PHYS 1443 – Section 003
Lecture #6

Monday, Feb. 8, 2021

Dr. Jaehoon Yu

- CH3: Motion in two dimensions
  - Vector and scalars, their operations
  - Motion under constant acceleration
  - Projectile Motion
  - Maximum range and height

Today's homework is homework #4, due 11pm, Tuesday, Feb. 23!!

Announcements

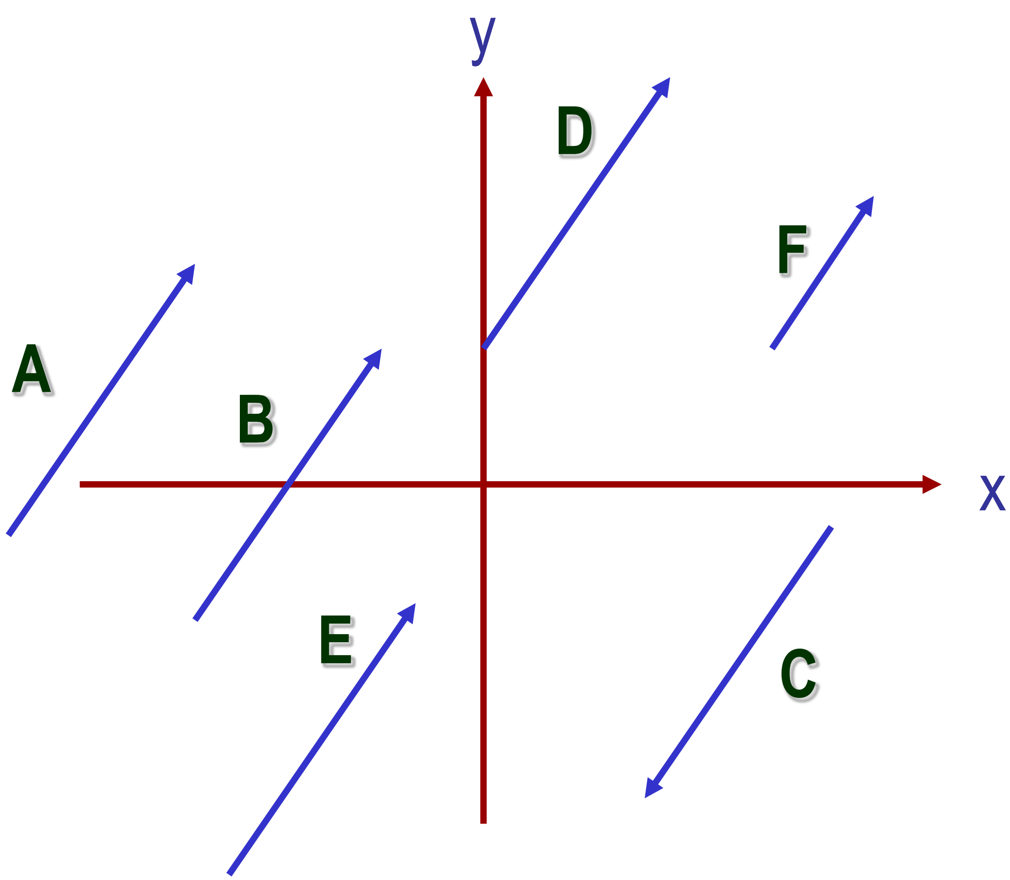
- First term exam in class this Wednesday, Feb. 10
  - Covers CH1.1 to what we finish today (CH3.5 or 3.6) + math refresher
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, setups or solutions of any problems, figures, pictures, diagrams or arrows, etc!
  - No additional formulae or values of constants will be provided!
  - Must email me the photos of front and back of the formula sheet, including the blank at jaehoonyu@uta.edu no later than **12:00pm the day of the test**
    - The subject of the email should be the same as your file name
    - File name must be FS-E1-LastName-FirstName-SP21.pdf
    - Once submitted, you cannot change, unless I ask you to delete part of the sheet!
- Reminder: Extra credit special COVID seminar at 4pm Saturday, March 20
  - Extra credit for participation and for asking the relevant questions
  - Dr. Linda Lee, a practicing physician from Wisconsin
  - Mark your calendars!!

Reminder: Special Project #2 for Extra Credit

- Show that the trajectory of a projectile motion is a parabola!! (20 points)
  - You MUST show full details of your OWN computations, including every step of the derivation, to obtain any credit
    - Beyond what was covered in this lecture note and in the book!
- You must **show your OWN work in detail** to obtain the full credit
  - Must be **handwritten** and in much more detail than in this lecture note or the book!
  - Please do not copy from the lecture note or from your friends. You will all get 0!!
  - BE SURE to show all the details of your own work, including all formulae, proper references to them and explanations
- Due at the beginning of the class 1:00pm Monday, Feb. 15 on **Canvas**
  - File name must be: SP2-LastName-FirstName-SP21.pdf

Properties of Vectors

- Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system!!



Which ones are the same vectors?

A=B=E=D

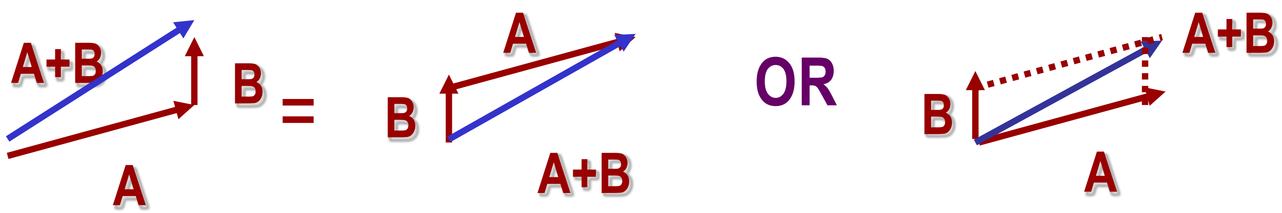
Why aren't the others?

