

PHYS 1443 – Section 003

Lecture #5

Wednesday, Feb. 3, 2021

Dr. Jaehoon Yu

- One Dimensional Motion
 - Motion under constant acceleration
 - One dimensional Kinematic Equations
 - How do we solve kinematic problems?
 - Falling motions
- Motion in two dimensions
 - Coordinate system



Announcements

- First term exam in class Wednesday, Feb. 10
 - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
 - No derivations, word definitions, setups or solutions of any problems, figures, pictures, diagrams or arrows, etc!
 - No additional formulae or values of constants will be provided!
 - Must email me the photos of front and back of the formula sheet, including the blank at jaehoonyu@uta.edu no later than **12:00pm the day of the test**
 - The subject of the email should be the same as your file name
 - File name must be FS-E1-LastName-FirstName-SP21.pdf
 - Once submitted, you cannot change, unless I ask you to delete part of the sheet!
- Extra credit special COVID seminar at 4pm Saturday, March 20
 - Extra credit for participation and for asking the relevant questions
 - Dr. Linda Lee, a practicing physician from Wisconsin



Reminder: Special Project #2 for Extra Credit

- Show that the trajectory of a projectile motion is a parabola!! (20 points)
 - You MUST show full details of your OWN computations, including every step of the derivation, to obtain any credit
 - Beyond what was covered in this lecture note and in the book!
- You must show your OWN work in detail to obtain the full credit
 - Must be handwritten and in much more detail than in this lecture note or the book!
 - Please do not copy from the lecture note or from your friends. You will all get 0!!
 - BE SURE to show all the details of your own work, including all formulae, proper references to them and explanations
- Due at the beginning of the class 1:00pm Monday, Feb. 15 on Canvas
 - File name must be: SP2-LastName-FirstName-SP21.pdf



One Dimensional Motion

- Let's focus on the simplest case: acceleration is a constant ($a=a_0$)
- Using the definitions of average acceleration and velocity, we can derive equations of motion (description of motion, velocity and position as a function of time)

$$\bar{a}_x = a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} \quad (\text{If } t_f=t \text{ and } t_i=0) \quad a_x = \frac{v_{xf} - v_{xi}}{t} \quad \Rightarrow \quad v_{xf} = v_{xi} + a_x t$$

For constant acceleration, average velocity is a simple numeric average

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2} a_x t$$

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i} \quad (\text{If } t_f=t \text{ and } t_i=0) \quad \bar{v}_x = \frac{x_f - x_i}{t} \quad \Rightarrow \quad x_f = x_i + \bar{v}_x t$$

Resulting Equation of Motion becomes

$$x_f = x_i + \bar{v}_x t = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v_{xf}(t) = v_{xi} + a_xt$$

Velocity as a function of time

$$x_f - x_i = \frac{1}{2} \bar{v}_x t = \frac{1}{2} (v_{xf} + v_{xi}) t$$

Displacement as a function of velocities and time

$$x_f = x_i + v_{xi}t + \frac{1}{2} a_xt^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinematic equations, depending on the information given to you in specific physical problems!!

How do we solve a problem using a kinematic formula under a constant acceleration?

- Identify what information is given in the problem.
 - Initial and final velocity?
 - Acceleration?
 - Distance, initial position or final position?
 - Time information?
- Convert the units of all quantities to SI units to be consistent.
- Identify what the problem is asking you to do in what unit
- Identify which kinematic formula is **most appropriate and easiest** to solve for what the problem wants.
 - Often multiple formulae can give you the answer for the quantities you are looking for. → Do not just use any formula but use the one that can be easiest to solve with the given set of information
- Solve the equations for the quantity or quantities wanted.

Example

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at the speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop?  As long as it takes for it to crumple.

