

Electric Energy Storage

- The work needed to add a small amount of charge, dq, when a potential difference across the plate is V: dW=Vdq.
- Since V=q/C, the work needed to store total charge Q is

$$W = \int_0^Q V\,dq = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

- Thus, the energy stored in a capacitor when the capacitor carries the charges +Q and –Q is

$$\boxed{U = \frac{Q^2}{2C}}$$

- Since Q=CV, we can rewrite

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Example 24 – 8

Energy store in a capacitor: A camera flash unit stores energy in a 150μF capacitor at 200V. How much electric energy can be stored?

Using the formula for stored energy. Umm.. Which one?

What do we know from the problem? C and V

So we use the one with C and V: $U = \frac{1}{2}CV^2$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(150 \times 10^{-6} F\right)\left(200V\right)^2 = 3.0J$$

How do we get J from FV²? $FV^2 = \left(\frac{C}{V}\right)V^2 = CV = C\left(\frac{J}{C}\right) = J$

Electric Energy Density

- The energy stored in a capacitor can be considered as being stored in the electric field between the two plates
- For a uniform field E between two plates, V=Ed and $C=\varepsilon_0 A/d$
- Thus the stored energy is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2 Ad$$

- Since Ad is the gap volume V, we can obtain the energy density, stored energy per unit volume, as

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

Valid for any space that is vacuum

Electric energy stored per unit volume in any region of space is proportional to the square of E in that region.

Dielectrics

- Capacitors have an insulating sheet of material, called dielectric, between the plates to
  - Increase breakdown voltage greater than that in air (3MV/m)
  - Apply higher voltage to the gap without the charge passing across
  - Allow the plates get closer together without touching
    - Increases capacitance (recall $C=\varepsilon_0 A/d$)
  - Increase the capacitance by the dielectric constant K

$$\boxed{C = KC_0}$$

  - Where C_0 is the intrinsic capacitance when the gap is vacuum

Dielectrics

- The value of dielectric constant K varies depending on the material (Table 24 – 1)
  – K for vacuum is 1.0000
  – K for air is 1.0006 (this is why permittivity of air and vacuum are used interchangeably.)
  – K for paper is 3.7
- **The maximum electric field before breakdown** occurs is called the **dielectric strength**. What is its unit?
  – V/m
- The capacitance of a parallel plate capacitor with a dielectric (K) filling the gap is

$$\boxed{C = KC_0 = K\varepsilon_0 \frac{A}{d}}$$

Dielectrics

- A new quantity of the **permittivity of a dielectric material** is defined as **$\varepsilon = K\varepsilon_0$**
- The capacitance of a parallel plate capacitor with a dielectric medium filling the gap is