C: The same magnitude but opposite direction: **C=-A:**A negative vector

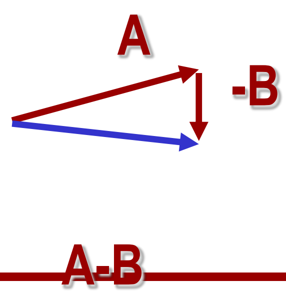
F: The same direction but different magnitude

Vector Operations

- Addition:
  - Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  - Parallelogram method: Connect the tails of the two vectors and extend
  - Addition is commutative: Changing order of operation does not affect the results **A+B=B+A**, **A+B+C+D+E=E+C+A+B+D**



- Subtraction:
  - The same as adding a negative vector: **A** - **B** = **A** + (-**B**)



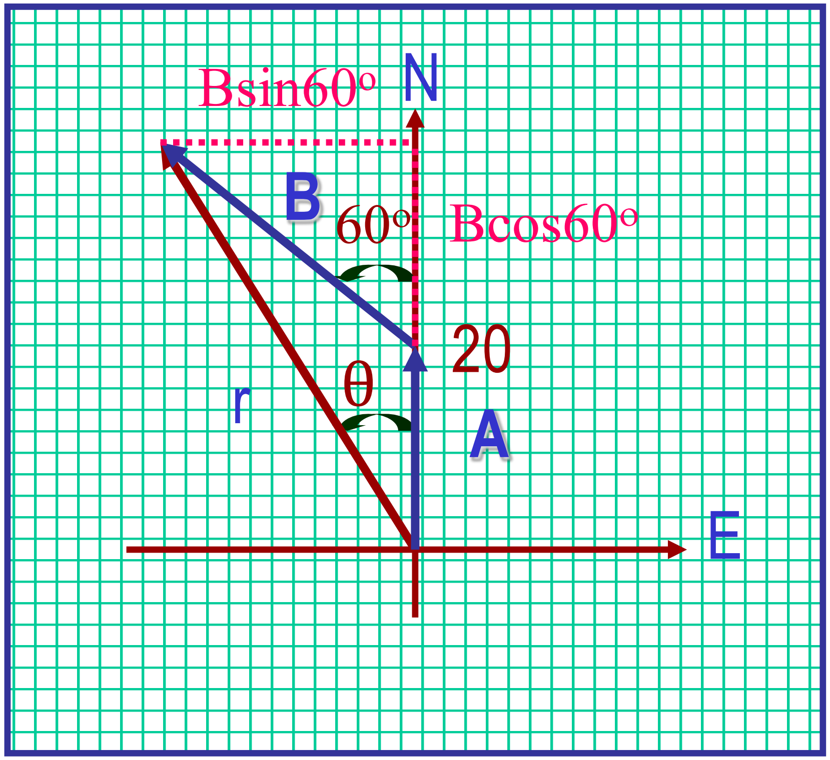
Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude **A, B**=2**A**

$$|\mathcal{B}| = 2|\mathcal{A}|$$

Example for Vector Addition

A car travels 20.0km due north followed by 35.0km in a direction 60.0° west of north. Find the magnitude and direction of resultant displacement.



$$r = \sqrt{\left(A + B\cos\theta\right)^2 + \left(B\sin\theta\right)^2}$$

$$= \sqrt{A^2 + B^2\left(\cos^2\theta + \sin^2\theta\right) + 2AB\cos\theta}$$

$$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{(20.0)^2 + (35.0)^2 + 2\times 20.0\times 35.0\cos 60}$$

$$= \sqrt{2325} = 48.2(km)$$

$$\theta = \tan^{-1}\frac{\left|\vec{B}\right|\sin 60}{\left|\vec{A}\right| + \left|\vec{B}\right|\cos 60}$$

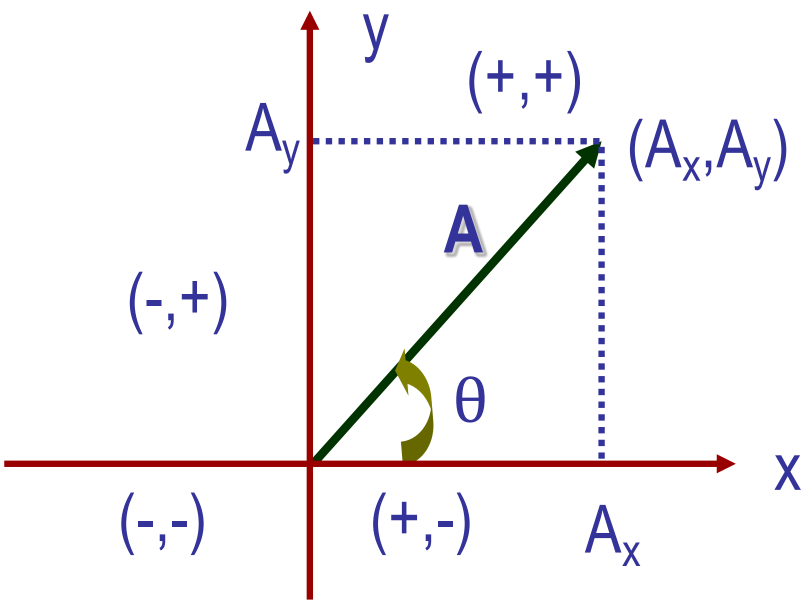
$$= \tan^{-1}\frac{35.0\sin 60}{20.0 + 35.0\cos 60}$$

$$= \tan^{-1}\frac{30.3}{37.5} = 38.9^\circ \text{ to W wrt N}$$

Do this using components!!

