PHYS 1444 – Section 002
Lecture #19

Monday, Apr. 20, 2020
Dr. Jaehoon Yu

CH 28:Sources of Magnetic Field

- Ampére's Law and Its Verification
- Solenoid and Toroidal Magnetic Field
- Magnetic Materials
- B in magnetic materials
- Hysteresis

- Chapter 29:EM Induction & Faraday's Law
  - Induced EMF and EM Induction

Today's homework is homework #12, due 11pm, Monday, May 4!!

Announcements

- Reading Assignments: 28.6 – 10, CH29.5 and 29.8
- Term 2 results
  - Class average: 61.8/104
    - Equivalent to 60.2/100
    - Previous exams: 59.7 and 54.5
  - Top score: 104/104
- Pass/fail grading choice
  - You can elect pass/fail for grades A – D by May 22
    - Pre-requisite requirements must be checked with your advisor
  - No change in drop date, still Apr. 28
- Final exam date: In class, Wed. May 6

Special Project #5 – COVID-19

- Make comparisons of COVID-19 statistics between the U.S., South Korea and Germany from https://coronaboard.com on spreadsheet
  - Total 27 points: 1 point for each of the top 15 cells and 2 points for each of the 6 cells for testing
- Make a timeline from Jan. 15, 2020 through Apr. 15 for The World Health Organization (WHO), U.S.A. and South Korea in their actions to COVID-19: One in Jan., one in Apr. and two each in Feb. and Mar.
  - 2 points each, totaling 30 points
- What are the 3 most fundamental requirements for opening back up (2 points each, total 6 points)? Identify the two of these U.S. is ready (1 point each, total 2 points; do NOT just take politician's words!) for opening.
- Due: 1pm Monday, May 4
  - Scan all pages of your special project into the pdf format
  - Save all pages into one file with the filename SP5-YourLastName-YourFirstName.pdf
- Spreadsheet has been posted on the class web page. Download ASAP.

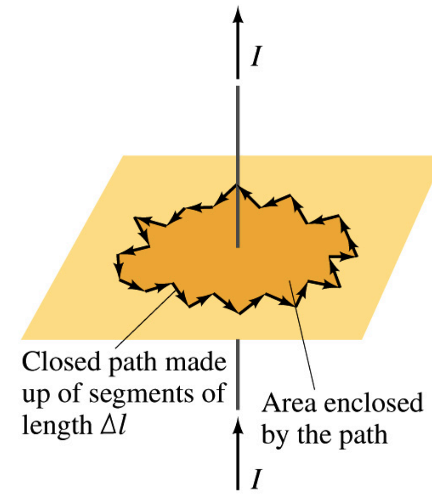
	Name:		Date of COVID-19 Data:	
Items		**U.S.A**	**South Korea**	**Germany**
Total Population				
COVID-19 Confirmed cases	Number			
	Cases per 1M people			
COVID-19 Deaths	Number			
	Death per 1M people			
COVID-19 Testing to date	Number			
	Per 1M people			

Ampére's Law

- What is the relationship between the magnetic field strength and the current? $B = \frac{\mu_0 I}{2\pi r}$
  - Does this work in all cases?
    - Nope!
    - OK, then when?
    - Only valid for a long straight wire (r<<l)
- Then what would be the more generalized relationship between the current and the magnetic field for any shapes of the wire?
  - French scientist André Marie Ampére proposed such a relationship soon after Oersted's discovery

Ampére's Law

- Let's consider an arbitrary closed path around the current as shown in the figure.
  - Let's cut this path in small segments, each Δl long.
  - The sum of all the products of the length of each segment and the component of B parallel to that segment is equal to μ_0 times the net current I_{encl} that passes through the surface enclosed by the path
  - $$\sum B_{||}\Delta l = \mu_0 I_{encl}$$
  - In the limit $\Delta l \rightarrow 0$, this relation becomes
  - $$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

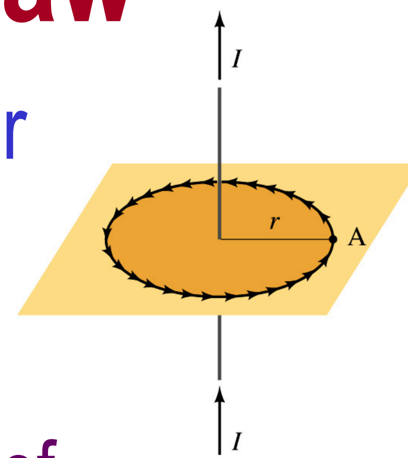


Ampére's Law

Looks very similar to a law in the electricity. Which law is it?

