PHYS 1444 – Section 002
Lecture #18

Monday, Apr. 13, 2020
Dr. Jaehoon Yu

CH 27: Magnetism & Magnetic Field

- Charged Particle Path in a Magnetic Field
- The cyclotron frequency
- Magnetic dipole Moment

CH 28:Sources of Magnetic Field

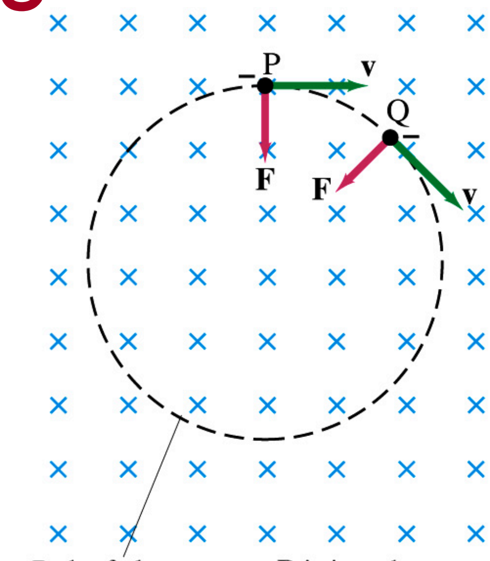
- Magnetic Field Due to Straight Wire
- Forces Between Two Parallel Wires

Today's homework is homework #11, due 11pm, Monday, Apr. 27!!

Announcements

- Reading Assignments: CH27.6, 27.8, 27.9 and 28.6 – 10
- 2nd Non-comprehensive term exam in class this Wed., Apr. 15
  - Do NOT miss the exam! **Must be in a quiet place to take the exam!**
  - This is one of the two exams that will be chosen for the final grade!
  - Covers CH25.1 through what we finish today (CH28.2)
  - Online exam based on Quest but **must join zoom class by 12:55pm**!
  - You can use your calculator but DO NOT input formula into it!
    - Cell phones or any types of computers cannot replace a calculator!
    - POWER OFF your phones!!
  - BYOF: You may prepare a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants
    - No derivations, plots, pictures, word definitions or solutions or setups of any problems!
  - Please send me the photos of your formula sheet by 12:55pm
    - If you don't have one, still send me email that you do not have one prepared!
  - Let's be fair to other students and not cheat!

Charged Particle's Path in Magnetic Field



- What shape do you think is the path of a charged particle on a plane perpendicular to a uniform magnetic field?
  - Circle!! Why?
  - An electron moving to right at the point P in the figure will be pulled downward
  - At a later time, the force is still perpendicular to the velocity
  - Since the force is always perpendicular to the velocity, the magnitude of the velocity is constant
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field
  - Thus, the electron moves in a circular path with a centripetal force F.

What about a proton? What would be the shape of its path?

Example 27 – 7

Electron's path in a uniform magnetic field. An electron travels at the speed of 2.0×10^7m/s in a plane perpendicular to a 100-G magnetic field. What is the radius of the electron's path?

What is formula for the centripetal force? $F = ma = m\frac{v^2}{r}$

What is the formula for the magnetic force on a moving charged particle? $\vec{F} = q\vec{v}\times\vec{B}$

Since the magnetic field is perpendicular to the motion of the electron, the magnitude of the magnetic force is $F = evB$

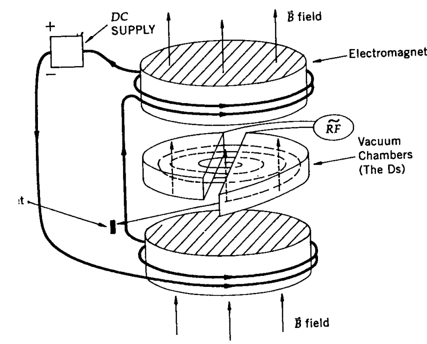
Since the magnetic force provides the centripetal force, we can establish an equation with the two forces $F = evB = m\frac{v^2}{r}$

Solving for r

$$r = \frac{mv}{eB} = \frac{\left(9.1\times10^{-31}kg\right)\cdot\left(2.0\times10^{7}\ m/s\right)}{\left(1.6\times10^{-19}C\right)\cdot\left(0.010T\right)} = 1.1\times10^{-2}m$$

