PHYS 1444 – Section 002
Lecture #23

Monday, May 4, 2020
Dr. Jaehoon Yu

CH30: Inductance

- Self Inductance
- Energy Stored in the Magnetic Field

CH31: Maxwell's equations

- Expansion of Ampere's Law
- Gauss' Law for Magnetism
- Production of EM Waves

Announcements – I

- Final comprehensive exam: 1:00 – 2:20pm this Wed. May 6
  - Do NOT miss the exam! **Must be in a quiet place to take the exam!**
    - You will get an F no matter how well you've been doing!!
  - Comprehensive: Covers CH21.1 – CH31-5 + Math Refresher
  - Online exam based on Quest but **must join zoom class by 12:55pm**!
  - You can use your calculator but DO NOT input formula into it!
    - Cell phones or any types of computers cannot replace a calculator!
    - POWER OFF your phones!!
  - BYOF: You may prepare a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants
    - No derivations, plots, pictures, word definitions or solutions or setups of any problems!
      - No names of the Maxwell's equations
  - Please send me the photos of your formula sheet by 12:00pm
    - If you don't have one, still send me email that you do not have one prepared!
  - Let's be fair to other students and not cheat!

Announcements – II

- Reading assignments: CH30.7 – 30.11 & CH31.5 – 10
- Planetarium Extra Credit: Due 1pm, May 6
  - Send me the photos of the sheet with the front of the ticket stubs and of the sheet with them flipped over, showing the back of them
  - Email subject line must be: SP-Planetarium
- Be sure to submit the course feedback survey ASAP!

Reminder: Special Project #6

- Special project #6: Fill out the survey at
  - https://s.surveyplanet.com/mw8bpHPyb
  - 15 points total for 7 questions
  - Deadline: End of the day, Wednesday, May 6

Self Inductance

- The concept of the inductance applies to a single isolated coil of N turns. How does this happen?
  - When a changing current passes through a coil
  - A changing magnetic flux is produced inside the coil
  - The changing magnetic flux in turn induces an emf in the same coil
  - This emf opposes the change in flux. Whose law is this? (Poll 14)
    - Lenz's law
- What would this do?
  - When the current through the coil is increasing?
    - The increasing magnetic flux induces an emf that opposes the original current
    - This tends to **impedes its increase**, trying to maintain the original current
  - When the current through the coil is decreasing?
    - The decreasing flux induces an emf in the same direction as the current
    - This tends to increase the flux, trying to maintain the original current

Self Inductance

- Since the magnetic flux Φ_B passing through N turn coil is proportional to current I in the coil, $N\Phi_B = LI$
- We define self-inductance, $\mathcal{L}$: $L = \frac{N\Phi_B}{I}$ **Self Inductance**
- The induced emf in a coil of self-inductance $\mathcal{L}$ is
  - $\varepsilon = -N\frac{d\Phi_B}{dt} = -\frac{d\left(N\Phi_B\right)}{dt} = -\frac{d\left(LI\right)}{dt} = -L\frac{dI}{dt}$
  - What is the unit for self-inductance? $1H = 1V \cdot s/A = 1\Omega \cdot s$
- What does magnitude of $\mathcal{L}$ depend on?
  - Geometry and the presence of a ferromagnetic material
- Self inductance can be defined for any circuit or part of a circuit

So what in the world is the Inductance?

- It is an **impediment** onto the electrical current due to the existence of changing magnetic flux
- So what?
- In other words, it **is like the resistance** to the varying current, such as AC, that causes the constant change of magnetic flux due to inductance
- But it also provides means to store energy, just like the capacitance

Inductor

- An electrical circuit always contains some inductance but is normally negligibly small
  - If a circuit contains a coil of many turns, it could have large inductance
- A coil that has significant inductance, $\mathcal{L}$, is called **an inductor** and is expressed with the symbol 
  - Precision resistors are normally wire wound
    - Would have both resistance and inductance
    - The inductance can be minimized by winding the wire back on itself in opposite direction to cancel magnetic flux
    - This is called a "non-inductive winding"
- If an inductor has negligible resistance, inductance controls the changing current
- For an AC current, the greater the inductance the less the AC current
  - An inductor thus acts like a resistor to impede the flow of alternating current (not to DC, though. Why? Poll 15)
  - The quality of an inductor is indicated by the term **reactance** or **impedance**

Example 30 – 3

Solenoid inductance. (a) Determine the formula for the self inductance $\mathcal{L}$ of a tightly wrapped solenoid (a long coil) containing **N** turns of wire in its length ℓ and whose cross-sectional area is **A**. (b) Calculate the value of $\mathcal{L}$ if **N=100**, **ℓ=5.0cm**, **A=0.30cm^2** and the solenoid is air filled. (c) calculate $\mathcal{L}$ if the solenoid has an iron core with **$\mu=4000\mu_0$**.