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$A_x = \left|\vec{A}\right| \cos\theta$$

$$A_y = \left|\vec{A}\right| \sin\theta$$

} Components

$$\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2}$$

} Magnitude

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right|\cos\theta\right)^2 + \left(\left|\vec{A}\right|\sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2\left(\cos^2\theta + \sin^2\theta\right)} \quad = \left|\vec{A}\right|$$

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
- **Dimensionless**
- **Magnitudes these vectors are exactly 1**
- Unit vectors are usually expressed in **i, j, k** or

$$\vec{i},\ \vec{j},\ \vec{k}$$

So a vector **A** can be expressed as

$$\vec{A} = A_x\vec{i} + A_y\vec{j} = \left|\vec{A}\right|\cos\theta\,\vec{i} + \left|\vec{A}\right|\sin\theta\,\vec{j}$$

Examples of Vector Operations

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C} = \vec{A} + \vec{B} = \left(2.0\vec{i} + 2.0\vec{j}\right) + \left(2.0\vec{i} - 4.0\vec{j}\right)$$

$$= \left(2.0 + 2.0\right)\vec{i} + \left(2.0 - 4.0\right)\vec{j} = 4.0\vec{i} - 2.0\vec{j}\left(m\right)$$

$$\left|\vec{C}\right| = \sqrt{\left(4.0\right)^2 + \left(-2.0\right)^2}$$

$$= \sqrt{16 + 4.0} = \sqrt{20} = 4.5(m)$$

$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{-2.0}{4.0} = -27^\circ$$

Find the resultant displacement of three consecutive displacements: **d**$_1$=(15**i**+30**j** +12**k**)cm, **d**$_2$=(23**i**+14**j** -5.0**k**)cm, and **d**$_3$=(-13**i**+15**j**)cm

$$\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = \left(15\vec{i} + 30\vec{j} + 12\vec{k}\right) + \left(23\vec{i} + 14\vec{j} - 5.0\vec{k}\right) + \left(-13\vec{i} + 15\vec{j}\right)$$

$$= \left(15 + 23 - 13\right)\vec{i} + \left(30 + 14 + 15\right)\vec{j} + \left(12 - 5.0\right)\vec{k} = 25\vec{i} + 59\vec{j} + 7.0\vec{k}(cm)$$

Magnitude

$$\left|\vec{D}\right| = \sqrt{\left(25\right)^2 + \left(59\right)^2 + \left(7.0\right)^2} = 65(cm)$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement:

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

- Average Velocity:

$$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

- Instantaneous Velocity:

$$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

- Average Acceleration

$$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

- Instantaneous Acceleration:

$$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

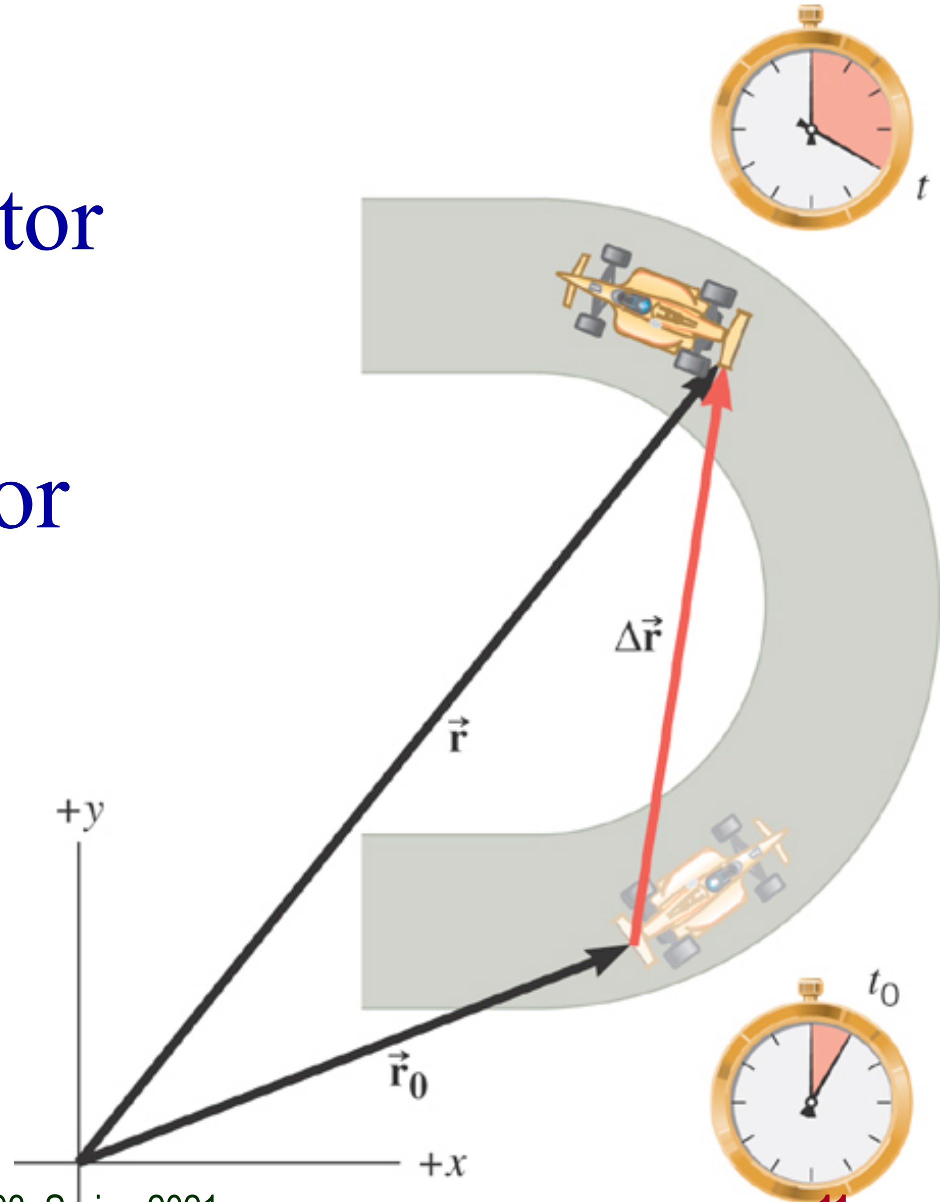
How is each of these quantities defined in 1-D?

