PHYS 1444 – Section 002
Lecture #17

Wednesday, Apr. 8, 2020
Dr. Jaehoon Yu

CH 27: Magnetism & Magnetic Field

- Electric Current and Magnetism
- Magnetic Field
- Magnetic Force on a Moving Charge
- Charged Particle Path in a Magnetic Field
- The cyclotron frequency

Announcements

- Reading Assignments: CH27.6, 27.8 and 27.9
- 2nd Non-comprehensive term exam in class Wednesday, Apr. 15
  - Do NOT miss the exam! Be in a quiet place to take the exam!
  - This is one of the two exams that will be chosen for the final grade!
  - Covers CH25.1 through what we finish Monday, Apr. 13
  - Online exam based on Quest but must join zoom class 12:55pm!
  - You can use your calculator but DO NOT input formula into it!
    - Cell phones or any types of computers cannot replace a calculator!
    - Turn off your phones!!
  - BYOF: You may prepare a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants
    - No derivations, plots, pictures, word definitions or solutions of any problems!
  - Please send me the photos of your formula sheet by 12:55pm 4/15
  - Let's be fair to other students and not cheat!
- Quiz 4 results
  - Class average: 33.5/60
    - Equivalent to 55.8/100
    - Previous results : 48.9, 48.1 and 52.6
  - Top score: 60/60

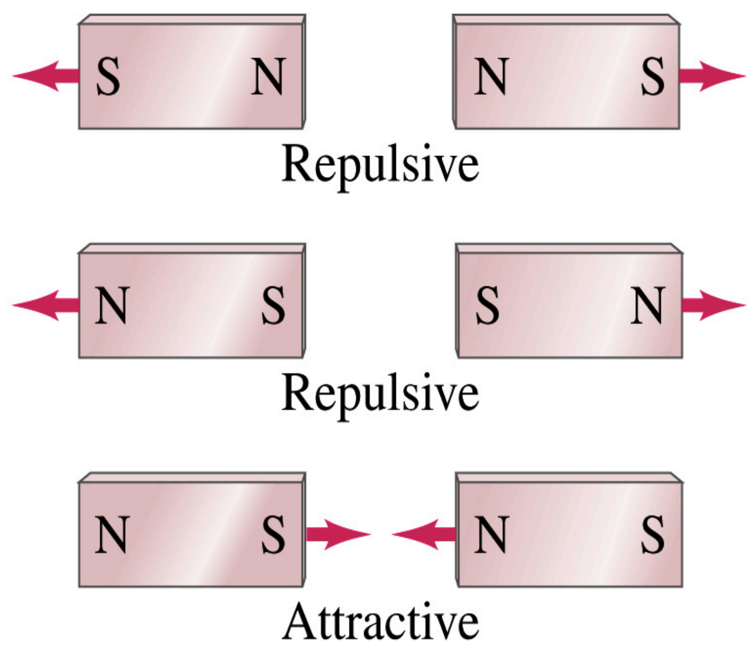
Reminder: Special Project #4

- Make a list of the power consumption and the resistance of all electric and electronic devices at your home and compile them in a table. (10 points total for the first 10 items and 0.5 points each additional item.)
- Estimate the cost of electricity for each of the items on the table using your own electric cost per kWh (if you don't find your own, use $0.12/kWh) and put them in the relevant column. (5 points total for the first 10 items and 0.2 points each additional items)
- Estimate the the total amount of energy in Joules and the total electricity cost per day, per month and per year for your home. (8 points)
- Due: Beginning of the class Monday, Apr. 13
  - Scan all pages of your special project into the pdf format
  - Save all pages into one file with the filename SP4-YourLastName-YourFirstName.pdf
  - Send me the file no later than 1pm Monday, Apr. 13
- Spreadsheet has been posted on the class web page. Download ASAP.

Your Name						Electricity Rate					c/kWh
Item Name	**Rated power (W)**	**Number of devices**	**Number of Hours per day**	**Daily Power Consumption (kWh)**	**Energy Cost per kWh (cents)**	**Daily Energy Consumption (J).**	**Daily Energy Cost ($)**	**Monthly Energy Consumption (J)**	**Monthly Energy Cost (S)**	**Yearly Energy Consumption (J)**	**Yearly Energy Cost ($)**
Light Bulbs	30	4									
	40	6									
	60	15									
Heaters	1000	2									
	1500	1									
	2000	1									
Home Appliances (Fans, vacuum cleaners, hair dryers, pool pumps, etc)											
Air Conditioners											
Kitchen Appliances (Fridges, freeezers, cook tops, microwave ovens, toaster ovens, etc)											
Computing devices (desktop, laptop, ipad, mobile phones, printers, chargers, etc))											
Tools (power tools, electric mower, electric cutter, etc)											
Medical Devices (blood pressure machine, thermometer, etc)											
Transporations (electric cars, electric bicycles, electric motor cycles, etc											
Total											

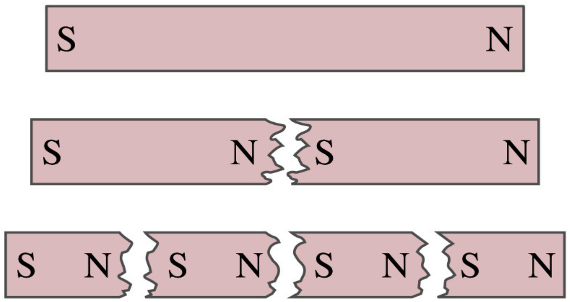
Magnetism

- What are magnets?
  - Objects with two poles, North and South poles
    - The pole that points to the geographical North is the North pole, and the other is the South pole
      - Principle of compass
  - They are called magnets due to the name of the region, Magnesia, where rocks that attract each other were found
- What happens when two magnets are brought to each other?
  - They exert force onto each other
  - What kind?
  - Both repulsive and attractive forces depending on their configurations
    - Like poles repel each other while the unlike poles attract



Magnetism

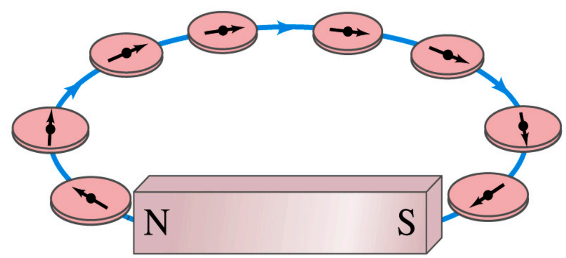
- So the magnetic poles are the same as the electric charge?
  - No. Why not?
  - While the electric charges (positive and negative) can be isolated, the magnetic poles cannot be isolated.
  - So what happens when a magnet is cut?
    - If a magnet is cut, two magnets are made.
    - The more they get cut, the more magnets are made