$$\boxed{C = \varepsilon \frac{A}{d}}$$

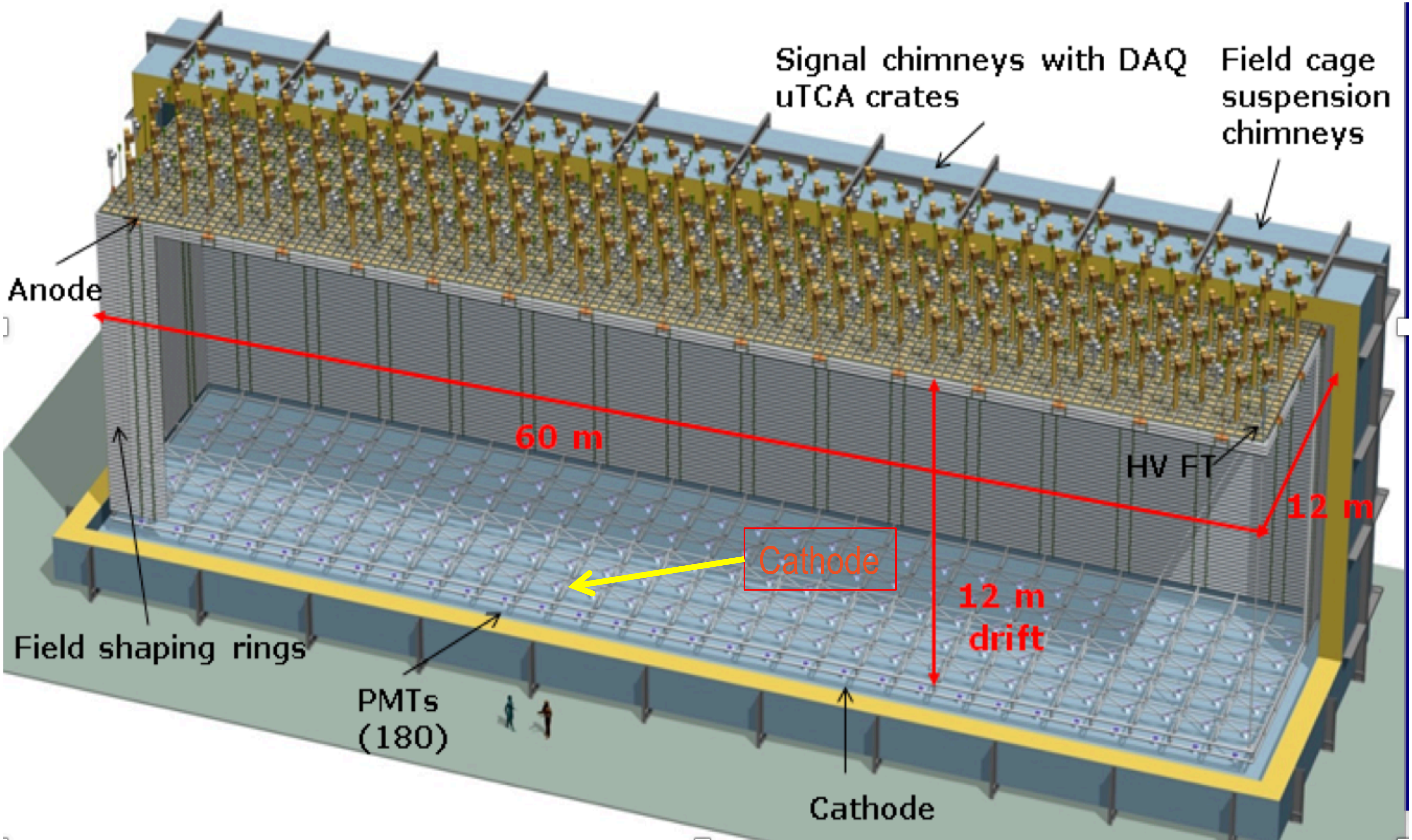
- The energy density stored in an electric field E in a dielectric is

$$\boxed{u = \frac{1}{2} K \varepsilon_0 E^2 = \frac{1}{2} \varepsilon E^2}$$

Valid for any space w/ dielectric w/ permittivity ε.

What is the stored energy in between ProtoDUNE cathode (-300kV) plate (6mx6m) and the cryostat wall (0V), d=1.5m, LAr K=1.6?

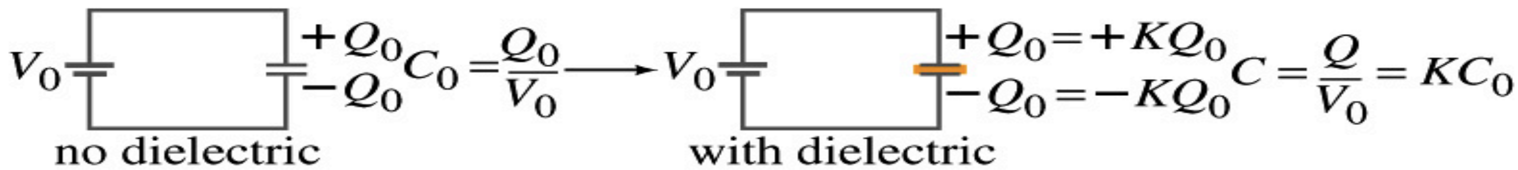
DUNE Dual Phase



Effect of a Dielectric Material on Capacitance

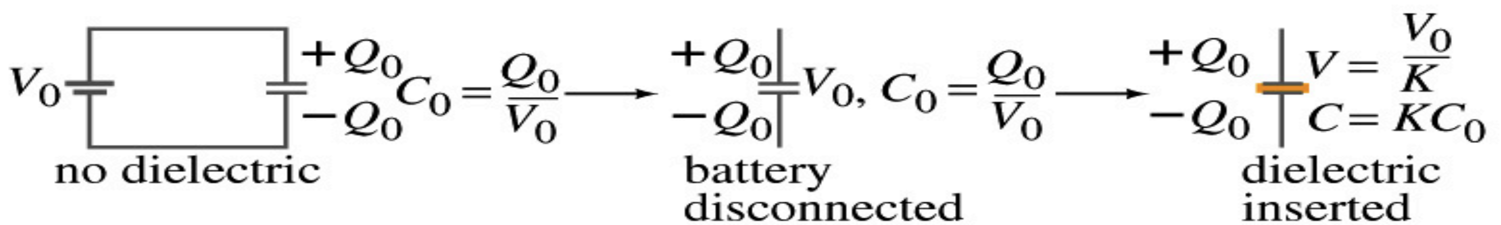
- Let's consider the two cases below:

Case #1 : constant V



(a) Voltage constant

Case #2 : constant Q



(b) Charge constant

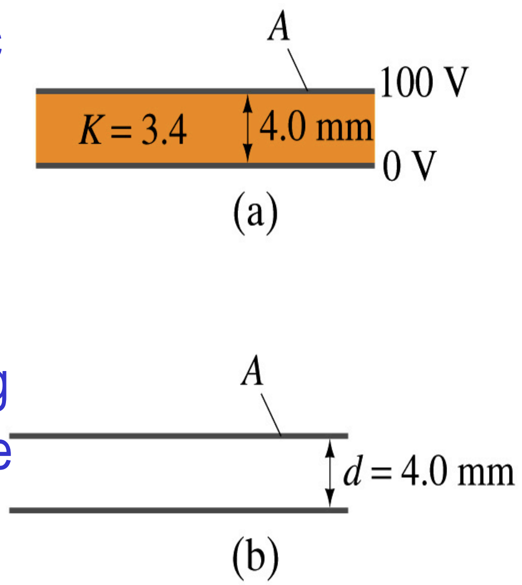
- Constant voltage: Experimentally observed that the total charge on each plate of the capacitor increases by K as a dielectric material is inserted between the gap ➔ $Q=KQ_0$
  - The capacitance increased to $C=Q/V_0=KQ_0/V_0=KC_0$
- Constant charge: Voltage found to drop by a factor K ➔ $V=V_0/K$
  - The capacitance increased to $C=Q_0/V=KQ_0/V_0=KC_0$

Effect of a Dielectric Material on Field

- What happens to the electric field within a dielectric?
- Without a dielectric, the field is $$E_0 = \frac{V_0}{d}$$
  - What are V_0 and d?
    - V_0: Potential difference between the two plates
    - d: separation between the two plates
- For a constant voltage, the electric field remains the same
- For the constant charge: the voltage drops to $V=V_0/K$, thus the field in the dielectric is $$E = E_D = \frac{V}{d} = \frac{V_0}{dK} = \frac{E_0}{K}$$
  - The field in the dielectric is reduced. $$E_D = \frac{E_0}{K}$$

Example 24 – 11

Dielectric Removal: A parallel-plate capacitor, filled with the dielectric of K=3.4, is connected to a 100-V battery. After the capacitor is fully charged, the battery is disconnected. The plates have area A=4.0m^2 and are separated by d=4.0mm. (a) Find the capacitance, the charge on the capacitor, the electric field strength, and the energy stored in the capacitor. (b) The dielectric is carefully removed, without changing the plate separation nor does any charge leave the capacitor. Find the new value of capacitance, electric field strength, voltage between the plates and the energy stored in the capacitor.



(a) $$C=\frac{\varepsilon A}{d}=\frac{K\varepsilon_0 A}{d}=\left(3.4\times 8.85\times 10^{-12}\,C^2/N\cdot m^2\right)\frac{4.0m^2}{4.0\times 10^{-3}\,m}=3.0\times 10^{-8}\,F=30nF$$

$$Q=CV=\left(3.0\times 10^{-8}\,F\right)\times 100V=3.0\times 10^{-6}\,C=3.0\mu C$$

$$E=\frac{V}{d}=\frac{100V}{4.0\times 10^{-3}\,m}=2.5\times 10^{4}\,V/m$$

$$U=\frac{1}{2}CV^2=\frac{1}{2}\left(3.0\times 10^{-8}\,F\right)\left(100V\right)^2=1.5\times 10^{-4}\,J$$

Example 24 – 11 cont'd

(b) Since the dielectric has been removed, the effect of dielectric constant must be removed as well.

$$C_0 = \frac{C}{K} = \left(8.85\times10^{-12}\,C^2/N\cdot m^2\right)\frac{4.0m^2}{4.0\times10^{-3}m} = 8.8\times10^{-9}F = 8.8nF$$

Since charge is the same ($Q_0 = Q$) before and after the removal of the dielectric, we obtain

$$V_0 = Q/C_0 = K\,Q/C = KV = 3.4\times100V = 340V$$

$$E_0 = \frac{V_0}{d} = \frac{340V}{4.0\times10^{-3}m} = 8.5\times10^4\,V/m = 84\,kV/m$$

$$U_0 = \frac{1}{2}C_0V_0^2 = \frac{1}{2}\frac{C}{K}(KV)^2 = \frac{1}{2}KCV^2 = KU = 3.4\times1.5\times10^{-4}J = 5.1\times10^{-4}J$$

Where did the extra energy come from?