Gauss' Law

Verification of Ampére's Law



- Let's find the magnitude of B at a distance r away from a long straight wire w/ current I
  - This is a verification of Ampere's Law
  - We can apply Ampere's law to a circular path of radius r.

$$\mu_0 I_{encl} = \oint \vec{B} \cdot d\vec{l} = \oint B\,dl = B\oint dl = 2\pi r B$$

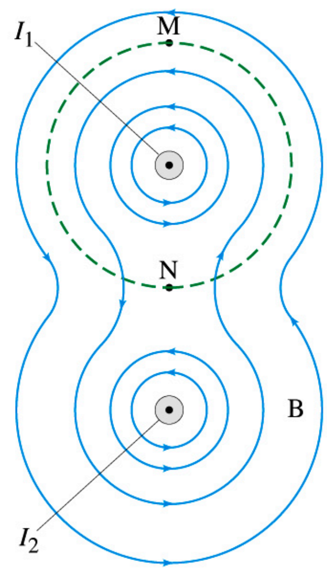
Solving for B

$$B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0}{2\pi}\frac{I}{r}$$

  - We just verified that Ampere's law works in a simple case
  - Experiments verified that it works for other cases too
  - The importance of this formula, however, is that it provides means to relate magnetic field to an electric current

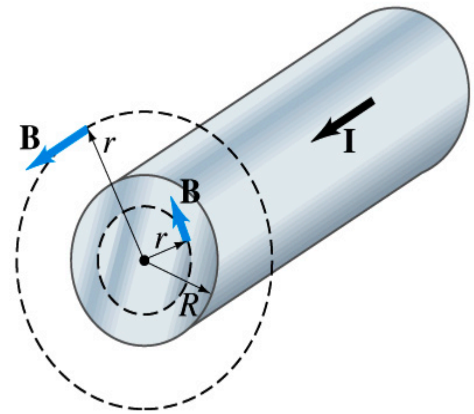
Verification of Ampére's Law

- Since Ampere's law is valid in general, B in Ampere's law is not just due to the current I_{encl}.
- B is the field at each point in space along the chosen path due to all sources
  - Including the current I enclosed by the path but also due to any other sources
  - How do you obtain B in the figure at any point?
    - Vector sum of the field by the two currents
  - The result of the closed path integral in Ampere's law for green dashed path is still $\mu_0 I_1$. Why?
  - While B in each point along the path varies, the integral over the closed path still comes out the same whether there is the second wire or not.



Example 28 – 6

Field inside and outside a wire. A long straight cylindrical wire conductor of radius R carries current I of uniform density in the conductor. Determine the magnetic field at (a) points outside the conductor (r>R) and (b) points inside the conductor (r<R). Assume that r, the radial distance from the axis, is much less than the length of the wire. (c) If R=2.0mm and I=60A, what is B at r=1.0mm, r=2.0mm and r=3.0mm?



Since the wire is long, straight and symmetric, the field should be the same at any point the same distance from the center of the wire.

Since B must be tangential to circles around the wire, let's choose a circular path of the closed-path integral outside the wire (r>R). What is I_{encl}? $I_{encl} = I$

So using Ampere's law

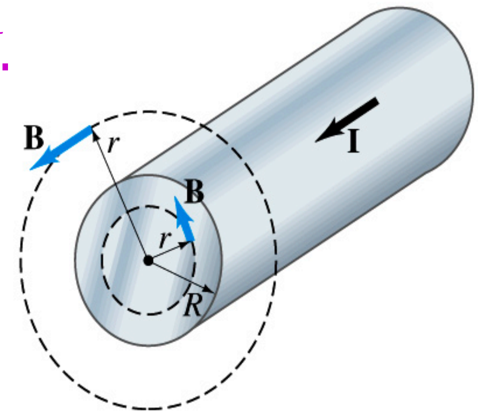
$$\mu_0 I = \oint \vec{B} \cdot d\vec{l} = 2\pi r B$$

Solving for B

$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

Example 28 – 6 cont'd

For r<R, the current inside the closed path is less than I. How much is it?