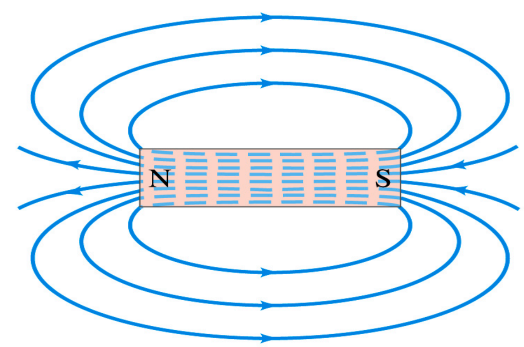


  - Single pole magnets are called the monopole but have not been seen yet
- **Ferromagnetic materials**: Materials that show strong magnetic effects
  - Iron, cobalt, nickel, gadolinium and certain alloys
- Other materials show very weak magnetic effects

Magnetic Field

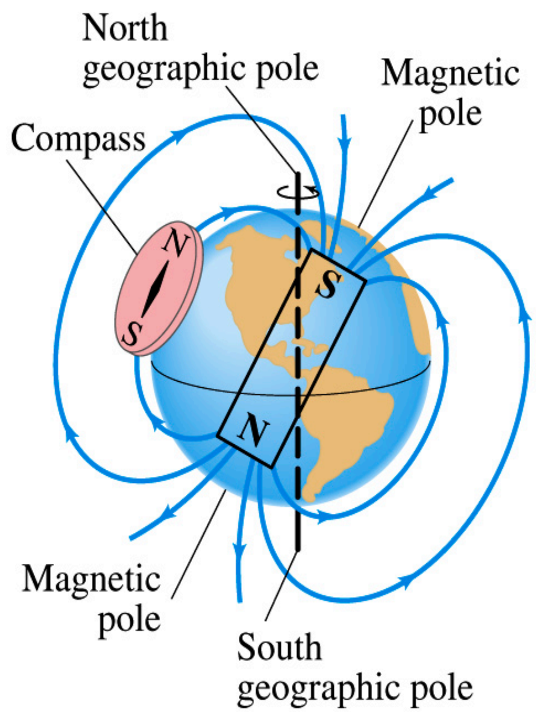
- Just like the electric field that surrounds electric charge, the magnetic field surrounds a magnet
- What does this mean?
  - Magnetic force is also **a field force**
  - The force one magnet exerts onto another can be viewed as the interaction between the magnet and the magnetic field produced by the other magnets
  - What kind of quantity is the magnetic field? Vector or Scalar? **Vector**
- So one can draw magnetic field lines, too.
  - The direction of the magnetic field at any given point along the line is **tangential to the line** at that point
  - The direction of the field is the direction the north pole of a compass would point to (from N to S)
  - The number of lines per unit area is proportional to the strength of the magnetic field
  - Magnetic field lines continue inside the magnet
  - Since magnets always have both the poles, magnetic field lines **form closed loops** unlike electric field lines





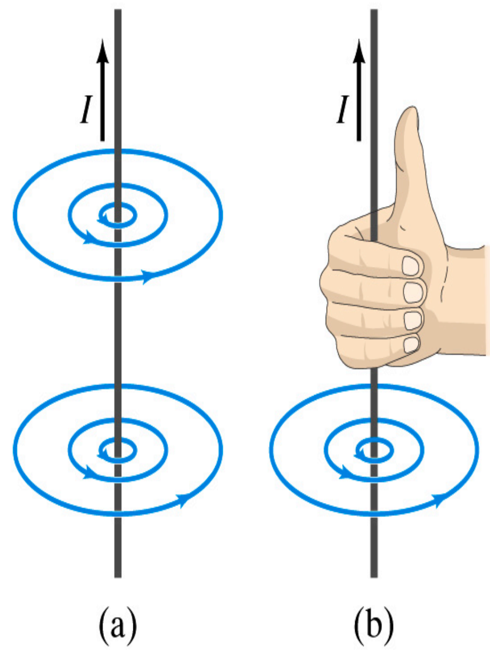
Earth's Magnetic Field

- What magnetic pole does the geographic North pole has to have?
  - Magnetic South pole. What? How do you know that?
  - Since the magnetic North pole points to the geographic North, the geographic north must have magnetic South pole
    - The pole in the North is still called geomagnetic North pole just because it is in the North
  - Similarly, South pole has magnetic North pole
- The Earth's magnetic poles do not coincide with the geographic poles ➔ **<u>magnetic declination</u>**
  - Geomagnetic North pole is in Northern Canada, some 900km off the true geographic North pole
- Earth's magnetic field line is not tangent to the earth's surface at all points
  - The angle the Earth's magnetic field makes to the horizontal line is called the angle dip