2D Displacement

$\vec{\mathbf{r}}_o =$ initial position vector

$\vec{\mathbf{r}} =$ final position vector

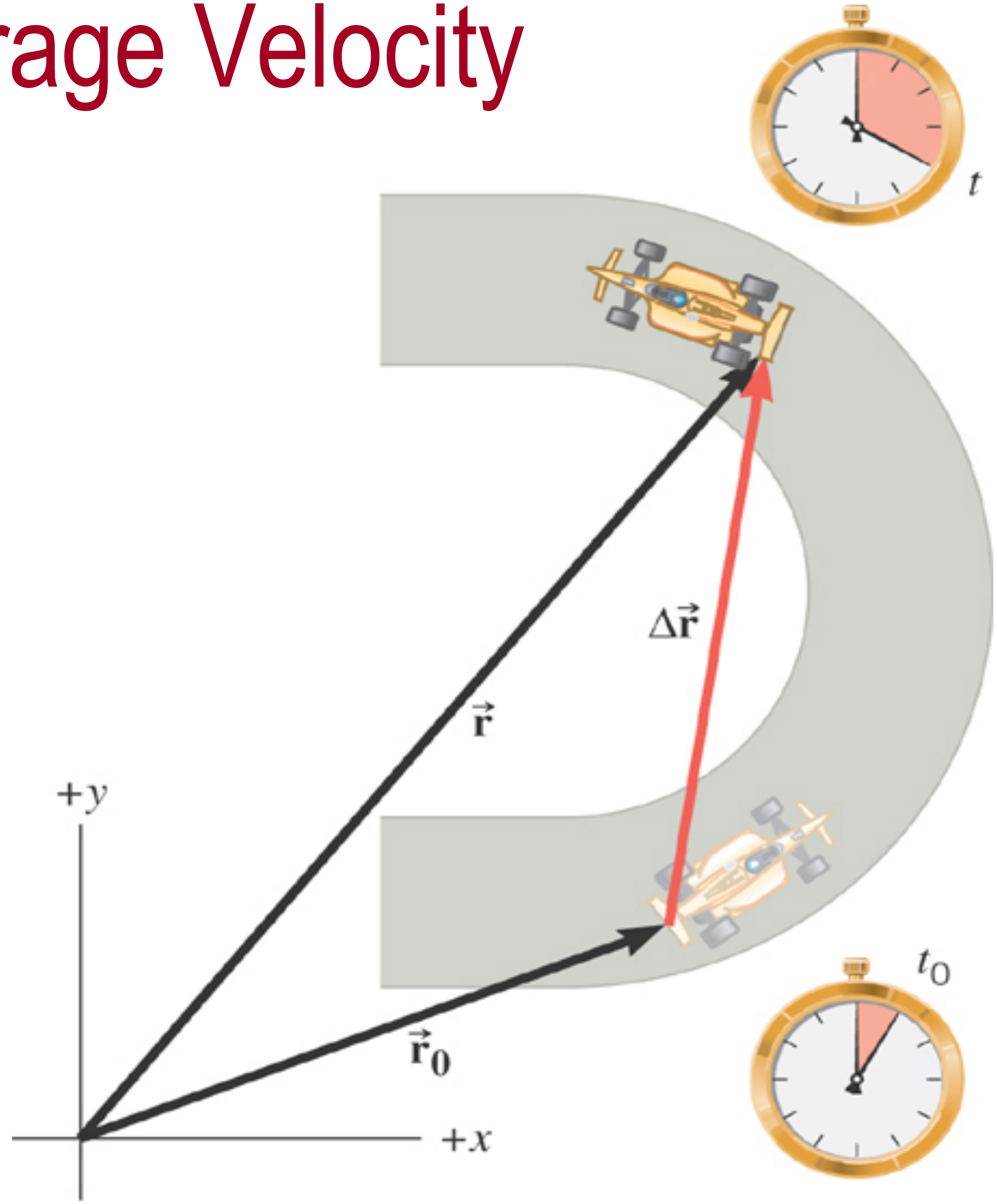
Displacement: $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_o$



2D Average Velocity

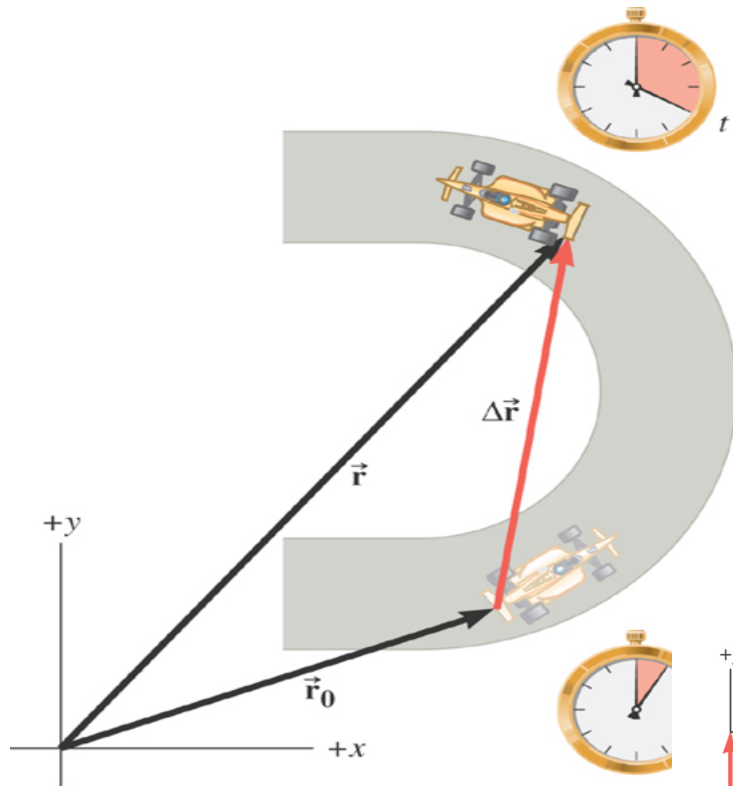
Average velocity is the displacement divided by the elapsed time.