What is the magnetic field inside a solenoid? $B=\mu_0 nI=\mu_0 NI/l$

The flux is, therefore, $\Phi_B=BA=\mu_0 NIA/l$

Using the formula for self inductance: $L=\frac{N\Phi_B}{I}=\frac{N\cdot\mu_0 NIA/l}{I}=\frac{\mu_0 N^2 A}{l}$

(b) Using the formula above

$$L=\frac{\mu_0 N^2 A}{l}=\frac{\left(4\pi\times10^{-7}\,T\cdot m/A\right)100^2\left(0.30\times10^{-4}\,m^2\right)}{5.0\times10^{-2}\,m}=7.5\mu H$$

(c) The magnetic field with an iron core solenoid is $B=\mu NI/l$

$$L=\frac{\mu N^2 A}{l}=\frac{4000\left(4\pi\times10^{-7}\,T\cdot m/A\right)100^2\left(0.30\times10^{-4}\,m^2\right)}{5.0\times10^{-2}\,m}=0.030H=30mH$$

Energy Stored in the Magnetic Field

- The work done onto the system is the same as the energy stored in the inductor when it is carrying current I
  - $U = \frac{1}{2} L I^2$

    Energy Stored in a magnetic field inside an inductor
  - This is compared to the energy stored in a capacitor, C, when the potential difference across it is V: $U = \frac{1}{2} C V^2$
  - Just like the energy stored in a capacitor is considered to reside in the electric field between its plates
  - The energy in an inductor can be considered to be stored in its magnetic field

Stored Energy in terms of B

- So how is the stored energy written in terms of magnetic field B?
  - Inductance of an ideal solenoid without the fringe effect

$$L = \mu_0 N^2 A/l$$

  - The magnetic field in a solenoid is $B = \mu_0 NI/l$
  - Thus the energy stored in an inductor is

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{l}\left(\frac{Bl}{\mu_0 N}\right)^2 = \frac{1}{2}\frac{B^2}{\mu_0}Al$$

What is this?

$$U = \frac{1}{2}\frac{B^2}{\mu_0}Al$$

E

  - Thus the energy density is

$$u = \frac{U}{V} = \frac{U}{Al} = \frac{1}{2}\frac{B^2}{\mu_0}$$

Volume V

$$u = \frac{1}{2}\frac{B^2}{\mu_0}$$

E density

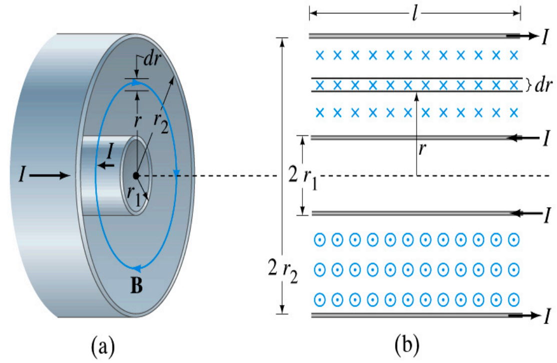
  - This formula is valid in any region of space
  - If a ferromagnetic material is present, μ_0 becomes μ.

What volume does Al represent?

The volume inside a solenoid!!

Example 30 – 5

Energy stored in a coaxial cable. (a) How much energy is being stored per unit length in a coaxial cable whose conductors have radii r_1 and r_2 and which carry a current I? (b) Where is the energy density highest?