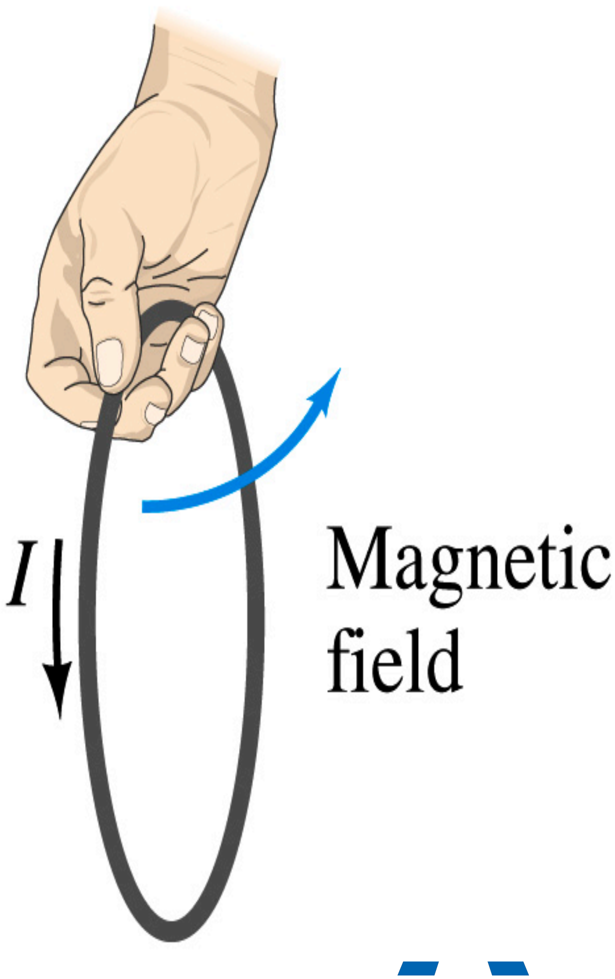
Electric Current and Magnetism

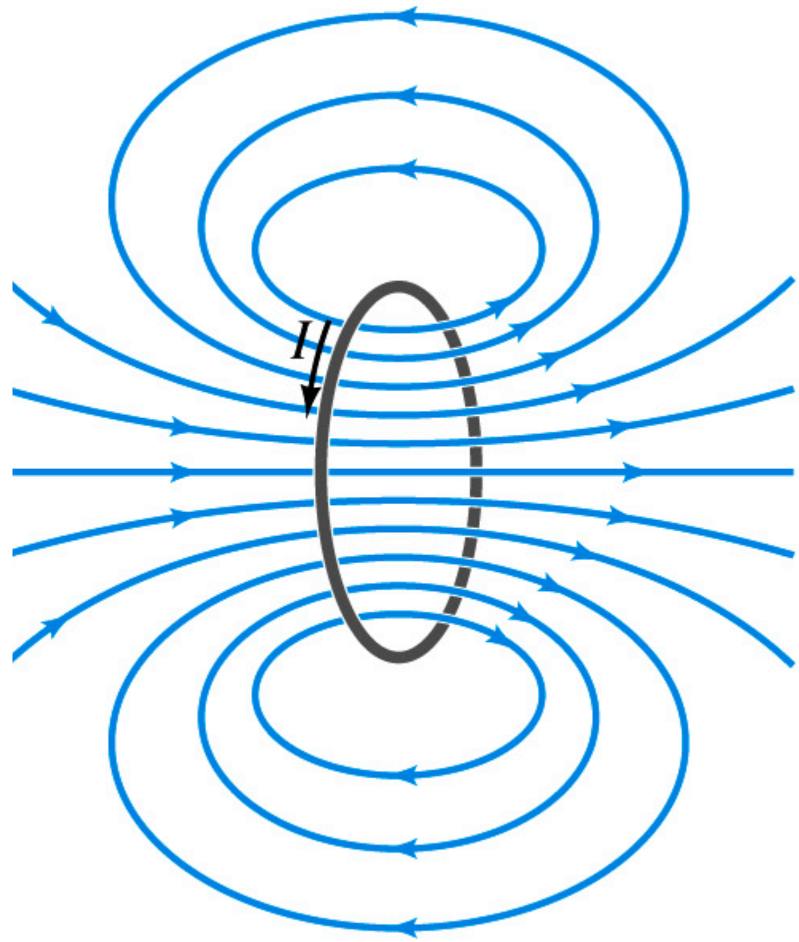
- In 1820, Oersted found that when a compass needle is placed near an electric wire, the needle deflects as soon as the wire is connected to a battery and the current flows
  - Electric current produces a magnetic field
    - The first indication that electricity and magnetism are of the same origin
  - What about a stationary electric charge and magnet?
    - They don't affect each other.
- The magnetic field lines produced by a current in a straight wire is in the form of circles following the "right-hand" rule
  - The field lines follow right-hand fingers wrapped around the wire when the thumb points to the direction of the electric current



Directions in a Circular Wire?

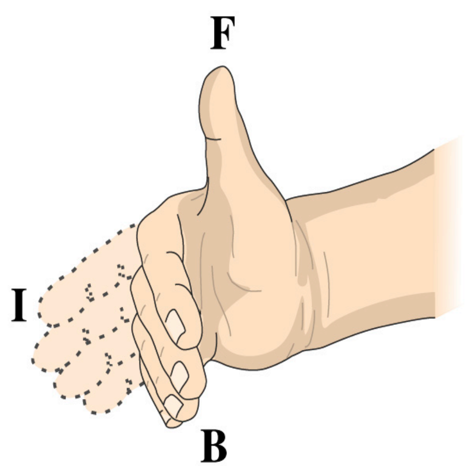
- OK, then what is the direction of the magnetic field generated by the current flowing through a circular loop?



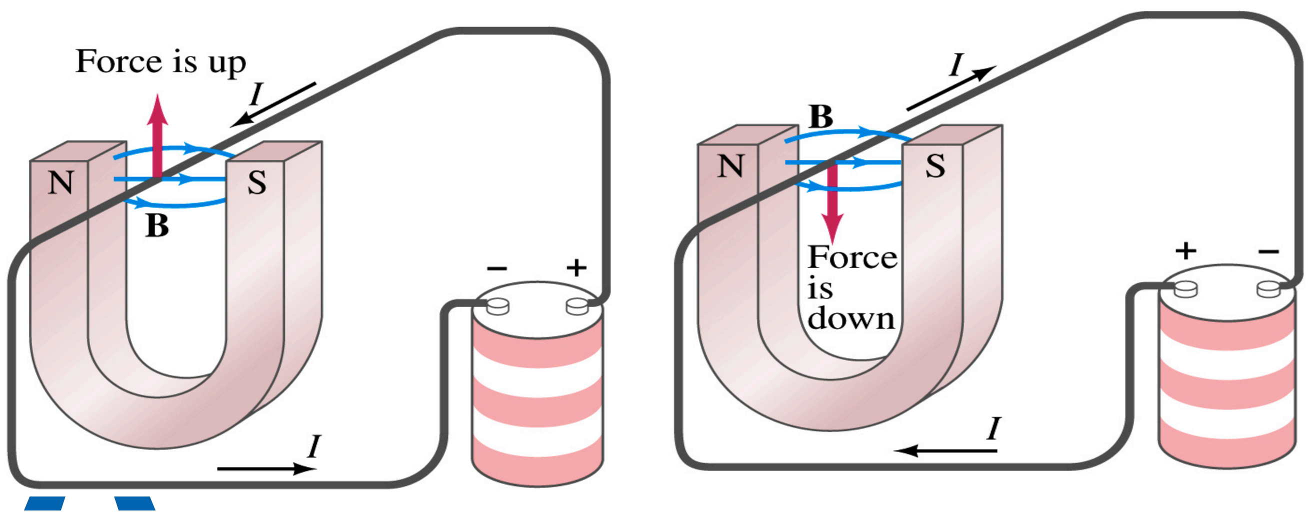


Magnetic Force on Electric Current

- Since the electric current exerts force on a magnet, a magnet should also exert force on the electric current
  - Which law justifies this?
    - Newton's 3rd law
  - This was also discovered by Oersted
- Direction of the force is always
  - **perpendicular** to the direction of **the current** and
  - **perpendicular** to the direction of **the magnetic field, B**
- Experimentally the direction of the force is given by another right-hand rule ➔ When the fingers of the right-hand points to the direction of the current and the finger tips bent to the direction of magnetic field B, the direction of thumb points to is the direction of the magnetic force



Right-hand rule



Magnetic Force on Electric Current

- OK, we are set for the direction but what about the magnitude?
- It is found that the magnitude of the force is directly proportional
  - To the **current** in the wire
  - To the **length** of the wire in the magnetic field (if the field is uniform)
  - To the strength of the **magnetic field**
- The force also depends on the angle θ between the directions of the current and the magnetic field
  - When the wire is perpendicular to the field, the force is the strongest
  - When the wire is parallel to the field, there is no force at all
- Thus the force on current I in the wire w/ length ℓ in a uniform field $\mathcal{B}$ is

$$F \propto IlB\sin\theta$$

Magnetic Forces on Electric Current

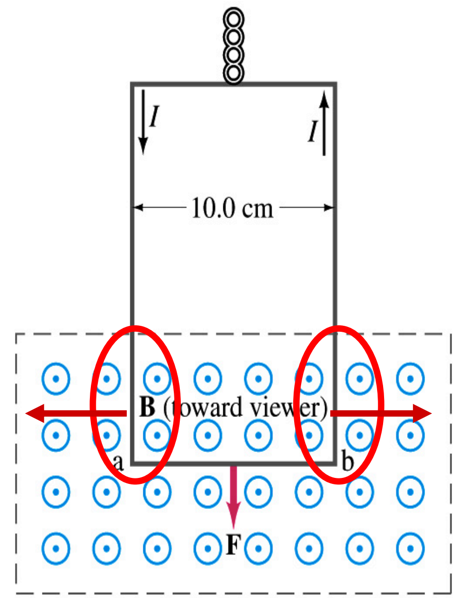
- Magnetic field strength B can be defined using the previous proportionality relationship w/ the constant 1: $F = IlB\sin\theta$
- if θ=90°, $F_{max} = IlB$ and if θ=0° $F_{min} = 0$
- So the magnitude of the magnetic field B can be defined as
  - $B = F_{max}/Il$ where F_{max} is the magnitude of the force on a straight length l of the wire carrying current I when the wire is perpendicular to **B**
- The relationship between F, B and I can be written in a vector formula: $\vec{F} = I\vec{l} \times \vec{B}$
  - l is the vector whose magnitude is the length of the wire, and its direction is along the wire in the direction of the **conventional current**
  - This formula works if **B** is uniform.
- If B is not uniform or l does not form the same angle with B everywhere, the infinitesimal force acting on a differential length dl is $d\vec{F} = Id\vec{l} \times \vec{B}$