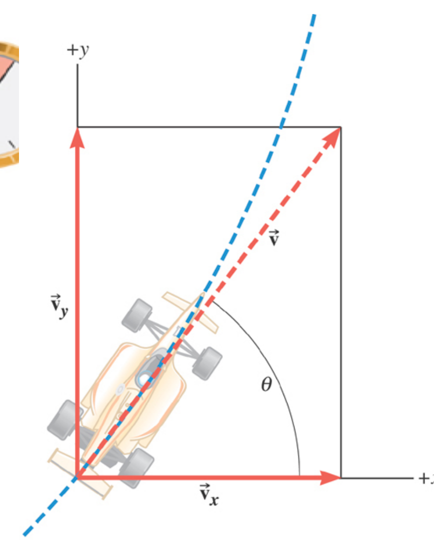
$$\overline{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$



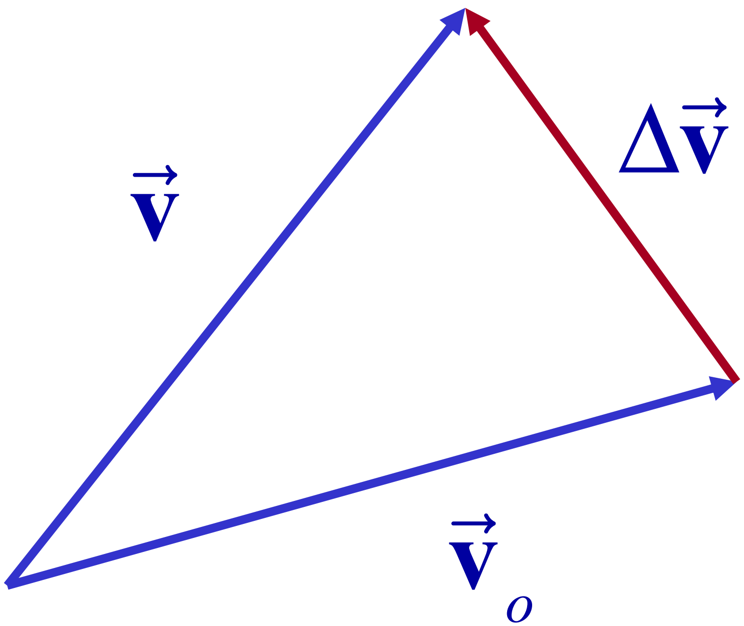
The ***instantaneous velocity*** indicates how fast the car moves and the direction of motion at each time instance.

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$





2D Average Acceleration



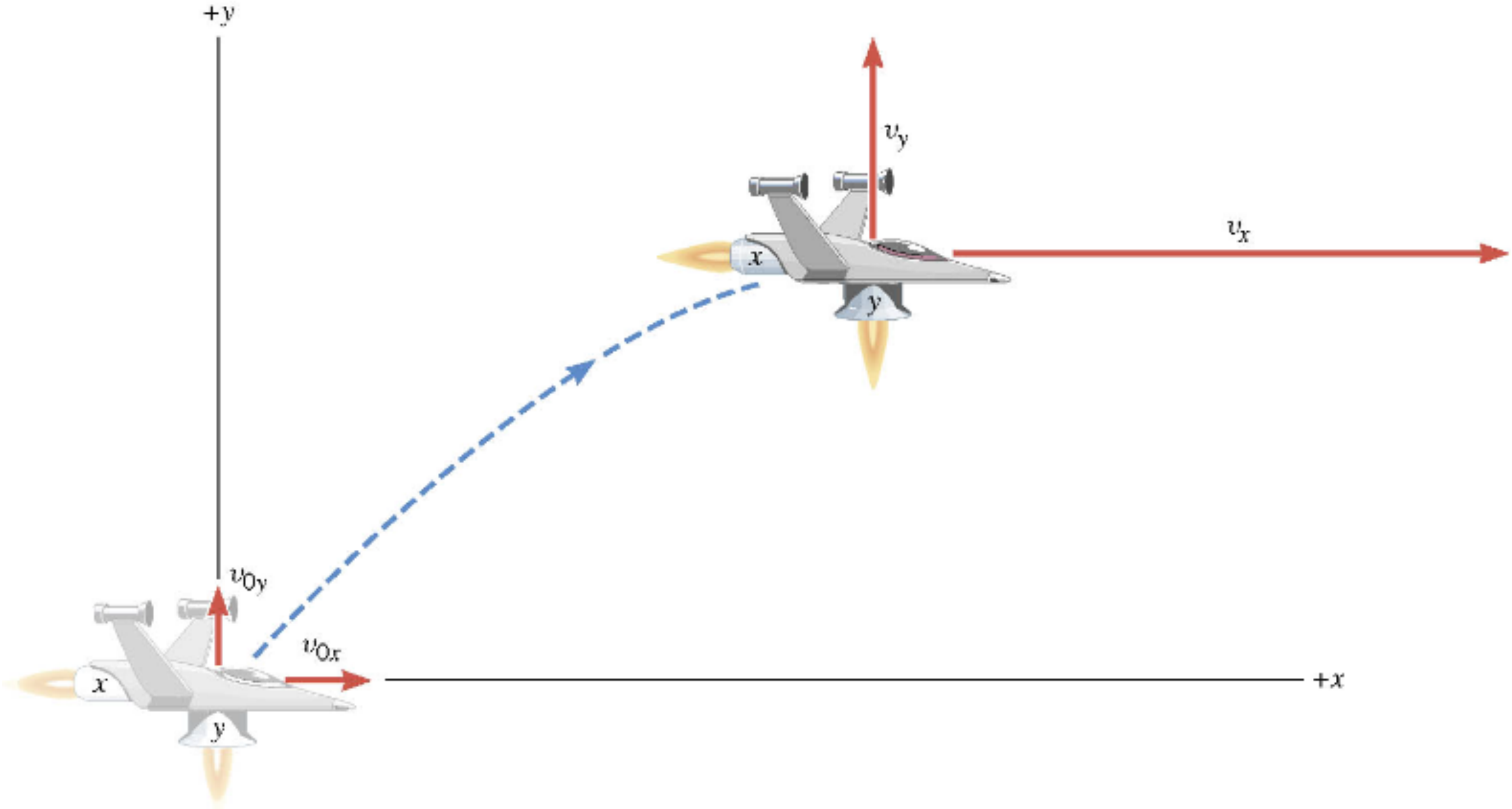
$$\overline{\vec{a}} = \frac{\vec{v} - \vec{v}_o}{t - t_o} = \frac{\Delta\vec{v}}{\Delta t}$$