(a) The total flux through l of the cable is $\Phi_B = \int B l\, dr = \frac{\mu_0 I l}{2\pi} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0 I l}{2\pi} \ln \frac{r_2}{r_1}$

Thus inductance per unit length for a coaxial cable is $\frac{L}{l} = \frac{\mu_0}{2\pi} \ln \frac{r_2}{r_1}$

Thus the energy stored per unit length is

$$\frac{U}{l} = \frac{1}{2} \frac{L I^2}{l} = \frac{\mu_0 I^2}{4\pi} \ln \frac{r_2}{r_1}$$

(b) Since the magnetic field is $B = \frac{\mu_0 I}{2\pi r}$

And the energy density is $u = \frac{1}{2} \frac{B^2}{\mu_0}$

The energy density is highest where B is highest. Since B is highest close to $r=r_1$, near the surface of the inner conductor.

Maxwell's Equations – I

- The development of EM theory by Oersted, Ampere and others was not done in terms of EM fields
  - The idea of fields was introduced somewhat by Faraday
- Scottish physicist James C. Maxwell unified all the phenomena of electricity and magnetism in one theory with only four equations (**Maxwell's Equations**) using the concept of fields
  - This theory provided the prediction of EM waves
  - As important as Newton's law, since it provides the dynamics of electromagnetism
  - This theory is also in agreement with Einstein's special relativity

Maxwell's Equations – II

- The biggest achievement of 19th century electromagnetic theory is the prediction and experimental verifications that the EM waves can travel through the empty space
  - What do you think this accomplishment did?
    - Open a new world of communication
    - It also yielded the prediction that the light is an EM wave
- Since all of Electromagnetism is contained in the four Maxwell's equations, this is considered as one of the greatest achievements of human intellect

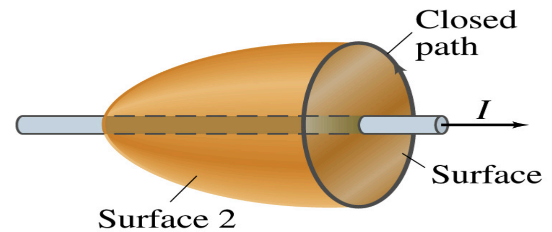
Ampere's Law

- Do you remember the mathematical expression of Oersted discovery of a magnetic field produced by an electric current, given by Ampere? (Poll 16)

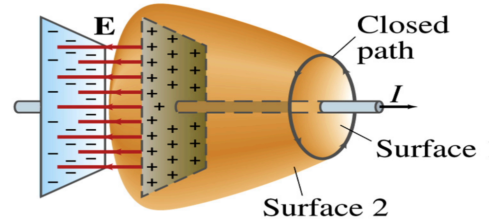
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- We've learned that a varying magnetic field produces an electric field (Poll 17)
- Then can the reverse phenomena, that a changing electric field producing a magnetic field, possible?
  - If this is the case, it would demonstrate a beautiful symmetry in nature between electricity and magnetism

Expanding Ampere's Law



- Let's consider a wire carrying current I
  - The current that is enclosed in the loop passes through the surface # 1 in the figure
  - We could imagine a different surface # 2 that shares the same enclosed path but cuts through the wire in a different location. What is the current that passes through this surface?
    - Still I.
  - So the Ampere's law still works



- We could then consider a capacitor being charged up or being discharged.
  - The current I is enclosed in the loop passes through the surface #1
  - However the surface #2 that shares the same closed loop do not have any current passing through it.
    - There, however, is a magnetic field present since there is a current ➔ In other words, there is a changing electric field in between the plates
    - Maxwell resolved this by adding an additional term to Ampere's law involving the changing electric field

Modifying Ampere's Law

- To determine what the extra term should be, we first have to figure out what the electric field between the two plates is
  - The charge Q on the capacitor with capacitance C is Q=CV
    - Where V is the potential difference between the plates
  - Since V=Ed
    - Where E is the uniform field between the plates, and d is the separation of the plates
  - And for parallel plate capacitor $C=\varepsilon_0 A/d$
  - We obtain

$$Q = CV = \left(\varepsilon_0 \frac{A}{d}\right) Ed = \varepsilon_0 AE$$

Modifying Ampere's Law

- If the charge on the plate changes with time, we can write

$$\frac{dQ}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

- Using the relationship between the current and charge we obtain

$$I = \frac{dQ}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{d(AE)}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$$

  - Where Φ_E=EA is the electric flux through the surface between the plates
- So in order to make Ampere's law work for the surface 2 in the figure, we must write it in the following form