Cyclotron Frequency

- The time required for a particle of charge **q** moving w/ a constant speed **v** to make one circular revolution in a uniform magnetic field, $\vec{B} \perp \vec{v}$, is



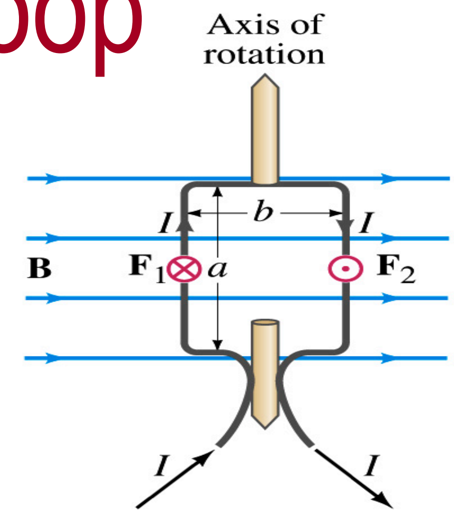
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

- Since T is the period of rotation, the frequency of the rotation is

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

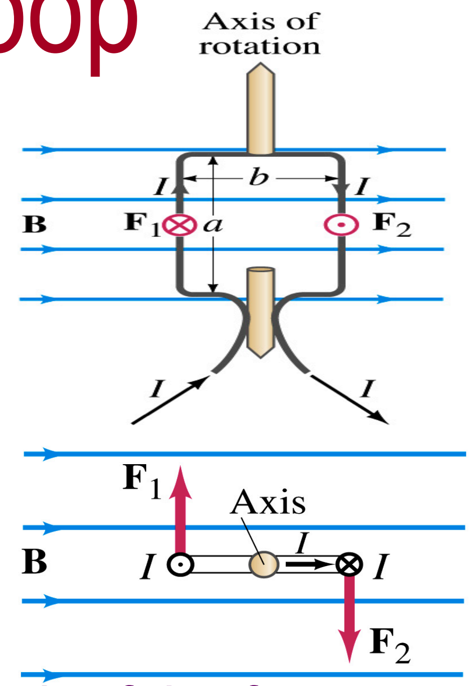
- This is the cyclotron frequency, the frequency of a particle with charge q in a cyclotron accelerator
  - While r depends on v, the frequency is independent of v and r.

Torque on a Current Loop



- What do you think will happen to a closed rectangular loop of wire with electric current as shown in the figure?
  - It will rotate! Why?
  - The magnetic field exerts a force on both vertical sections of wire.
  - Where is this principle used in?
    - Ammeters, motors, volt-meters, speedometers, etc
- The two forces on the different sections of the wire exerts net torque in the same direction about the rotational axis along the symmetry axis of the wire.
- What happens when the wire turns 90 degrees?
  - It will not turn unless there is more torque generated

Torque on a Current Loop



- So what would be the magnitude of this torque?
  - What is the magnitude of the force on the section of the wire with length a?
    - $F_a = IaB$
    - The moment arm of the coil is $b/2$
  - So the total torque is the sum of the torques by each of the forces

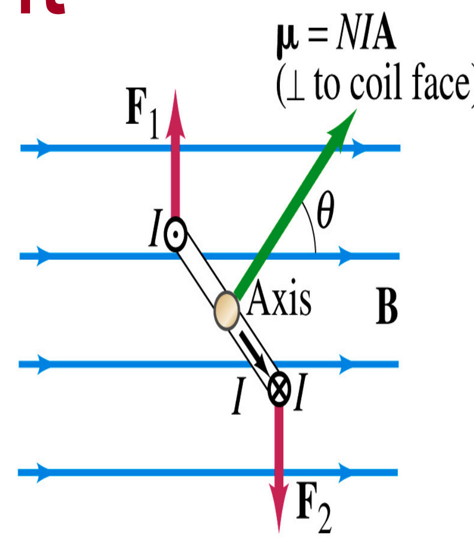
$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB$$

    - Where $A = ab$ is the area of the coil loop
  - What is the total net torque if the coil consists of N loops of wire?