The initial speed of the car is $v_{xi} = 100km / h = \frac{100000m}{3600s} = 28m / s$

We also know that $v_{xf} = 0m / s$ and $x_f - x_i = 1m$

Using the kinematic formula $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$

The acceleration is $a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{0 - (28m / s)^2}{2 \times 1m} = -390m / s^2$

Thus the time for air-bag to deploy is $t = \frac{v_{xf} - v_{xi}}{a} = \frac{0 - 28m / s}{-390m / s^2} = 0.07s$

Check point for conceptual understanding

- Which of the following equations for positions of a particle as a function of time can the four kinematic equations under constant acceleration applicable?
- (a) $x = 3t - 4$
- (b) $x = -5t^3 + 4t^2 + 6$
- (c) $x = 2/t^2 - 4/t$
- (d) $x = 5t^2 - 3$
- What is the key here? Finding which equation gives a constant acceleration using its definition!
- Yes, you are right! The answers are (a) and (d)!

Falling Motion

- Falling motion is a motion under the influence of the gravitational pull (gravity) only (how many dimensions? poll 5) ; Which direction is a freely falling object moving? (poll 7)

Yes, down to the center of the Earth!!

 - A motion under a constant acceleration
 - All kinematic formula we've learned can be used to solve for falling motions.
- Gravitational acceleration is inversely proportional to the distance between the object and the center of the Earth
- The magnitude of the gravitational acceleration is $g=9.80\text{m/s}^2$ on the surface of the earth, most of the time.
- The direction of the gravitational acceleration is ALWAYS toward the center of the earth, which we normally call $(-y)$; where up and down direction are indicated as the variable “y”
- Thus, the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80\text{m/s}^2$ when $+y$ points upward

Example for Using 1D Kinematic Equations on a Falling object

A stone was thrown straight upward at $t=0$ with $+20.0\text{m/s}$ initial velocity on the roof of a 50.0m tall building,



What is the acceleration in this motion (poll 8) ? $g=-9.80\text{m/s}^2$

(a) Find the time the stone reaches at the maximum height.

What happens to the speed at the maximum height? $V=0$

(b) Find the maximum height.

(c) Find the time the stone reaches back to its original height.

(d) Find the velocity of the stone when it reaches its original height.

(e) Find the velocity and position of the stone at $t=5.00\text{s}$.

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What happens to the speed at the maximum height? $V=0$

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00\text{m/s} \quad \text{Solve for } t \quad t = \frac{20.0}{9.80} = 2.04\text{s}$$

(b) Find the maximum height.

$$\begin{aligned} y_f &= y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2 \\ &= 50.0 + 20.4 = 70.4(\text{m}) \end{aligned}$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

(e) Find the velocity and position of the stone at $t=5.00s$.

Velocity

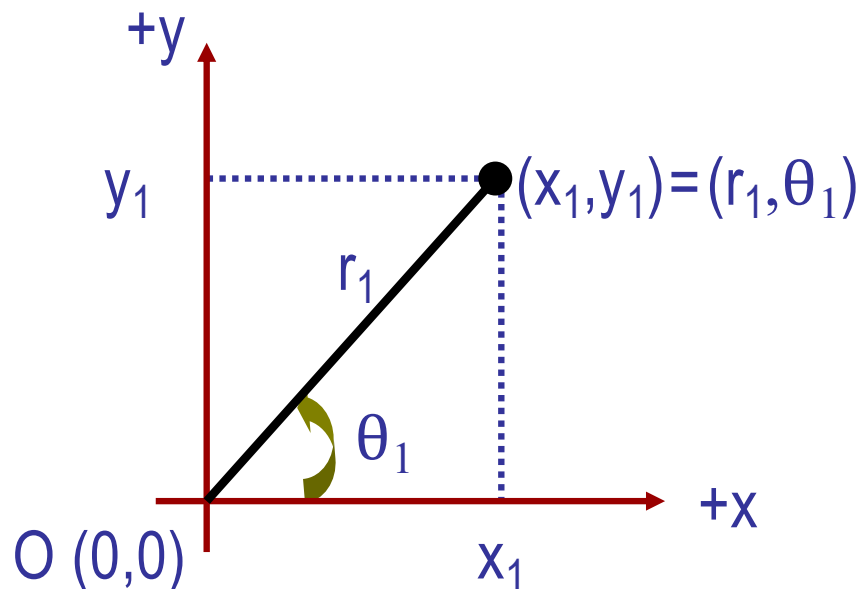
$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0(m/s)$$

Position

$$\begin{aligned} y_f &= y_i + v_{yi} t + \frac{1}{2} a_y t^2 \\ &= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m) \end{aligned}$$

2D Coordinate Systems

- They make it easy and consistent to express locations or positions
- Two commonly used systems, depending on convenience, are
 - Cartesian (Rectangular) Coordinate System
 - Coordinates are expressed in (x,y)
 - Polar Coordinate System
 - Coordinates are expressed in distance from the origin \textcircled{O} and the angle measured from the x-axis, θ (r, θ)
- Vectors become a lot easier to express and compute



How are Cartesian and Polar coordinates related?