Kinematic Quantities in 1D and 2D

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

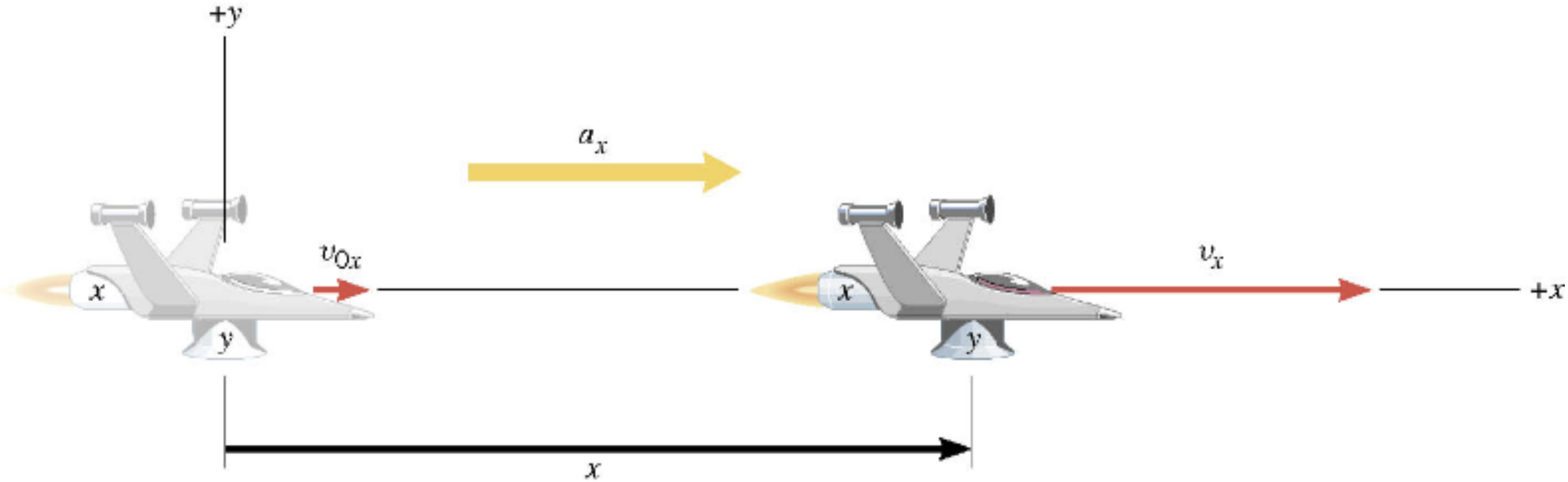
What is the difference between 1D and 2D quantities?

A Motion in 2 Dimension



This is a motion that could be viewed as two motions combined into one. (superposition…)

Motion in horizontal direction (x)