Fundamentals on the Magnetic Field, B

- The magnetic field is a vector quantity
- The SI unit for B is tesla (T)
  - What is the definition of 1 Tesla in terms of other known units?
  - 1T=1N/A-m
  - In older names, tesla is the same as weber per meter-squared
    - $1Wb/m^2=1T$
- The cgs unit for B is gauss (G)
  - How many T is one G?
    - $1G=10^{-4}$ T
  - For computation, one MUST convert G to T at all times
- Magnetic field on the Earth's surface is about $0.5G=0.5\times10^{-4}T$
- On a diagram, $\odot$ for field coming out and $\otimes$ for going in.

Example 27 – 2

Measuring a magnetic field. A rectangular loop of wire hangs vertically as shown in the figure. A magnetic field **B** is directed horizontally perpendicular to the wire, and points out of the page. The magnetic field **B** is very nearly uniform along the horizontal portion of wire *ab* (length l=10.0cm) which is near the center of a large magnet producing the field. The top portion of the wire loop is free of the field. The loop hangs from a balance which measures a downward force (in addition to the gravitational force) of $F=3.48\times10^{-2}$N when the wire carries a current I=0.245A. What is the magnitude of the magnetic field B at the center of the magnet?



Magnetic force exerted on the wire due to the uniform field is $\vec{F} = I\vec{l} \times \vec{B}$

Since $\vec{B} \perp \vec{l}$ Magnitude of the force is $F = IlB$

Solving for B

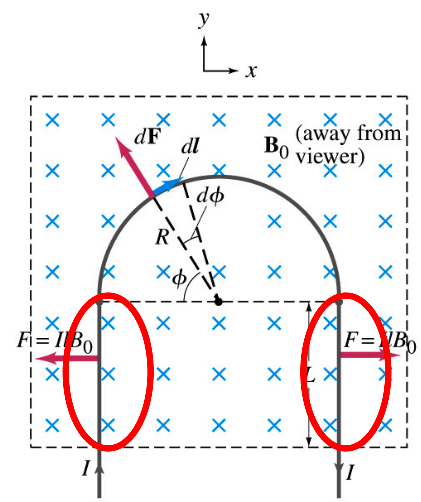
$$B = \frac{F}{Il} = \frac{3.48\times10^{-2}\,N}{0.245A\cdot0.10m} = 1.42T$$

Something is not right! What happened to the forces on the loop on the sides?

The two forces cancel out since they are in opposite direction with the same magnitude.

Example 27 – 3

Magnetic force on a semi-circular wire. A rigid wire, carrying the current I, consists of a semicircle of radius R and two straight portions as shown in the figure. The wire lies in a plane perpendicular to the uniform magnetic field $\mathbf{B_0}$. The straight portions each have length ℓ within the field. Determine the net force on the wire due to the magnetic field $\mathbf{B_0}$.



As in the previous example, the forces on the straight sections of the wire is equal and in opposite direction. Thus they cancel.

What do we use to figure out the net force on the semicircle? $d\vec{F} = Id\vec{l} \times \vec{B}$

We divide the semicircle into infinitesimal straight sections. $dl = Rd\phi$

What is the net x component of the force exerting on the circular section? 0 **Why?**

Because the forces on left and the right-hand sides of the semicircle balance.

Since $\vec{B}_0 \perp d\vec{l}$ Y-component of the force dF is $dF_y = d\left(F\sin\phi\right) = IRB_0 d\phi$

Integrating over ϕ=0 - π

$$F = \int_0^{\pi} d\left(F\sin\phi\right) = IB_0 R\int_0^{\pi}\sin\phi d\phi = -IB_0 R\left[\cos\phi\right]_0^{\pi} = 2RIB_0$$

Which direction?

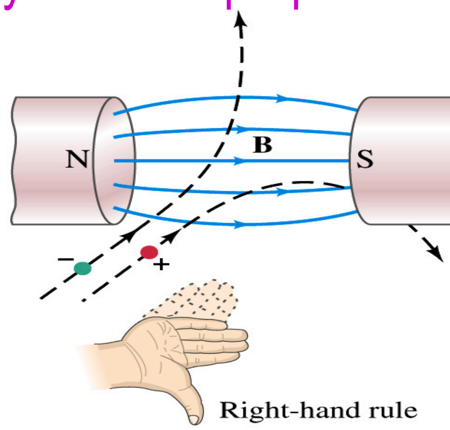
Vertically upward direction. The wire will be pulled deeper into the field.

Magnetic Forces on a Moving Charge

- Will moving charge in a magnetic field experience force?
  - Yes
  - Why?
  - Since the wire carrying current (moving charge) experiences force in a magnetic field, a free moving charge must feel the same kind of force…☺
- OK, then how much force would it experience?
  - Let's consider **N** moving particles with charge **q** each, and they pass by a given point in a time interval **t**.
    - What is the current? $I = Nq/t$
  - Let **t** be the time for the charge **q** to travel a distance ℓ in a magnetic field **B**
    - Then, the length vector ℓ can be written as $\vec{l} = \vec{v}t$
    - Where **v** is the velocity of the particle
- Thus the force on N particles by the field is $\vec{F} = I\vec{l} \times \vec{B} = Nq\vec{v} \times \vec{B}$
- The force on one particle with charge q, $\vec{F} = q\vec{v} \times \vec{B}$

Magnetic Forces on a Moving Charge

- This can be an alternate way of defining the magnetic field.
  - How?
  - The magnitude of the magnetic force on a particle with charge **q** moving with a velocity **v** in a field **B** is
    - $F = qvB\sin\theta$
    - What is θ?
      - The angle between the magnetic field and the direction of particle's movement
    - When is the force maximum?
      - When the angle between the field and the velocity vector is perpendicular.
    - $F_{max} = qvB$ ➔ $B = \frac{F_{max}}{qv}$
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field



Example 27 – 5

Magnetic force on a proton. A proton with the speed of $5x10^6$m/s in a magnetic field feels the force of $F=8.0x10^{-14}$N toward West when it moves vertically upward. When moving horizontally in a northerly direction, it feels zero force. What is the magnitude and the direction of the magnetic field in this region?