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Extra term by Maxwell

- This equation represents the general form of Ampere's law
  - This means that a magnetic field can be caused not only by an ordinary electric current but also by a changing electric flux

Example 31 – 1

Charging capacitor. A 30-pF air-gap capacitor has circular plates of area A=100cm^2. It is charged by a 70-V battery through a 2.0-Ω resistor. At the instance the battery is connected, the electric field between the plates is changing most rapidly. At this instance, calculate (a) the current into the plates, and (b) the rate of change of electric field between the plates. (c) Determine the magnetic field induced between the plates. Assume **E** is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

Since this is an RC circuit, the charge on the plates is: $Q = CV_0\left(1 - e^{-t/RC}\right)$

For the initial current (t=0), we differentiate the charge with respect to time.

$$I_0 = \left.\frac{dQ}{dt}\right|_{t=0} = \left.\frac{CV_0}{RC}e^{-t/RC}\right|_{t=0} = \frac{V_0}{R} = \frac{70V}{2.0\Omega\cdot} = 35A$$

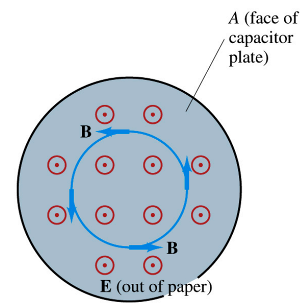
The electric field is $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q/A}{\varepsilon_0}$

Change of the electric field is $\dfrac{dE}{dt} = \dfrac{dQ/dt}{A\varepsilon_0} = \dfrac{35A}{\left(8.85\times10^{-12}\,C^2/N\cdot m^2\right)\cdot\left(1.0\times10^{-2}\,m^2\right)} = 4.0\times10^{14}\,V/m\cdot s$

Example 31 – 1

(c) Determine the magnetic field induced between the plates. Assume **E** is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

The magnetic field lines generated by changing electric field is perpendicular to E and is circular due to symmetry



Whose law can we use to determine B?

Extended Ampere's Law w/ I_{encl}=0!

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

We choose a circular path of radius r, centered at the center of the plane, following the B.

For $r<r_{plate}$, the electric flux is $\Phi_E = EA = E\pi r^2$ since E is uniform throughout the plate

So from Ampere's law, we obtain $$B\cdot(2\pi r) = \mu_0\varepsilon_0 \frac{d\left(E\pi r^2\right)}{dt} = \mu_0\varepsilon_0\pi r^2\frac{dE}{dt}$$

Solving for B

$$B = \mu_0\varepsilon_0 \frac{r}{2}\frac{dE}{dt}$$

For $r<r_{plate}$

Since we assume E=0 for $r>r_{plate}$, the electric flux beyond the plate is fully contained inside the surface.

$$\Phi_E = EA = E\pi r_{plate}^2$$

So from Ampere's law, we obtain $$B\cdot(2\pi r) = \mu_0\varepsilon_0 \frac{d\left(E\pi r_{plate}^2\right)}{dt} = \mu_0\varepsilon_0\pi r_{plate}^2\frac{dE}{dt}$$

Solving for B

$$B = \frac{\mu_0\varepsilon_0 r_{plate}^2}{2r}\frac{dE}{dt}$$

For $r>r_{plate}$

Displacement Current

- Maxwell interpreted the second term in the generalized Ampere's law equivalent to an electric current
  - He called this term as **the displacement current, I_D**
  - While the other term is called as **the conduction current, I**
- Ampere's law then can be written as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_{encl} + I_D \right)$$

  - Where

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

  - While it is in effect equivalent to an electric current, a flow of electric charge, this actually does not have anything to do with the flow itself

Gauss' Law for Magnetism

- If there is a symmetry between electricity and magnetism, there must be an equivalent law in magnetism as the Gauss' Law in electricity
- For a magnetic field **B**, the magnetic flux Φ_B through the surface is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

  - Where the integration is over the area of either an open or a closed surface
- The magnetic flux through a closed surface which completely encloses a volume is

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

- What was the Gauss' law in the electric case?
  - The electric flux through a closed surface is equal to the total net charge Q enclosed by the surface divided by ε_0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