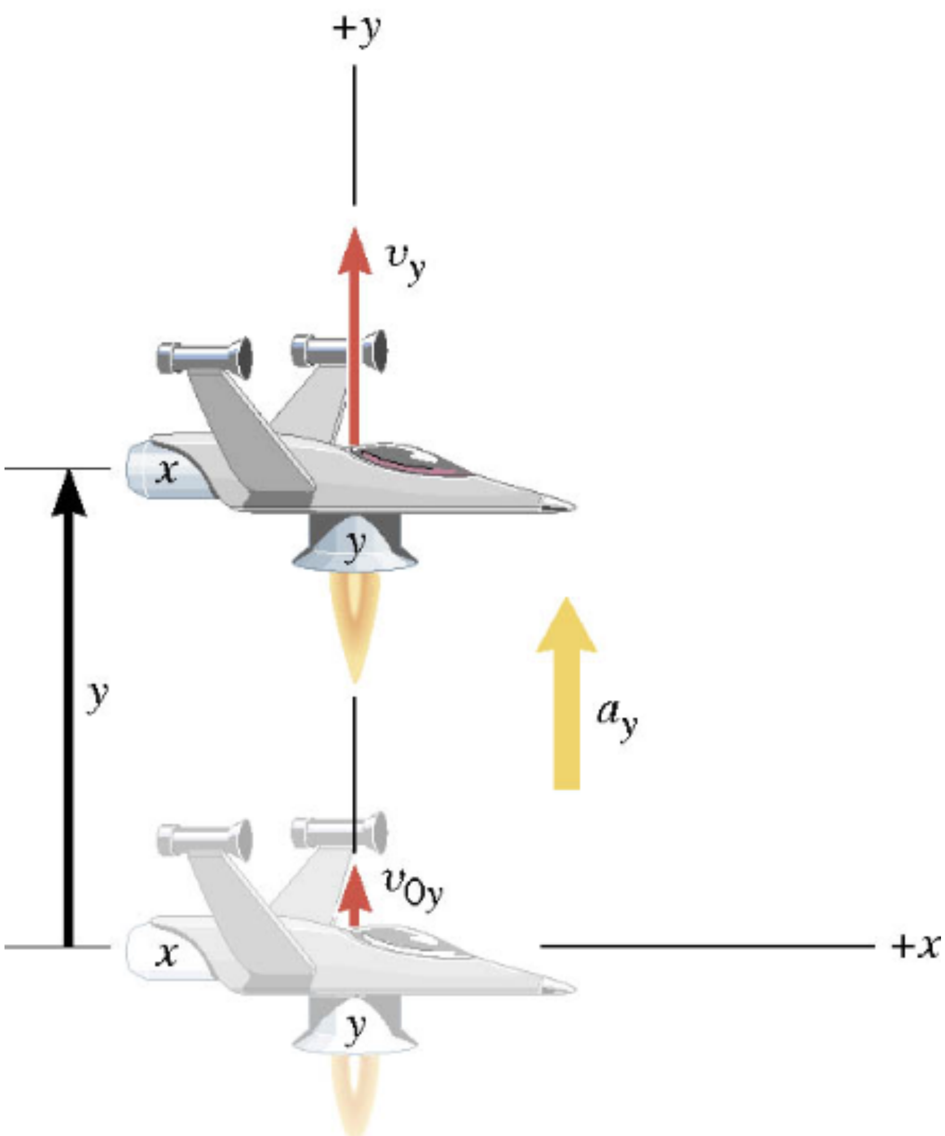
$$v_x = v_{xo} + a_x t$$

$$x = \frac{1}{2}\left(v_{xo} + v_x\right)t$$

$$v_x^2 = v_{xo}^2 + 2a_x x$$

$$x = v_{xo}t + \frac{1}{2}a_x t^2$$

Motion in vertical direction (y)



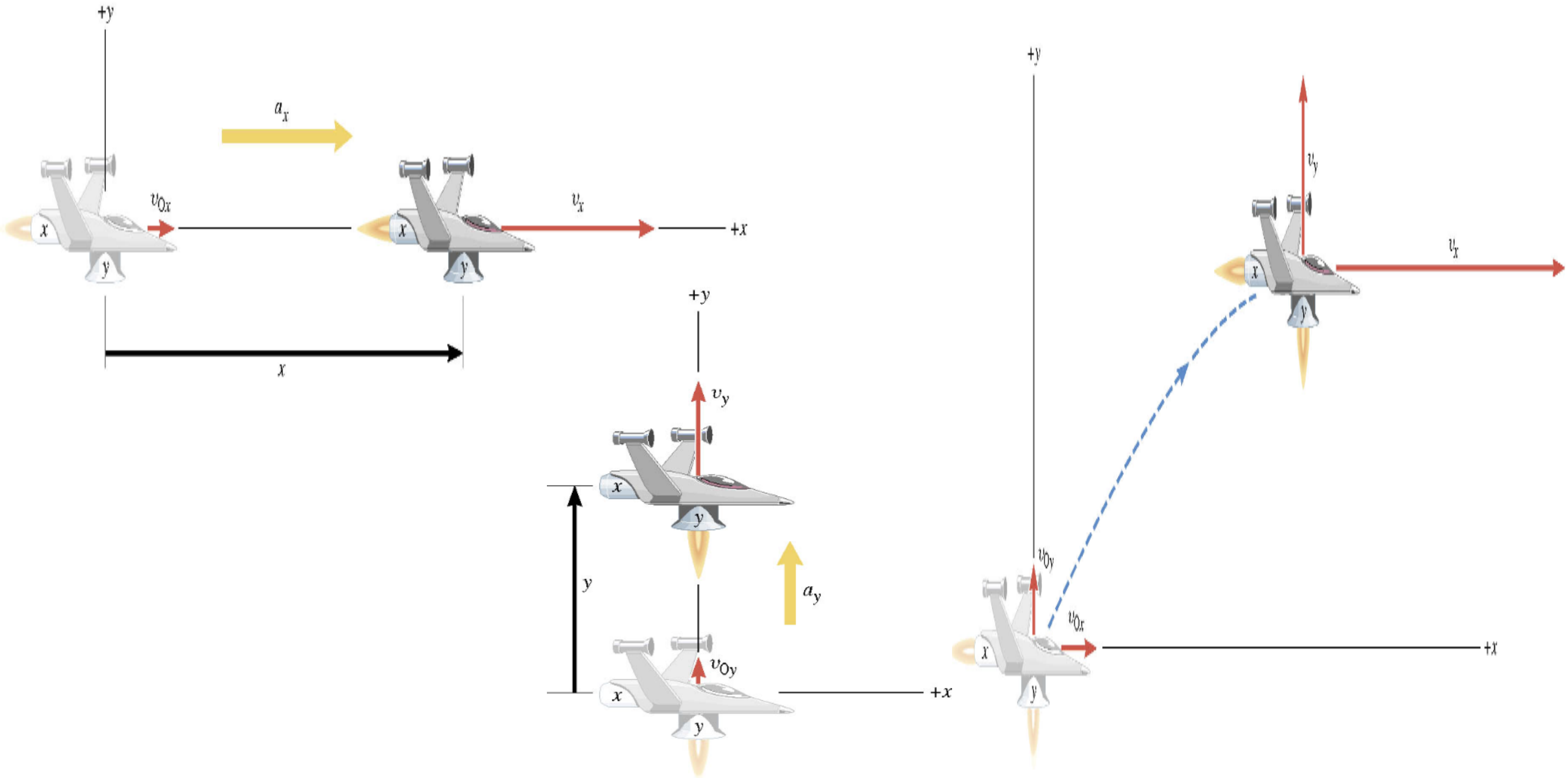
$$v_y = v_{yo} + a_y t$$

$$y = \tfrac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$y = v_{yo}t + \tfrac{1}{2}a_y t^2$$

A Motion in 2 Dimension



Imagine you adding the two 1 dimensional motions on the left. It would make up a one 2 dimensional motion on the right.