$$\tau = NIAB$$

  - If the coil makes an angle θ w/ the field $\tau = NIAB\sin\theta$

Magnetic Dipole Moment



- The formula derived in the previous page for a rectangular coil is valid for any shape of the coil
- The quantity N$I\mathcal{A}$ is called the **magnetic dipole moment of the coil**
  - It is considered a vector $\vec{\mu} = NI\overrightarrow{A}$
    - Its direction is the same as that of the area vector **A** and is perpendicular to the plane of the coil consistent with the right-hand rule
      - Your thumb points to the direction of the magnetic moment when your finer cups around the loop in the direction of the current
  - Using the definition of magnetic moment, the torque can be written in vector form $\vec{\tau} = NI\overrightarrow{A} \times \vec{B} = \vec{\mu} \times \vec{B}$

Magnetic Dipole Potential Energy

- Where else did you see the same form of the torque?
  - Remember the torque due to electric field on an electric dipole? $\vec{\tau} = \vec{p} \times \vec{E}$
  - The potential energy of the electric dipole is
  - $U = -\vec{p} \cdot \vec{E}$
- How about the potential energy of a magnetic dipole?
  - The work done by the torque is
  - $U = \int \tau d\theta = \int NIAB \sin\theta \, d\theta = -\mu B \cos\theta + C$
  - If we chose U=0 at θ=π/2, then C=0
  - Thus the potential energy is $U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$
    - Very similar to the electric dipole

Example 27 – 12

Magnetic moment of a hydrogen atom. Determine the magnetic dipole moment of the electron orbiting the proton of a hydrogen atom, assuming (in the Bohr model) it is in its ground state with a circular orbit of radius 0.529×10^{-10}m.

What provides the centripetal force? The Coulomb force

So we can obtain the speed of the electron from

$$F=\frac{e^2}{4\pi\varepsilon_0 r^2}=\frac{m_e v^2}{r}$$

Solving for v

$$v=\sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e r}}=\sqrt{\frac{\left(8.99\times10^9\,N\cdot m^2/C^2\right)\cdot\left(1.6\times10^{-19}\,C\right)^2}{\left(9.1\times10^{-31}\,kg\right)\cdot\left(0.529\times10^{-10}\,m\right)}}=2.19\times10^6\,m/s$$

Since the electric current is the charge that passes through the given point per unit time, we can obtain the current

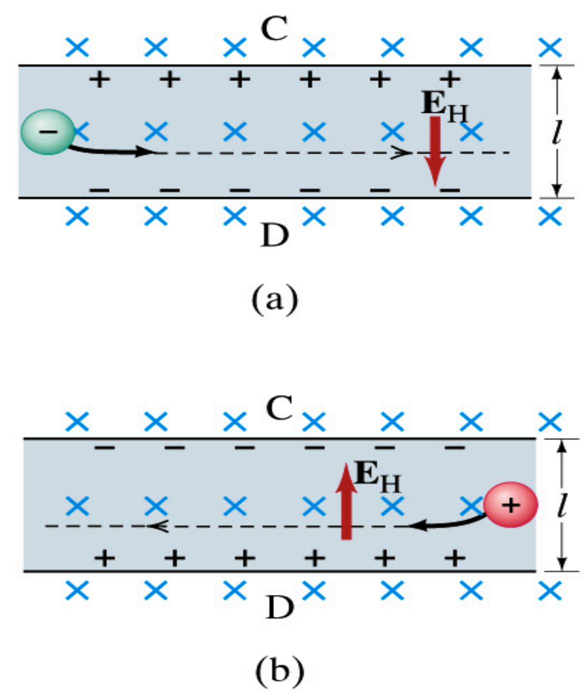
$$I=\frac{e}{T}=\frac{ev}{2\pi r}$$

Since the area of the orbit is $A=\pi r^2$, we obtain the hydrogen magnetic moment

$$\mu=IA=\frac{ev}{2\pi r}\pi r^2=\frac{evr}{2}=\frac{er}{2}\sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e r}}=\frac{e^2}{4}\sqrt{\frac{r}{\pi\varepsilon_0 m_e}}$$