- Similarly, we can write Gauss' law for magnetism as

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for magnetism

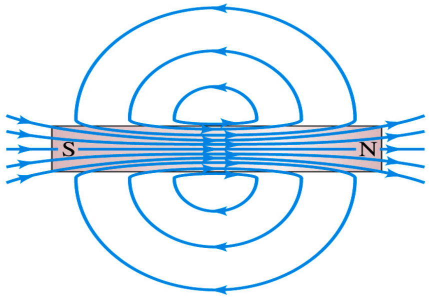
- Why is result of the integral zero? (poll 18)
  - There is no isolated magnetic poles, the magnetic equivalent of single electric charges

Gauss' Law for Magnetism

- What does the Gauss' law in magnetism mean physically?

$$\oint \vec{B} \cdot d\vec{A} = 0$$

  - There are as many magnetic flux lines that enter the enclosed volume as leave it
  - If magnetic monopole does not exist, there is no starting or stopping point of the flux lines
    - Electricity do have the source and the sink
  - Magnetic field lines must be continuous
  - Even for a bar magnet, the field lines exist both insides and outside of the magnet



Maxwell's Equations

- In the absence of dielectric or magnetic materials, the four equations developed by Maxwell are:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

A generalized form of Coulomb's law relating electric field to its sources, the electric charge

$$\oint \vec{B}\cdot d\vec{A} = 0$$

Gauss' Law for magnetism

A magnetic equivalent of Coulomb's law relating magnetic field to its sources. This says there are no magnetic monopoles.

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's Law

An electric field is produced by a changing magnetic field

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampére's Law

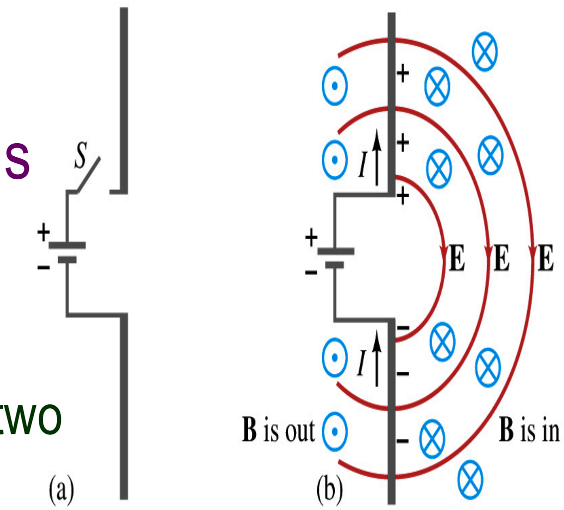
A magnetic field is produced by an electric current or by a changing electric field

Maxwell's Amazing Leap of Faith

- According to Maxwell, a magnetic field will be produced even in an empty space if there is a changing electric field
  - He then took this concept one step further & concluded that
    - If a changing magnetic field produces an electric field, the electric field is also changing in time.
    - This changing electric field in turn produces a magnetic field that changes.
    - This changing magnetic field then in turn produces an electric field that changes.
    - This process continues.
  - With a manipulation of the equations, Maxwell found that the net result of this interacting changing fields is a wave of electric and magnetic fields that can actually propagate (travel) through the empty space

Production of EM Waves

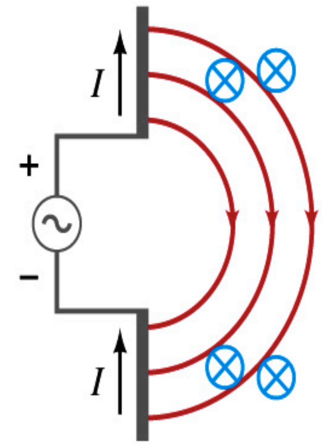
- Consider two conducting rods that will serve as an antenna are connected to a DC power source
  - What do you think will happen when the switch is closed?
    - The rod connected to the positive terminal is charged positively and the other negatively
    - Then the electric field will be generated between the two rods
    - Since there is the current that flows through, the rods generates a magnetic field around them