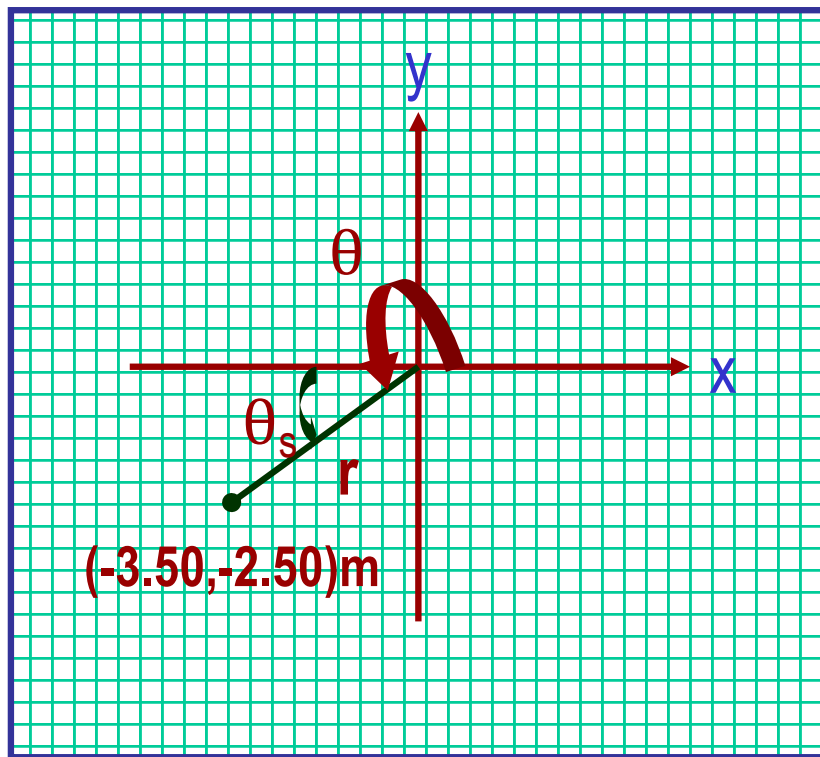
$$x_1 = r_1 \cos \theta_1 \quad r_1 = \sqrt{(x_1^2 + y_1^2)}$$

$$y_1 = r_1 \sin \theta_1 \quad \tan \theta_1 = \frac{y_1}{x_1}$$

$$\theta_1 = \tan^{-1} \left(\frac{y_1}{x_1} \right) \quad 13$$

Example

Cartesian Coordinate of a point in the xy plane are $(x,y) = (-3.50,-2.50)\text{m}$. Find the equivalent polar coordinates of this point.



$$\begin{aligned} r &= \sqrt{(x^2 + y^2)} \\ &= \sqrt{((-3.50)^2 + (-2.50)^2)} \\ &= \sqrt{18.5} = 4.30(m) \end{aligned}$$

$$\theta = 180 + \theta_s$$

$$\tan \theta_s = \frac{-2.50}{-3.50} = \frac{5}{7}$$

$$\theta_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$$

$$\therefore \theta = 180 + \theta_s = 180^\circ + 35.5^\circ = 216^\circ$$

Vector and Scalar

Vector quantities have both magnitudes (sizes) and directions

Force, gravitational acceleration, momentum

Normally denoted in **BOLD** letters, \mathbf{F} , or a letter with arrow on top \vec{F}

Their sizes or magnitudes are denoted with normal letters, F , or absolute values: $|\vec{F}|$ or $|\mathbf{F}|$

Scalar quantities have magnitudes only

Energy, heat, mass, time

Can be completely specified with a value and its unit

Normally denoted in normal letters, E

Both have units!!!