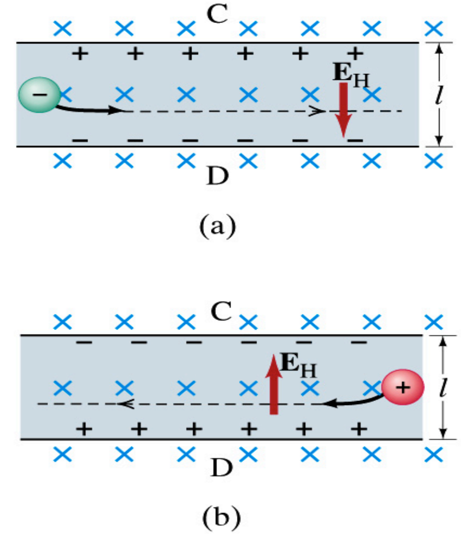
The Hall Effect

- What do you think will happen to the electrons flowing through a conductor immersed in a magnetic field?
  - Magnetic force will push the electrons toward one side of the conductor. Then what happens?
    - $\vec{F}_B = -e\vec{v}_d \times \vec{B}$
  - A potential difference will be created due to continued accumulation of electrons on one side. Till when? Forever?
  - Nope. Till the electric force inside the conductor is equal and opposite to the magnetic force
- This is called the **Hall Effect**
  - The potential difference produced is called
    - The Hall emf
  - The electric field due to the separation of the charge is called the Hall field, $\mathbf{E}_H$, and it points to the direction opposite to the magnetic force



The Hall Effect

- In the equilibrium, the force due to Hall field is balanced by the magnetic force $ev_d B$, so we obtain



- $eE_H = ev_d B$ and $E_H = v_d B$
- The Hall emf is then $\varepsilon_H = E_H l = v_d Bl$
  - Where l is the width of the conductor
- What do we use the Hall effect for?
  - The current of negative charge moving to right is equivalent to the positive charge moving to the left
  - The Hall effect can distinguish these since the direction of the Hall field or direction of the Hall emf is opposite
  - Since the magnitude of the Hall emf is proportional to the magnetic field strength ➔ can measure the B-field strength
    - Hall probe

Sources of Magnetic Field

- We have learned so far about the effects of magnetic field on electric current and a moving charge
- We will now learn about the dynamics of magnetism
  - How do we determine magnetic field strengths in certain situations?
  - How do two wires with electric current interact?
  - What is the general approach to finding the connection between the electric current and the magnetic field?

Magnetic Field due to a Straight Wire

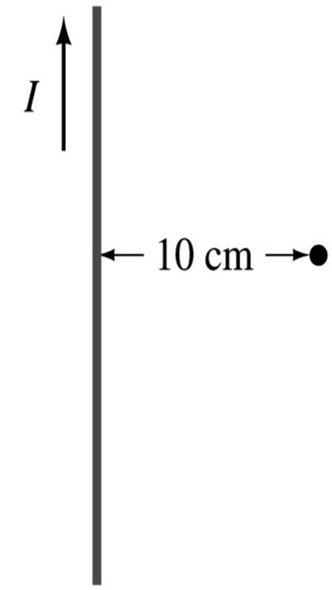
- The magnetic field due to the current flowing through a straight wire forms a circular pattern around the wire
  - What do you imagine the strength of the field is as a function of the distance from the wire?
    - It must be weaker as the distance increases
  - How about as a function of the electric current?
    - Directly proportional to the current
  - Indeed, the above are experimentally verified $B \propto \frac{I}{r}$
    - This is valid as long as r << the length of the wire
  - The proportionality constant is $\mu_0/2\pi$, thus the field strength becomes

$$B = \frac{\mu_0 I}{2\pi r}$$

  - μ_0 is the permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$

Merriam-Webster: Permeability is the property of a magnetizable substance that determines the degree in which it modifies the magnetic flux in the region occupied by it in a magnetic field

Example 28 – 1



Calculation of B near wire. A vertical electric wire in the wall of a building carries a DC current of 25A upward. What is the magnetic field at a point 10cm due East of this wire?