What is the charge of a proton? $q_p = +e = 1.6 \times 10^{-19} C$

What does the fact that the proton does not feel any force in a northerly direction tell you about the magnetic field?

The field is along the north-south direction. Why?

Because the particle does not feel any magnetic force when it is moving along the direction of the field.

Since the particle feels force toward West, the field should be pointing to …. North

Using the formula for the magnitude of the field B, we obtain

$$B = \frac{F}{qv} = \frac{8.0 \times 10^{-14} N}{1.6 \times 10^{-19} C \cdot 5.0 \times 10^{6} m/s} = 0.10T$$

We can use magnetic field to measure the momentum of a particle. How?

This print-out should have 6 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Kirchhoff's loop rule for circuit analysis is an expression of which of the following?

1. Ohm's law

2. Conservation of charge

3. Ampère's law

4. Faraday's law

5. Conservation of energy **correct**

Explanation:

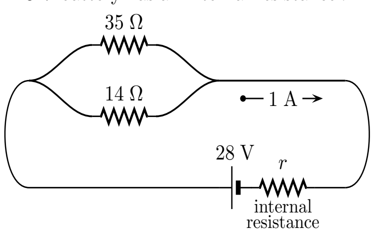
Kirchhoff's loop rule

$$\sum V = V_1 + V_2 + V_3 + \cdots = 0$$

follows from the conservation of energy.

002 (part 1 of 2) 10.0 points

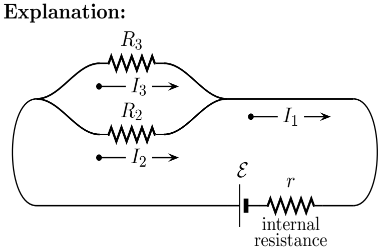
A 28 V battery has an internal resistance r.



What is the value of r?

Correct answer: 18 Ω.

Explanation:



$$\text{Let}: \quad \mathcal{E} = 28\text{ V}, \quad R_2 = 14\,\Omega, \quad R_3 = 35\,\Omega, \quad \text{and} \quad I_1 = 1\text{ A}.$$

Since R_2 and R_3 are connected parallel, their equivalent resistance R_{23} is

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_3 + R_2}{R_2\,R_3}$$
$$R_{23} = \frac{R_2\,R_3}{R_2 + R_3} = \frac{(14\,\Omega)\,(35\,\Omega)}{14\,\Omega + 35\,\Omega} = 10\,\Omega.$$

Using Ohm's law, we have

$$\mathcal{E} = I_1\,r + I_1\,R_{23}$$
$$r = \frac{\mathcal{E} - I_1\,R_{23}}{I_1} = \frac{28\text{ V} - (1\text{ A})\,(10\,\Omega)}{1\text{ A}} = 28\,\Omega - 10\,\Omega = \boxed{18\,\Omega}.$$

003 (part 2 of 2) 10.0 points

Determine the magnitude of the current through the 35 Ω resistor in the upper left of the circuit.

Correct answer: 0.285714 A.

Explanation:

The potential drop across the 35 Ω resistor on the left-hand side of the circuit is

$$\mathcal{E}_3 = \mathcal{E} - I_1\,r = 28\text{ V} - (1\text{ A})\,(18\,\Omega) = 28\text{ V} - 18\text{ V} = 10\text{ V},$$

so the current through the resistor is

$$I_3 = \frac{\mathcal{E}_3}{r_3} = \frac{10\text{ V}}{35\,\Omega} = \frac{2}{7}\text{ A} = \boxed{0.285714\text{ A}}.$$

004 10.0 points

The current in an AC circuit is measured with an ammeter, which gives a reading of 4.4 A.

Calculate the maximum AC current.

Correct answer: 6.22254 A.

Explanation:

$$\text{Let}: \quad I_{rms} = 4.4\text{ A}.$$

The *rms* current is

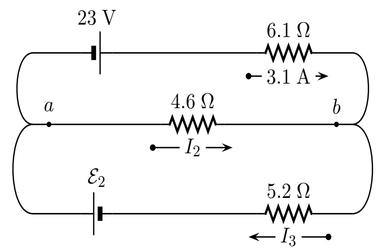
$$I_{rms} = \frac{\sqrt{2}}{2}\,I_{max} = \frac{I_{max}}{\sqrt{2}},$$

so the maximum AC current is

$$I_{max} = I_{rms}\sqrt{2} = (4.4\text{ A})\sqrt{2} = \boxed{6.22254\text{ A}}.$$

005 (part 1 of 2) 10.0 points

In the circuit of the figure, the current I_1 is 3.1 A and the value of $\mathcal{E}_2$ is unknown.

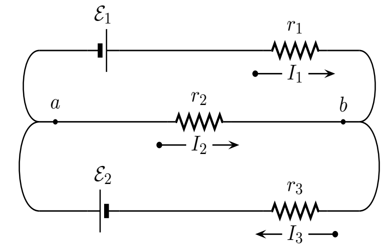


What is the magnitude of the current I_2?

Correct answer: 0.88913 A.

Explanation:

$$\text{Let}: \quad r_1 = 6.1\,\Omega, \quad r_2 = 4.6\,\Omega, \quad r_3 = 5.2\,\Omega, \quad I_1 = 3.1\text{ A}, \quad \text{and} \quad \mathcal{E}_1 = 23\text{ V}.$$



Applying Kirchhoff's loop rule clockwise around the right loop,

$$\mathcal{E}_1 - r_1\,I_1 + r_2\,I_2 = 0$$

$$I_2 = \frac{r_1\,I_1 - \mathcal{E}_1}{r_2} = \frac{(6.1\,\Omega)\,(3.1\text{ A}) - 23\text{ V}}{(4.6\,\Omega)} = -0.88913\text{ A},$$

directed from b toward a with magnitude $\boxed{0.88913\text{ A}}$.

006 (part 2 of 2) 10.0 points

Find I_3.

Correct answer: 2.21087 A.

Explanation:

Applying Kirchhoff's junction rule at point a gives

$$I_3 = I_1 + I_2 = 3.1\text{ A} + (-0.88913\text{ A}) = \boxed{2.21087\text{ A}}.$$

This print-out should have 18 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 5.0 points

A neutral copper ball is suspended by a string. A positively charged insulating rod is placed near the ball, which is observed to be attracted to the rod.

Why is this?

1. There is a rearrangement of the electrons in the ball. **correct**

2. The number of electrons in the ball is greater than in the rod.

3. The ball becomes positively charged by induction.

4. The string is not a perfect conductor.

5. The ball becomes negatively charged by induction.

Explanation:

If a positively charged insulator is brought close to a conductor, it will attract some of the free electrons of the conductor closer to the insulator. The attraction between the insulator and the conductor results from the rearrangement of electrons.

002 6.0 points

Two electrons in an atom are separated by 1.5×10^{-10} m, the typical size of an atom.

What is the force between them? The Coulomb constant is $9 \times 10^9\ \mathrm{N \cdot m^2/C^2}$.

Correct answer: 1.02656×10^{-8} N.