$$I_{encl} = I\frac{\pi r^2}{\pi R^2} = I\left(\frac{r}{R}\right)^2$$

So using Ampere's law

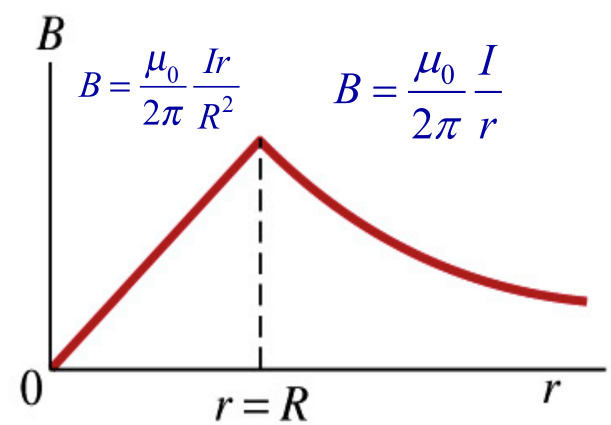
$$\mu_0 I\left(\frac{r}{R}\right)^2 = \oint \vec{B}\cdot d\vec{l} = 2\pi r B$$

Solving for B

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}\left(\frac{r}{R}\right)^2 = \frac{\mu_0}{2\pi}\frac{Ir}{R^2}$$

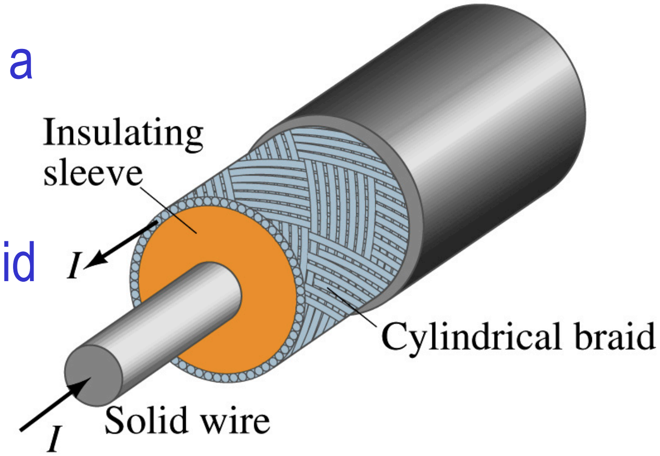
What does this mean?

The field is 0 at r=0 and increases linearly as a function of the distance from the center of the wire up to r=R then decreases as 1/r beyond the radius of the conductor.



Example 28 – 7

Coaxial cable. A coaxial cable is a single wire surrounded by a cylindrical metallic braid, as shown in the figure. The two conductors are separated by an insulator. The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground. Describe the magnetic field (a) in the space between the conductors and (b) outside the cable.



(a) The magnetic field between the conductors is the same as the long, straight wire case since the current in the outer conductor does not impact the enclosed current.

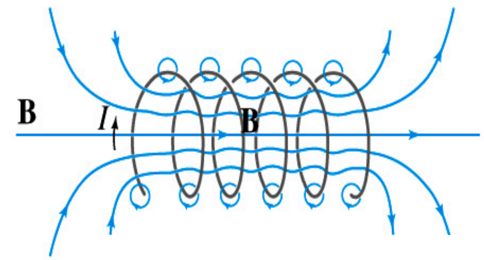
$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

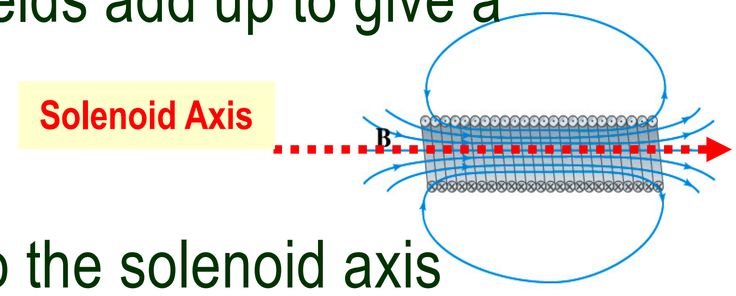
(b) Outside the cable, we can draw a similar circular path, since we expect the field to have a circular symmetry. What is the sum of the total current inside the closed path? $I_{encl} = I - I = 0.$

So there is no magnetic field outside a coaxial cable. In other words, the coaxial cable self-shields. The outer conductor also shields against an external electric field. Cleaner signal and less noise.