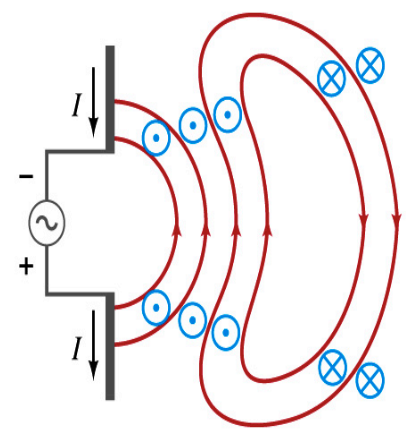


- How far would the electric and magnetic fields extend?
  - In static case, the field extends indefinitely
  - When the switch is closed, the fields are formed nearby the rods quickly but
  - The stored energy in the fields won't propagate w/ infinite speed

Production of EM Waves

- What happens if the antenna is connected to an AC power source?
  - When the connection was initially made, the rods are charging up quickly w/ the current flowing in one direction as shown in the figure
    - The field lines form as in the dc case
    - The field lines propagate away from the antenna
  - Then the direction of the voltage reverses
    - The new field lines with the opposite direction forms
    - While the original field lines still propagates away from the rod reaching out far
      - Since the original field propagates through an empty space, the field lines must form a closed loop (no charge exist)
    - Since changing electric and magnetic fields produce changing magnetic and electric fields, the fields moving outward is self supporting and do not need antenna with flowing charge
  - The fields far from the antenna is called the **radiation field**
  - Both electric and magnetic fields form closed loops perpendicular to each other





Properties of Radiation Fields – I

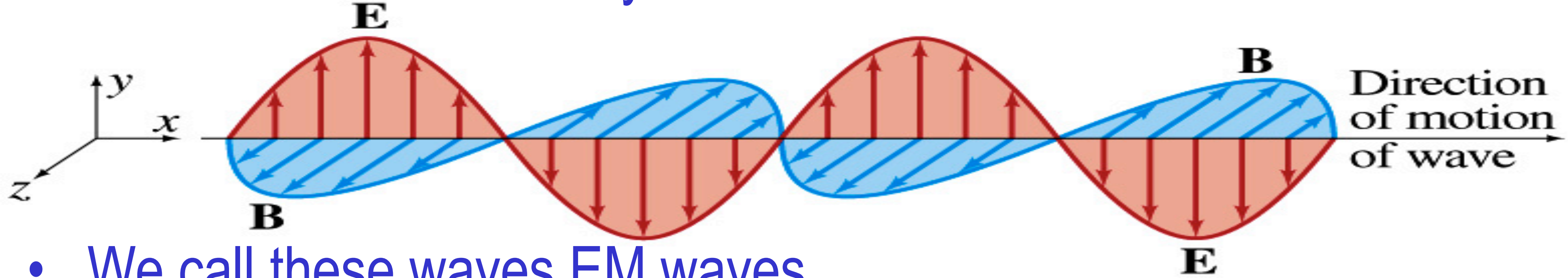
- The fields travel on the other side of the antenna as well
- The field strength is the greatest in the direction perpendicular to the oscillating charge while it is 0 along the direction of the current
- The magnitude of E and B in the radiation field decrease with distance as 1/r
- The energy carried by the EM wave is proportional to the square of the amplitude, E^2 or B^2
  – So the intensity of wave decreases as $1/r^2$

Properties of Radiation Fields – II

- The electric and magnetic fields at any point are perpendicular to each other and to the direction of the propagation (or motion)
- The fields alternate in direction
  - The field strengths vary from maximum in one direction, to 0 and to max in the opposite direction
- The electric and magnetic fields are in phase
  - B is maximum when E is maximum, vise versa
- Very far from the antenna, the field lines are quite flat over a reasonably large area
  - Called plane waves

EM Waves

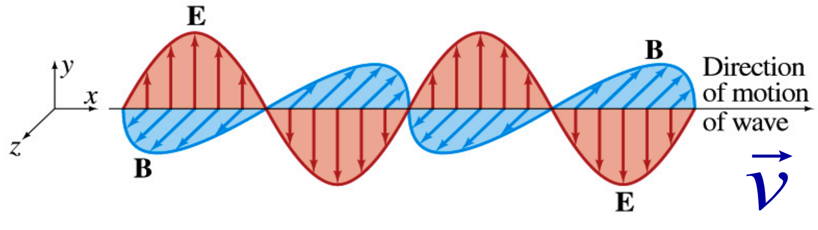
- If the voltage of the source varies sinusoidal, the field strengths of the radiation field vary sinusoidal