Explanation:

$$\text{Let}: \quad d = 1.5 \times 10^{-10}\ \mathrm{m} \quad \text{and} \quad k_e = 9 \times 10^9\ \mathrm{N \cdot m^2/C^2}.$$

The force between the electrons is

$$F = \frac{k_e\, q_1\, q_2}{d^2} = \frac{k_e\, q_e^2}{d^2}$$

$$= \frac{9 \times 10^9\ \mathrm{N \cdot m^2/C^2}}{1.5 \times 10^{-10}\ \mathrm{m^2}} \times (1.602 \times 10^{-19}\ \mathrm{C})^2$$

$$= \boxed{1.02656 \times 10^{-8}\ \mathrm{N}}.$$

003 5.0 points

How is Coulomb's law similar to Newton's law of gravitation? How is it different?

1. Both forces are proportional to the same constant; electrical forces are only present on earth, whereas gravitational forces exist everywhere.

2. Both forces proportional to the same constant; electrical forces may be either attractive or repulsive, whereas gravitational forces are always attractive.

3. Both forces vary inversely as the square of the separation distance between the two objects; electrical forces are only present on earth, whereas gravitational forces can exist everywhere.

4. Both forces vary inversely as the square of the separation distance between the two objects; electrical forces may be either attractive or repulsive, whereas gravitational forces are always attractive. **correct**

5. Both forces are proportional to the product of the mass of the two objects; electrical forces may be either attractive or repulsive, whereas gravitational forces are always attractive.

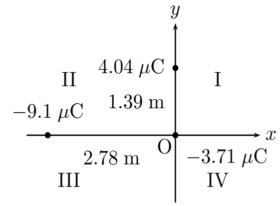
6. Both forces are proportional to the product of the masses of the two objects; electrical forces are only present on earth, whereas gravitational forces exist everywhere.

Explanation:

Both forces obey the inverse square law $F \propto \dfrac{1}{r^2}$. The most important difference between gravitational and electrical forces is that electrical forces may be either attractive or repulsive, whereas gravitational forces are always attractive.

004 (part 1 of 2) 6.0 points

Consider 3 charges arranged as shown.



Find the direction of the resultant electric force on the $-3.71\ \mu$C charge at the origin due to the other two charges. The value of the Coulomb constant is $8.9875 \times 10^9\ \mathrm{N \cdot m^2/C^2}$.

1. In quadrant III

2. In quadrant IV

3. Along the negative x-axis

4. In quadrant II

5. Along the negative y-axis

6. Along the positive x-axis

7. Along the positive y-axis

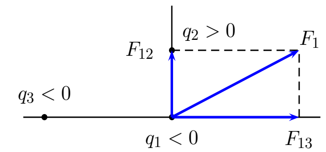
8. In quadrant I **correct**

Explanation:

From Coulomb's law, the magnitude of the force from point charge q_1 on point charge q_2 at distance x is

$$F = k_e\, \frac{q_1\, q_2}{x^2}.$$

The electric field from charge q_2 points along the positive y-axis, since q_2 and q_1 are of opposite sign (attractive force). The electric field from charge q_3 points along the positive x-axis, since q_3 and q_1 are of the same sign (repulsive force).



By inspection, the resultant points into quadrant I.

005 (part 2 of 2) 6.0 points

Determine the magnitude of the resultant electric force on the $-3.71\ \mu$C charge at the origin due to the other two charges.

Correct answer: 0.0800156 N.

Explanation:

$$\begin{aligned} \text{Let}: \quad q_1 &= -3.71\ \mu\mathrm{C}, \\ q_2 &= 4.04\ \mu\mathrm{C}, \\ q_3 &= -9.1\ \mu\mathrm{C}, \\ a &= 1.39\ \mathrm{m}, \quad \text{and} \\ b &= 2.78\ \mathrm{m}, \end{aligned}$$

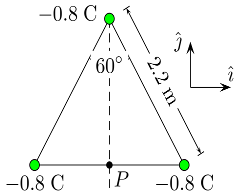
$$\begin{aligned} F_{12} &= k_e\, \frac{q_1\, q_2}{a^2} \\ &= \left(8.9875 \times 10^9\ \mathrm{N \cdot m^2/C^2}\right) \times \frac{(-3.71\ \mu\mathrm{C})\,(4.04\ \mu\mathrm{C})}{(1.39\ \mathrm{m})^2} \\ &= -0.0697212\ \mathrm{N} \quad \text{and} \end{aligned}$$

$$\begin{aligned} F_{13} &= k_e\, \frac{q_1\, q_3}{b^2} \\ &= \left(8.9875 \times 10^9\ \mathrm{N \cdot m^2/C^2}\right) \times \frac{(-3.71\ \mu\mathrm{C})\,(-9.1\ \mu\mathrm{C})}{(2.78\ \mathrm{m})^2} \\ &= 0.0392613\ \mathrm{N}, \quad \text{so} \end{aligned}$$

$$\begin{aligned} F_1 &= \sqrt{F_{12}^2 + F_{13}^2} \\ &= \sqrt{(-0.0697212\ \mathrm{N})^2 + (0.0392613\ \mathrm{N})^2} \\ &= \boxed{0.0800156\ \mathrm{N}}. \end{aligned}$$

006 (part 1 of 2) 6.0 points

Three point charges are placed at the vertices of an equilateral triangle.

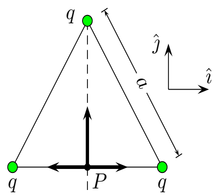


Find the magnitude of the electric field vector $\|\vec{E}\|$ at P. The value of the Coulomb constant is 8.9875×10^9 N · m²/C².

Correct answer: 1.98072×10^9 N/C.

Explanation:

$$\text{Let:}\quad a = 2.2\ \text{m}\,,\quad q = -0.8\ \text{C}\,,\quad \text{and}\quad k = 8.9875 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\,.$$



$h = a\cos(30°) = \dfrac{\sqrt{3}}{2}\,a$ is the height of the triangle. Electric field vectors due to bottom two charges cancel out each other and the magnitude of the field vector due to charge at the top of the triangle is

$$\|\vec{E}\| = \frac{k\,q}{\left(\frac{\sqrt{3}}{2}\,a\right)^2} = \frac{4}{3}\,\frac{k\,q}{a^2}$$
$$= \frac{4}{3}\,\frac{(8.9875\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\,(-0.8\ \text{C})}{(2.2\ \text{m})^2}$$
$$= \boxed{1.98072\times10^9\ \text{N/C}}\,.$$

007 (part 2 of 2) 5.0 points

Find the direction of the field vector $\vec{E}$ at P.

1. $\dfrac{1}{\sqrt{2}}\,(\hat{\imath}+\hat{\jmath})$
2. $\hat{\imath}$
3. $-\hat{\imath}$
4. $-\dfrac{1}{\sqrt{2}}\,(\hat{\imath}-\hat{\jmath})$
5. $-\dfrac{1}{\sqrt{2}}\,(\hat{\imath}+\hat{\jmath})$
6. $\hat{\jmath}$ **correct**
7. $-\hat{k}$
8. $\hat{k}$
9. $-\hat{\jmath}$
10. $\dfrac{1}{\sqrt{2}}\,(\hat{\imath}-\hat{\jmath})$

Explanation:

By inspection, $\vec{E}$ at P is along the $\hat{\jmath}$ direction.