Solenoid and Its Magnetic Field

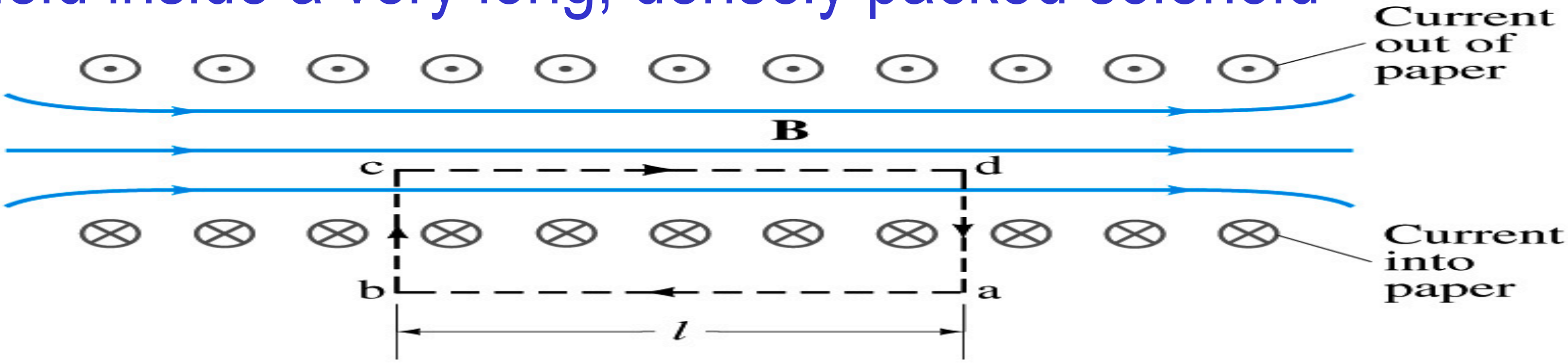
- What is a solenoid?
  - A long coil of wire consisting of many loops
  - If the space between loops are wide
    - The field near the wires are nearly circular
    - Between any two wires, the fields due to each loop cancel
    - Toward the center of the solenoid, the fields add up to give a field that can be fairly large and uniform
  - For a long, densely packed loops
    - The field is nearly uniform and parallel to the solenoid axis within the entire cross section
    - The field outside the solenoid is very small compared to the field inside, except at the ends
      - The same number of field lines spread out to an open space





Solenoid Magnetic Field

- Now let's use Ampere's law to determine the magnetic field inside a very long, densely packed solenoid

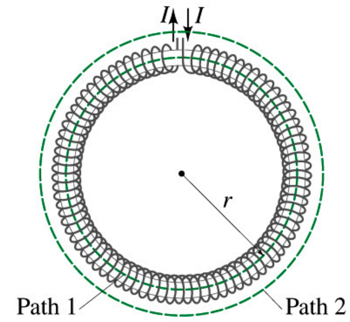


- Let's choose the path *abcd*, far away from the ends
  - We can consider four segments of the loop for integral
  - $\oint \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$
  - Since the field outside the solenoid is negligible, the integral on *a*→ *b* is 0.
  - Now the field B is perpendicular to the *bc* and *da* segments. So these integrals become 0, also.

Solenoid Magnetic Field

- Therefore, the sum becomes: $\oint \vec{B} \cdot d\vec{l} = \int_c^d \vec{B} \cdot d\vec{l} = Bl$
- If the current I flows in the wire of the solenoid, the total current enclosed by the closed path is $\mathcal{N}I$
  - Where $\mathcal{N}$ is the number of loops (or turns of the coil) enclosed
- Thus Ampere's law gives us $Bl = \mu_0 NI$
- If we let $n=\mathcal{N}/\ell$ be the number of loops per unit length, the magnitude of the magnetic field within the solenoid becomes
- $$B = \mu_0 nI$$
  - B depends on the number of loops per unit length, n, and the current I
    - B does not depend on the position within the solenoid but uniform inside it, like a bar magnet

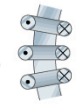
Example 28 – 10



(a)

Toroid. Use Ampere's law to determine the magnetic field (a) inside and (b) outside a toroid, which is like a solenoid bent into the shape of a circle.

(a) What do you think the magnetic field lines inside the toroid look?