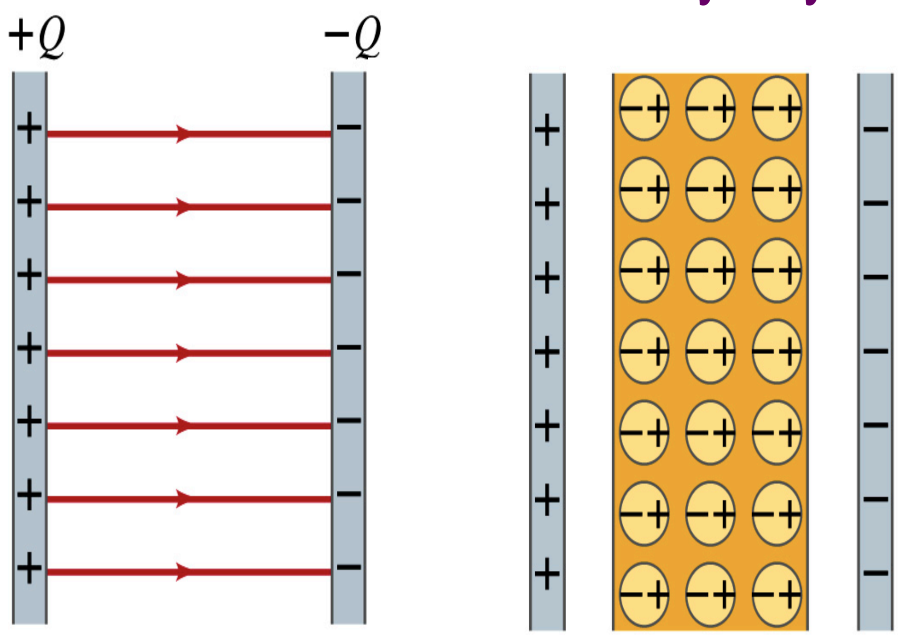
~~Th… ergy conservation law is violated in elec… ??~~

Wrong!
Wrong!
Wrong!

An external force has done the work of 3.6x10⁻⁴J on the system to remove dielectric!!

Molecular Description of Dielectric

- So what in the world makes dielectrics behave the way they do?
- We need to examine this in a microscopic scale.
- Let's consider a parallel plate capacitor that is charged up $+Q(=C_0V_0)$ and $-Q$ with air in between.
  - Assume there is no way any charge can flow in or out

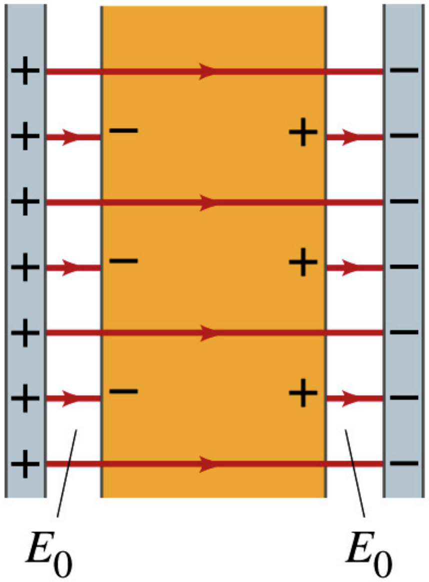


(a) (b)

- Now insert a dielectric
  - Dielectric can be polar ➔ could have permanent dipole moment. What will happen?
- Due to the electric field molecules will be aligned.

Molecular Description of Dielectric

- OK. Then what happens?
- Then effectively, there will be some negative charges close to the surface of the positive plate and positive charge on the negative plate
  - Some electric field do not pass through the whole dielectric but stops at the negative charge



(c)

  - So the field inside dielectric is smaller than the air
- Since electric field is smaller, the force is smaller
  - The work need to move a test charge inside the dielectric is smaller
  - Thus the potential difference across the dielectric is smaller than across the air