- We call these waves EM waves
- They are transverse waves
- EM waves are always those of fields
  - Since these are fields, they can propagate through an empty space
- In general **accelerating electric charges give rise to electromagnetic waves**
- This prediction from Maxwell's equations was experimentally proven by Heinrich Hertz through the **discovery of radio waves**

EM Waves and Their Speeds

- Let's consider a region of free space. What's a free space?
  - An area of space where there is no charges or conduction currents
  - In other words, far from emf sources so that the wave fronts are essentially flat or not distorted over a reasonable area
  - What are these flat waves called?
    - **Plane waves**
    - At any instance **E** and **B** are uniform over a large plane perpendicular to the direction of propagation
  - So we can also assume that the wave is traveling in the x-direction w/ velocity, **v**=v**i**, and that **E** is parallel to y axis and **B** is parallel to z axis



Maxwell's Equations w/ Q=I=0

- In this region of free space, Q=0 and I=0, thus the four Maxwell's equations become

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \quad \xrightarrow{Q_{encl}=0} \quad \oint \vec{E}\cdot d\vec{A} = 0$$

$$\oint \vec{B}\cdot d\vec{A} = 0 \quad \xrightarrow{\text{No Changes}} \quad \oint \vec{B}\cdot d\vec{A} = 0$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \xrightarrow{\text{No Changes}} \quad \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\varepsilon_0\frac{d\Phi_E}{dt} \quad \xrightarrow{I_{encl}=0} \quad \oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$

One can observe the symmetry between electricity and magnetism.

The last equation is the most important one for EM waves and their propagation!!

EM Waves from Maxwell's Equations

- If the wave is sinusoidal w/ wavelength λ and frequency *f*, such traveling wave can be written as

$$E = E_y = E_0 \sin\left(kx - \omega t\right)$$
$$B = B_z = B_0 \sin\left(kx - \omega t\right)$$

  – Where

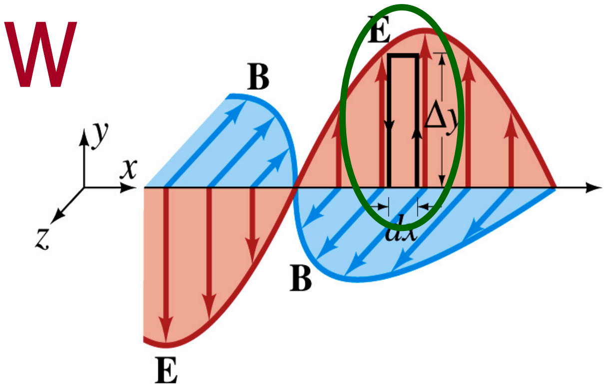
$$k = \frac{2\pi}{\lambda} \qquad \varpi = 2\pi f$$

Thus

$$f\lambda = \frac{\omega}{k} = v$$

  – What is v?
    - It is the speed of the traveling wave
  – What are E_0 and B_0?
    - The amplitudes of the EM wave. Maximum values of **E** and **B** field strengths.

From Faraday's Law



- Let's apply Faraday's law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

  – to the rectangular loop of height Δy and width dx

- $\vec{E} \cdot d\vec{l}$ along the top and bottom of the loop is 0. Why?
  – Since **E** is perpendicular to d*ℓ*.
  – So the result of the integral through the loop counterclockwise becomes $\oint \vec{E} \cdot d\vec{l} = \vec{E} \cdot d\vec{x} + \left(\vec{E} + d\vec{E}\right) \cdot \Delta\vec{y} + \vec{E} \cdot d\vec{x} + \vec{E} \cdot \Delta\vec{y} =$

$$= 0 + \left(E + dE\right)\Delta y - 0 - E\Delta y = dE\Delta y$$

  – For the right-hand side of Faraday's law, the magnetic flux through the loop changes as

$$-\frac{d\Phi_B}{dt} = \frac{dB}{dt} dx\Delta y$$

Thus

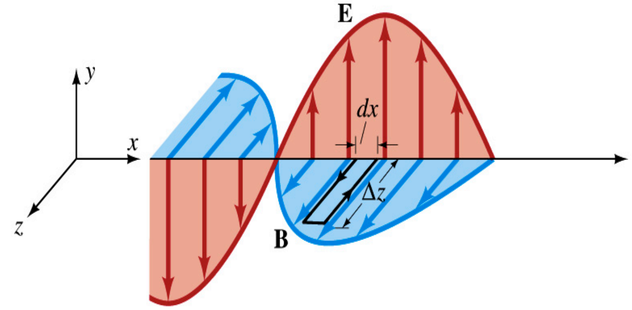
$$dE\Delta y = -\frac{dB}{dt} dx\Delta y$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