(b)

Since it is a bent solenoid, it should be a circle concentric with the toroid.

If we choose the path of integration one of these field lines of radius r inside the toroid, path 1, to use the symmetry of the situation, making B the same at all points on the path, we obtain from Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r) = \mu_0 I_{encl} = \mu_0 NI$$

Solving for B

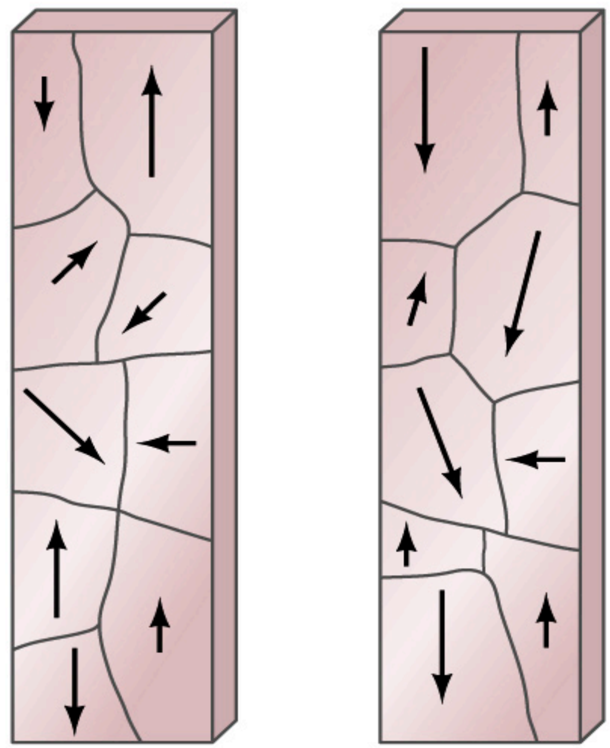
$$B = \frac{\mu_0 NI}{2\pi r}$$

So the magnetic field inside a toroid is not uniform. It is larger on the inner edge. However, the field will be uniform if the radius is large and the toroid is thin. The field in this case is B = $\mu_0 n I$.

(b) Outside the solenoid, the field is 0 since the net enclosed current is 0.

Magnetic Materials - Ferromagnetism

- Iron is a material that can turn into a strong magnet
  - This kind of material is called the **ferromagnetic** material
- In microscopic sense, ferromagnetic materials consist of many tiny regions called **domains**
  - Domains are like little magnets usually smaller than 1mm in length or width
- What do you think the alignment of domains are like when they are not magnetized?
  - Randomly arranged
- What if they are magnetized?
  - The size of the domains aligned with the external magnetic field direction grows while those of the domains not aligned reduce
  - This gives magnetization to the material
- How do we demagnetize a bar magnet?
  - Hit the magnet hard or heat it over the Curie temperature



(b)

B in Magnetic Materials

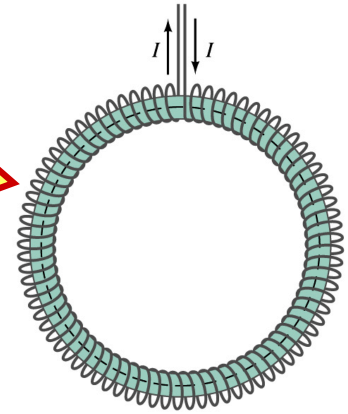
- What is the magnetic field inside a solenoid?
- $$B_0 = \mu_0 n I$$
  - Magnetic field in a long solenoid is directly proportional to the current.
  - This is valid only if air is inside the coil
- What do you think will happen to B if we have something other than the air inside the solenoid?
  - B will be increased dramatically, when the current flows
    - Especially if a ferromagnetic material such as an iron is put inside, the field could increase by several orders of magnitude
- Why?
  - Since the domains in the iron aligns permanently by the external field.
  - The resulting magnetic field is the sum of that due to current and due to the iron