Using the formula for the magnetic field near a straight wire

$$B = \frac{\mu_0 I}{2\pi r}$$

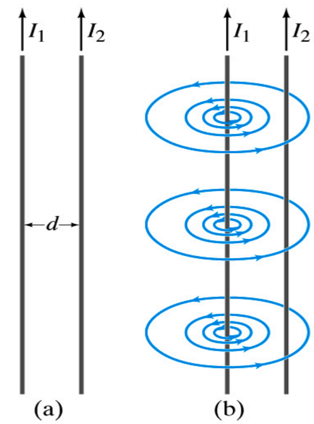
So we can obtain the magnitude of the magnetic field at 10cm away as

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ T \cdot m/A\right) \cdot (25A)}{(2\pi) \cdot (0.01m)} = 5.0 \times 10^{-5}\ T$$

Whaich direction? Into the page!

Force Between Two Parallel Wires

- We have learned that a wire carrying electric current produces magnetic field
- Now what do you think will happen if we place two current carrying wires next to each other?
  - They will exert force onto each other. Repel or attract?
  - Depending on the direction of the currents

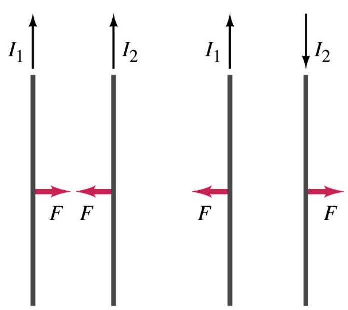


- This was first pointed out by Ampére.
- Let's consider two long parallel conductors separated by a distance **d**, carrying currents I_1 and I_2.
- At the location of the second conductor, the magnitude of the magnetic field produced by I_1 is

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Force Between Two Parallel Wires

- The force **F** by a magnetic field $\mathbf{B}_1$ on a wire of length l, carrying the current I_2 when the field and the current are perpendicular to each other is: $F = I_2 B_1 l$
  - So the force per unit length is $$\frac{F}{l} = I_2 B_1 = I_2 \frac{\mu_0}{2\pi}\frac{I_1}{d}$$
  - This force is only due to the magnetic field generated by the wire carrying the current I_1
    - There is the force exerted on the wire carrying the current I_1 by the wire carrying current I_2 of the same magnitude but in the opposite direction
- So the force per unit length is $$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$
- How about the direction of the force?

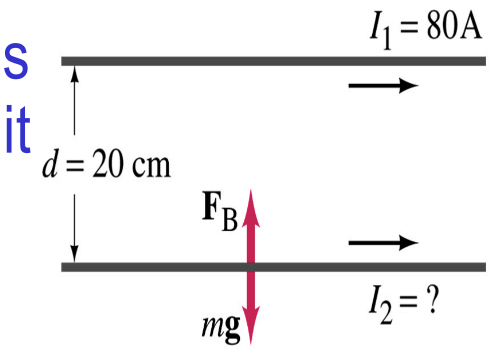


(a) (b)

If the currents are in the same direction, the attractive force. If opposite, repulsive.

Example 28 – 5

Suspending a wire with current. A horizontal wire carries a current I_1=80A DC. A second parallel wire 20cm below it must carry how much current I_2 so that it doesn't fall due to the gravity and in which direction? The lower has a mass of 0.12g per meter of length.



Which direction is the gravitational force? Down to the center of the Earth

This force must be balanced by the magnetic force exerted on the wire by the first wire.

$$\frac{F_g}{l} = \frac{mg}{l} = \frac{F_M}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

Solving for I_2

$$I_2 = \frac{mg}{l}\frac{2\pi d}{\mu_0 I_1} = \frac{2\pi\left(9.8\, m/s^2\right)\cdot\left(0.12\times 10^{-3}\, kg\right)\cdot\left(0.20m\right)}{\left(4\pi\times 10^{-7}\; T\cdot m/A\right)\cdot\left(80A\right)} = 15A$$

Operational Definition of Ampere and Coulomb

- The permeability of free space is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$$

- The unit of current, ampere, is defined using the definition of the force between two wires, each carrying **1A** of current and separated by **1m**

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{4\pi \times 10^{-7}\ T \cdot m/A}{2\pi}\frac{1A \cdot 1A}{1m} = 2 \times 10^{-7}\ N/m$$

  – So 1A is defined as: the current flowing each of two long parallel conductors 1m apart, which results in a force of exactly $2\text{x}10^{-7}$N/m.

- Coulomb is then defined as exactly 1C=1A s.
- We do it this way since the electric current is measured more accurately and controlled more easily than the charge.