Since E and B depend on x and t

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

From Modified Ampére's Law



- Let's apply Maxwell's 4th equation

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

  – to the rectangular loop of length Δz and width dx

- $\vec{B}\cdot d\vec{l}$ along the x-axis of the loop is 0
  – Since **B** is perpendicular to dl.
  – So the result of the integral through the loop counterclockwise becomes

$$\oint \vec{B}\cdot d\vec{l} = B\Delta z - \left(B + dB\right)\Delta z = -dB\Delta z$$

  – For the right-hand side of the equation is

$$\mu_0\varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0\varepsilon_0 \frac{dE}{dt} dx\Delta z$$

Thus

$$-dB\Delta z = \mu_0\varepsilon_0 \frac{dE}{dt} dx\Delta z$$

  –

$$\frac{dB}{dx} = -\mu_0\varepsilon_0 \frac{dE}{dt}$$

Since E and B depend on x and t

$$\frac{\partial B}{\partial x} = -\mu_0\varepsilon_0 \frac{\partial E}{\partial t}$$

Relationship between **E**, **B** and **v**

- Let's now use the relationship from Faraday's law $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\partial E}{\partial x} = \frac{\partial}{\partial x}\left(E_0 \sin\left(kx - \omega t\right)\right) = kE_0 \cos\left(kx - \omega t\right)$$

$$\frac{\partial B}{\partial t} = \frac{\partial}{\partial t}\left(B_0 \sin\left(kx - \omega t\right)\right) = -\omega B_0 \cos\left(kx - \omega t\right)$$

Since $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$ **We obtain** $kE_0 \cos\left(kx - \omega t\right) = \omega B_0 \cos\left(kx - \omega t\right)$

Thus $\frac{E_0}{B_0} = \frac{\omega}{k} = \mathrm{v}$

- Since E and B are in phase, we can write $E/B = \mathrm{v}$
  - This is valid at any point and time in space. What is v?
    - The velocity of the wave

Speed of EM Waves

- Let's now use the relationship from Apmere's law $\frac{\partial B}{\partial x}=-\varepsilon_0\mu_0\frac{\partial E}{\partial t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\partial B}{\partial x}=\frac{\partial}{\partial x}\left(B_0\sin\left(kx-\omega t\right)\right)=kB_0\cos\left(kx-\omega t\right)$$

$$\frac{\partial E}{\partial t}=\frac{\partial}{\partial t}\left(E_0\sin\left(kx-\omega t\right)\right)=-\omega E_0\cos\left(kx-\omega t\right)$$

Since $\frac{\partial B}{\partial x}=-\varepsilon_0\mu_0\frac{\partial E}{\partial t}$ **We obtain** $kB_0\cos\left(kx-\omega t\right)=\varepsilon_0\mu_0\omega E_0\cos\left(kx-\omega t\right)$

Thus $\frac{B_0}{E_0}=\frac{\varepsilon_0\mu_0\omega}{k}=\varepsilon_0\mu_0 v$

- However, from the previous page we obtain $E_0/B_0=\mathrm{v}=\frac{1}{\varepsilon_0\mu_0\mathrm{v}}$
- Thus $\mathrm{v}^2=\frac{1}{\varepsilon_0\mu_0}$

$$\mathrm{v}=\frac{1}{\sqrt{\varepsilon_0\mu_0}}=\frac{1}{\sqrt{\left(8.85\times10^{-12}\,C^2/N\cdot m^2\right)\cdot\left(4\pi\times10^{-7}\,T\cdot m/A\right)}}=3.00\times10^8\,m/s$$

The speed of EM waves is the same as the speed of light. EM waves behaves like the light.