B in Magnetic Materials

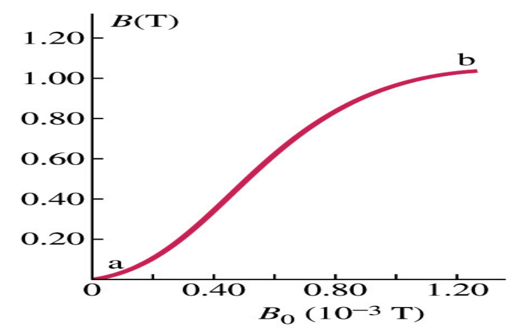
- It is sometimes convenient to write the total field as the sum of two terms
- $$\vec{B} = \vec{B}_0 + \vec{B}_M$$
  - $\mathbf{B_0}$ is the field due only to the current in the wire, namely the external field
    - The field that would be present without a ferromagnetic material
  - $\mathbf{B_M}$ is the additional field due to the ferromagnetic material itself; often $\mathbf{B_M} >> \mathbf{B_0}$
- The total field in this case can be written by replacing μ_0 with another proportionality constant μ, the magnetic permeability of the material $B = \mu n I$
  - μ is a property of the magnetic material
  - μ is not a constant but varies with the external field

Hysteresis

Iron Core Toroid

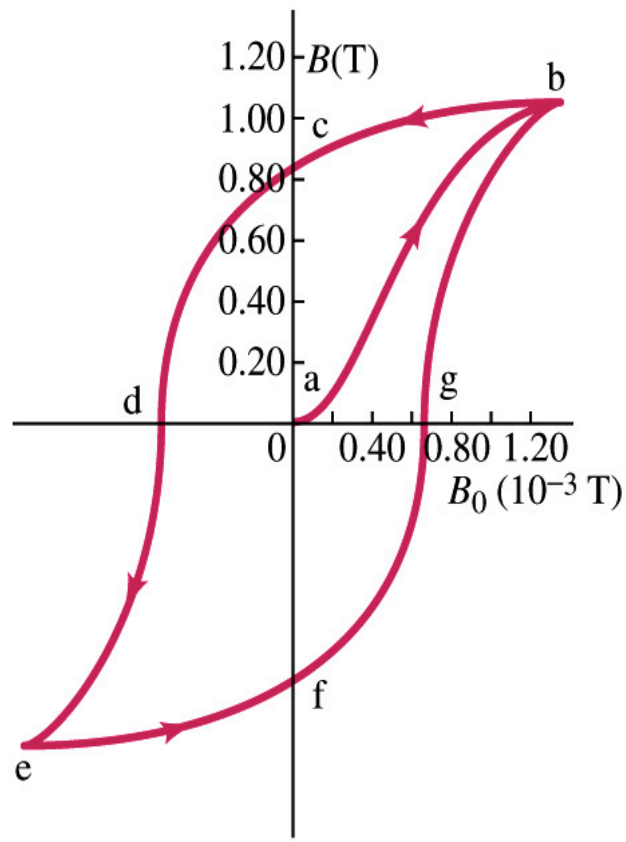


- What is a Toroid?
  - A solenoid bent into a shape
- Toroid can be used for magnetic field measurement
  - Why?
  - Since it does not leak magnetic field outside of itself, it fully contains all the magnetic field created within it.
- Consider an un-magnetized iron core Toroid, without any current flowing in the wire
  - What do you think will happen if the current slowly increases?
  - B_0 increases linearly with the current.
  - And B increases also but follows the curved line shown in the graph
  - As B_0 increases, the domains become more aligned until nearly all are aligned (point b on the graph)
    - The iron is said to be approaching saturation
    - Point b is typically at 70% of the max



Hysteresis

- What do you think will happen to B if the external field B_0 is reduced to 0 by decreasing the current in the coil?
  – Of course it goes to 0!!
  – Wrong! Wrong! Wrong! They do not go to 0. Why not?
  – The domains do not completely return to random alignment state
- Now if the current direction is reversed, the external magnetic field direction is reversed, causing the total field B passed 0, and the direction reverses to the opposite side
  – If the current is reversed again, the total field B will increase but never goes through the origin
- This kind of curve whose path does not retrace themselves and does not go through the origin is called the **Hysteresis**.



Magnetically Soft Material

- In a hysteresis cycle, much energy is transformed to thermal energy. Why?
  - Due to the microscopic friction between domains as they change directions to align with the external field
- The energy dissipated in the hysteresis cycle is proportional to the area of the hysteresis loop
- Ferromagnetic material with a large hysteresis area is called magnetically hard while the small ones are called soft
  - Which one do you think are preferred in electromagnets or transformers?
    - Soft. Why?
    - Since the energy loss is small and much easier to switch off the field
- Then how do we demagnetize a ferromagnetic material?
  - Keep repeating the Hysteresis loop, reducing the range of B_0.