008 6.0 points

A particle of mass 2.8 g and charge 19.8 mC moves in a region of space where the electric field is uniform and is given by $E_x = -3.5$ N/C, $E_y = 0$, and $E_z = 0$.

If the velocity of the particle at $t = 0$ is $v_{x0} = 50$ m/s, $v_{y0} = 0$, and $v_{z0} = 0$, what is the speed of the particle at 1.4 s?

Correct answer: 15.35 m/s.

Explanation:

$$\text{Let:}\quad m = 2.8\ \text{g} = 0.0028\ \text{kg}\,,\quad q = 19.8\ \text{mC} = 0.0198\ \text{C}\,,\quad E_x = -3.5\ \text{N/C}\,,\quad E_y = E_z = 0\,,\quad v_{x0} = 50\ \text{m/s}\,,\quad \text{and}\quad v_{y0} = v_{z0} = 0\,.$$

The force on the particle is

$$F = q\,E = m\,a$$
$$a = \frac{q\,E}{m}$$

and the velocity is given by

$$v = v_0 + a\,t = v_0 + \frac{q\,E}{m}\,t$$
$$= 50\ \text{m/s} + \frac{(0.0198\ \text{C})(-3.5\ \text{N/C})(1.4\ \text{s})}{0.0028\ \text{kg}}$$
$$= 15.35\ \text{m/s}\,,$$

corresponding to a speed of $\boxed{15.35\ \text{m/s}}$.

009 6.0 points

A cylindrical shell of radius 9.7 cm and length 298 cm has its charge density uniformly distributed on its surface. The electric field intensity at a point 26.3 cm radially outward from its axis (measured from the midpoint of the shell) is 57000 N/C.

What is the net charge on the shell? The Coulomb constant is 8.99×10^9 N · m²/C².

Correct answer: 2.4846×10^{-6} C.

Explanation:

$$\text{Let:}\quad a = 0.097\ \text{m}\,,\quad \ell = 2.98\ \text{m}\,,\quad E = 57000\ \text{N/C}\,,\quad r = 0.263\ \text{m}\,,\quad \text{and}\quad k = 8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\,.$$

Applying Gauss' law

$$\oint E\cdot dA = \frac{Q}{\epsilon_0}$$
$$2\,\pi\,r\,\ell\,E = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{2\,\pi\,\epsilon_0\,r\,\ell} = \frac{2\,Q}{4\,\pi\,\epsilon_0\,r\,\ell} = \frac{2\,k\,Q}{r\,\ell}$$
$$Q = \frac{E\,r\,\ell}{2\,k}$$
$$= \frac{(57000\ \text{N/C})\,(0.263\ \text{m})\,(2.98\ \text{m})}{2\,(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)}$$
$$= \boxed{2.4846\times10^{-6}\ \text{C}}\,.$$

010 5.0 points

Three positive charges lie on the x axis:

$$q_1 = 1\times10^{-8}\ \text{C at } x_1 = 1\ \text{cm},$$
$$q_2 = 2\times10^{-8}\ \text{C at } x_2 = 2\ \text{cm},$$

and

$$q_3 = 3\times10^{-8}\ \text{C at } x_3 = 3\ \text{cm}.$$

The potential energy of this arrangement, relative to the potential energy for infinite separation, is about:

1. 0.079 J
2. 0.00085 J **correct**
3. 0 J
4. 0.16 J
5. 0.0017 J

Explanation:

The key to solving this problem is to remember two things:

First, the electrostatic potential energy between any two point charges is $U = k\dfrac{q_1q_2}{r}$ where r is the separation between the two charges. This also means that an isolated point charge has exactly zero potential energy.

Second, electrostatic potential energy only depends on the relative positions of the particles, not on the manner in which a configuration was assembled.

Therefore, we can put together the system particle by particle, calculating its potential energy, and then just compare this total energy to our initial energy.

Let's start with nothing, and place q_1 at x_1. We can do this for free, as there is no previous field to work against, and it is the only involved charge. So, the energy cost of adding charge q_1, U_1, is zero.

Now, add the charge q_2 at x_2. Here, we have only one pair of charges, so it costs us $U_2 = k\dfrac{q_1q_2}{|r_1 - r_2|}$ to place this charge here while holding q_1 fixed.

Finally, add the charge q_3. For this charge, we have both q_1 and q_2 to work against, so the net energy we spend putting q_3 in place while holding the other two charges fixed is $U_3 = k\frac{q_1 q_3}{|r_1 - r_3|} + k\frac{q_2 q_3}{|r_2 - r_3|}$.

So, our net energy is:

$$\begin{aligned} U_{tot} &= U_1 + U_2 + U_3 \\ &= 0 + k\frac{q_1 q_2}{|r_1 - r_2|} \\ &\quad + k\frac{q_1 q_3}{|r_1 - r_3|} + k\frac{q_2 q_3}{|r_2 - r_3|} \\ &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(100 \text{ cm/m}) \\ &\quad \times \left(\frac{(1 \times 10^{-8} \text{ C})(2 \times 10^{-8} \text{ C})}{2 \text{ cm} - 1 \text{ cm}} \right. \\ &\quad + \frac{(1 \times 10^{-8} \text{ C})(3 \times 10^{-8} \text{ C})}{3 \text{ cm} - 1 \text{ cm}} \\ &\quad \left. + \frac{(2 \times 10^{-8} \text{ C})(3 \times 10^{-8} \text{ C})}{3 \text{ cm} - 2 \text{ cm}} \right) \\ &= 0.00085 \text{ J} \end{aligned}$$

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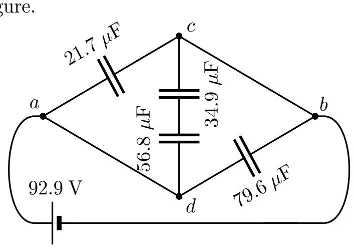
011 (part 1 of 2) 6.0 points

Four capacitors are connected as shown in the figure.

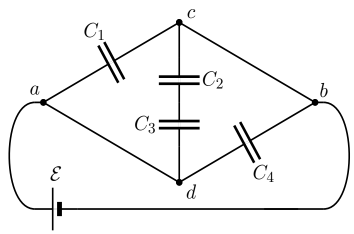


Find the capacitance between points a and b of the entire capacitor network. Answer in units of μ F.

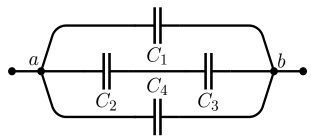
Correct answer: 122.917.

Explanation:

$$\begin{aligned} \text{Let :} \quad C_1 &= 21.7 \ \mu\text{F}, \\ C_2 &= 34.9 \ \mu\text{F}, \\ C_3 &= 56.8 \ \mu\text{F}, \\ C_4 &= 79.6 \ \mu\text{F}, \quad \text{and} \\ \mathcal{E} &= 92.9 \text{ V}. \end{aligned}$$



A good rule of thumb is to eliminate junctions connected by zero capacitance.



The definition of capacitance is $C \equiv \frac{Q}{V}$.

The series connection of C_2 and C_3 gives the equivalent capacitance

$$\begin{aligned} C_{23} &= \frac{1}{\frac{1}{C_2} + \frac{1}{C_3}} \\ &= \frac{C_2 C_3}{C_2 + C_3} \\ &= \frac{(34.9 \ \mu\text{F})(56.8 \ \mu\text{F})}{34.9 \ \mu\text{F} + 56.8 \ \mu\text{F}} \\ &= 21.6174 \ \mu\text{F}. \end{aligned}$$

The total capacitance C_{ab} between a and b can be obtained by calculating the capacitance in the parallel combination of the capacitors C_1, C_4, and C_{23}; *i.e.*,

$$\begin{aligned} C_{ab} &= C_1 + C_4 + C_{23} \\ &= 21.7 \ \mu\text{F} + 79.6 \ \mu\text{F} + 21.6174 \ \mu\text{F} \\ &= \boxed{122.917}. \end{aligned}$$

---

012 (part 2 of 2) 6.0 points

A dielectric with dielectric constant 2.97 is inserted into the 56.8 μF capacitor (lower-centered capacitor) while the battery is connected.

What is the charge on the 79.6 μF lower-right capacitor? Answer in units of μ C.

Correct answer: 7394.84.

Explanation:

Since the battery is still connected, the voltage will remain the same. Thus the charge is simply

$$\begin{aligned} Q_4 &= V_{ab} C_4 \\ &= (92.9 \text{ V})(79.6 \ \mu\text{F}) \\ &= \boxed{7394.84}. \end{aligned}$$

---

013 6.0 points

The compressor on an air conditioner draws 103 A when it starts up. The start-up time is about 0.34 s.

How much charge passes a cross-sectional area of the circuit in this time?

Correct answer: 35.02 C.

Explanation:

$$\begin{aligned} \text{Let :} \quad I &= 103 \text{ A} \quad \text{and} \\ \Delta t &= 0.34 \text{ s}. \end{aligned}$$

Current is

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} \\ \Delta Q &= I \, \Delta t = (103 \text{ A})(0.34 \text{ s}) \\ &= \boxed{35.02 \text{ C}}. \end{aligned}$$

---

014 5.0 points

The current in a section of a conductor depends on time as

$$I = a\,t^2 - b\,t + c.$$

What quantity of charge moves across the section of the conductor from $t = 0$ to $t = t_1$?

1. $q = \frac{a}{3} t_1^3 - \frac{b}{2} t_1^2 + c\,t_1$ **correct**

2. $q = \frac{a}{3} t_1^3 - \frac{b}{2} t_1^2 + c$

3. $q = a\,t_1^3 - b\,t_1^2 + c\,t_1$

4. $q = a\,t_1^3 - \frac{b}{2} t_1^2 + c t_1$

5. $q = a\,t_1^2 - b\,t_1 + c$

Explanation:

The unit of current is Coulomb per second: $I = \frac{d\,q}{dt}$ or $dq = I\,dt$.

To find the total charge that passes through the conductor, one must integrate the current over the time interval.

$$\begin{aligned} q &= \int_0^{t_1} dq = \int_0^{t_1} I\,dt \\ &= \int_0^{t_1} (a\,t^2 - b\,t + c)\,dt \\ &= \left[\frac{a\,t^3}{3} - \frac{b\,t^2}{2} + c\,t \right] \Bigg|_0^{t_1} \\ &= \frac{a\,t_1^3}{3} - \frac{b\,t_1^2}{2} + c\,t_1. \end{aligned}$$

---

015 5.0 points

Which of the following copper conductor conditions has the least resistance?

1. Thin, long, and cool

2. Thin, short, and cool

3. Thick, short, and cool **correct**

4. Thin, long, and hot

5. Thick, long, and cool

6. Thick, short, and hot

7. Thin, short, and hot

8. Thick, long, and hot

Explanation:

The resistance R of a conductor is determined by the resistivity ρ, cross-sectional area A and length ℓ:

$$R = \rho\,\frac{\ell}{A}.$$

Thus we need smaller ρ and ℓ and larger A. A cool copper conductor has a lower resistivity than a hot one does, namely a smaller ρ. A thicker conductor means a larger A. A short conductor gives a smaller ℓ.

---

016 (part 1 of 2) 6.0 points

The damage caused by electric shock depends on the current flowing through the body; 1 mA can be felt and 5 mA is painful. Above 15 mA, a person loses muscle control, and 70 mA can be fatal. A person with dry skin has a resistance from one arm to the other of about 60000 Ω. When skin is wet, the resistance drops to about 4900 Ω.

What is the minimum voltage placed across the arms that would produce a current that could be felt by a person with dry skin?

Correct answer: 60 V.

Explanation:

$$\text{Let}: \quad I_{min} = 1 \text{ mA} \quad \text{and}$$
$$R_{dry} = 60000\,\Omega\,.$$

The minimum voltage depends on the minimum current for a given resistance, so

$$\begin{aligned} V_{min} &= I_{min}\,R_{dry} \\ &= (1\text{ mA})\left(\frac{1\text{ A}}{1000\text{ mA}}\right)(60000\ \Omega) \\ &= \boxed{60\text{ V}}\,. \end{aligned}$$

---

017 (part 2 of 2) 6.0 points

For the same electric potential what would be the current if the person had wet skin?

Correct answer: 12.2449 mA.

Explanation:

$$\text{Let}: \quad R_{wet} = 4900\,\Omega\,.$$

$$\begin{aligned} I = \frac{V_{min}}{R_{wet}} &= \frac{60\text{ V}}{4900\ \Omega}\left(\frac{1000\text{ mA}}{1\text{ A}}\right) \\ &= \boxed{12.2449\text{ mA}}\,. \end{aligned}$$

---

018 6.0 points

An electric air conditioning unit draws 15 amps of direct current from a 109 V direct voltage source, and is used 24 hours a day during 20.8 days in July.

How much will the electricity cost for the month if the local electrical rate is 11.4 cents/kW · hr?

Correct answer: \$93.05.

Explanation:

$$\begin{aligned} \text{Let}: \quad I &= 15\text{ A}\,, \\ V &= 109\text{ V}\,, \\ t &= 20.8\text{ days}\,, \quad \text{and} \\ \mathcal{R} &= \ \$0.114\ /\text{kW}\cdot\text{hr}. \end{aligned}$$

The power is $P = I\,V$ and the energy used is $E = P\,t = I\,V\,t$, so the cost will be

$$\begin{aligned} C &= E\,r = (I\,V\,t)\,\mathcal{R} \\ &= (15\text{ A})\,(109\text{ V})\,(20.8\text{ days}) \\ &\quad \times (\ \$0.114\ /\text{kW}\cdot\text{hr})\,\frac{1\,\text{kW}}{1000\,\text{W}}\,\frac{24\text{ hrs}}{1\text{ day}} \\ &= \boxed{\$93.05}\,. \end{aligned}$$