Kinematic Equations in 2-Dim

x-component

$$v_x = v_{x0} + a_x t$$

$$x = \tfrac{1}{2}\left(v_{x0} + v_x\right)t$$

$$v_x^2 = v_{x0}^2 + 2a_x x$$

$$\Delta x = v_{x0}t + \tfrac{1}{2}a_x t^2$$

y-component

$$v_y = v_{y0} + a_y t$$

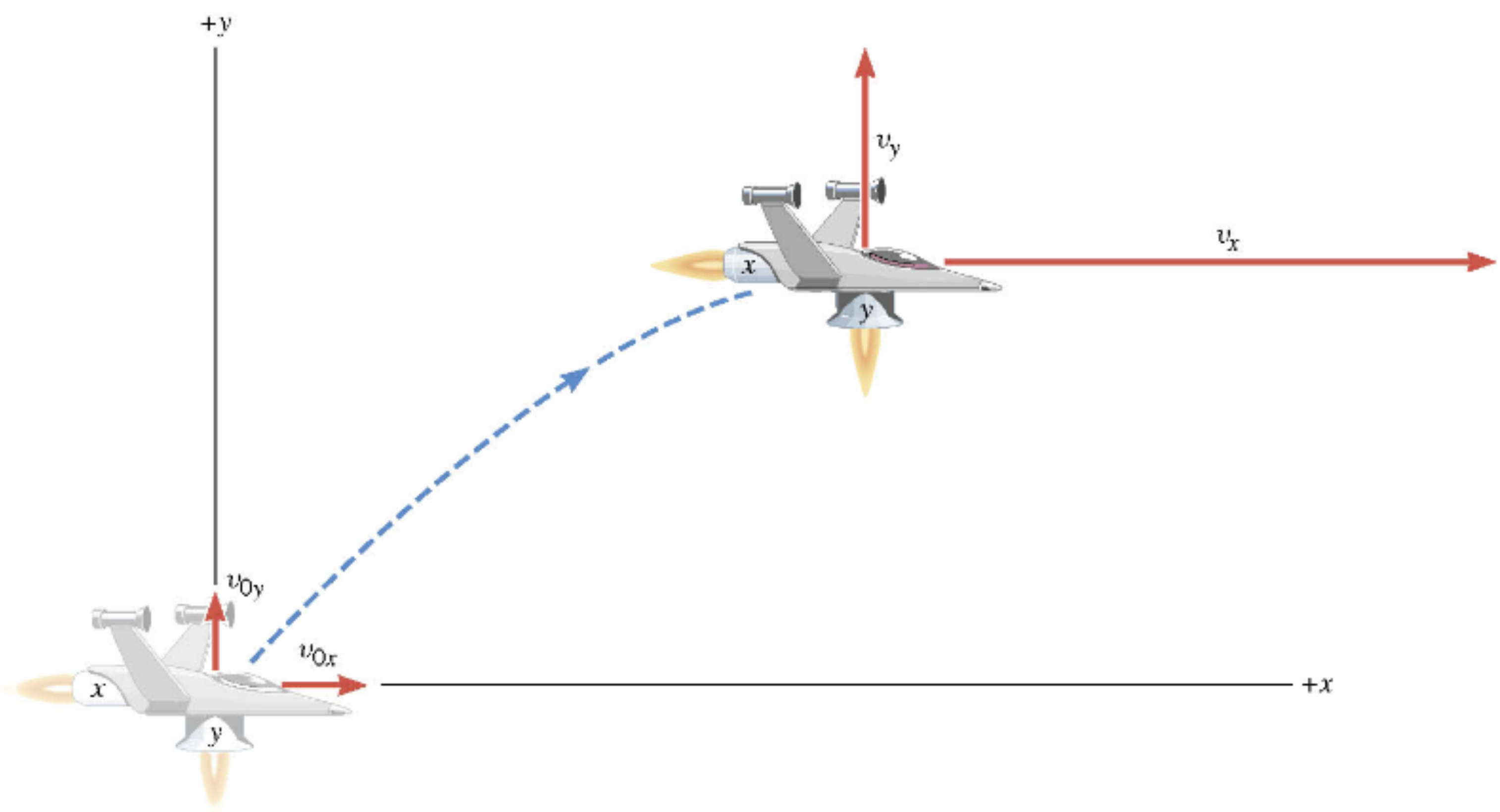
$$y = \tfrac{1}{2}\left(v_{y0} + v_y\right)t$$

$$v_y^2 = v_{y0}^2 + 2a_y y$$

$$\Delta y = v_{y0}t + \tfrac{1}{2}a_y t^2$$

Ex. A Moving Spacecraft

In the *x* direction, the spacecraft in zero-gravity zone has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.



How do we solve this problem?

1. Visualize the problem → Draw a picture!
2. Decide which directions are to be called positive (+) and negative (-).
3. Write down the values that are given for any of the five kinematic variables associated with each direction.
4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y separately.* Select the appropriate equation.
5. When the motion is divided into segments in time, remember that the final velocity of one time segment is the initial velocity for the next.
6. Keep in mind that there may be two possible answers to a kinematics problem.

Ex. continued

In the *x* direction, the spacecraft in a zero gravity zone has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22.0 m/s	7.0 s

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14.0 m/s	7.0 s

First, the motion in x-direciton…

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$\Delta x = v_{ox}t + \tfrac{1}{2}a_x t^2$$

$$= (22\,\mathrm{m/s})(7.0\ \mathrm{s}) + \tfrac{1}{2}(24\,\mathrm{m/s^2})(7.0\ \mathrm{s})^2 = +740\ \mathrm{m}$$

$$v_x = v_{ox} + a_x t$$

$$= (22\,\mathrm{m/s}) + (24\,\mathrm{m/s^2})(7.0\ \mathrm{s}) = +190\,\mathrm{m/s}$$

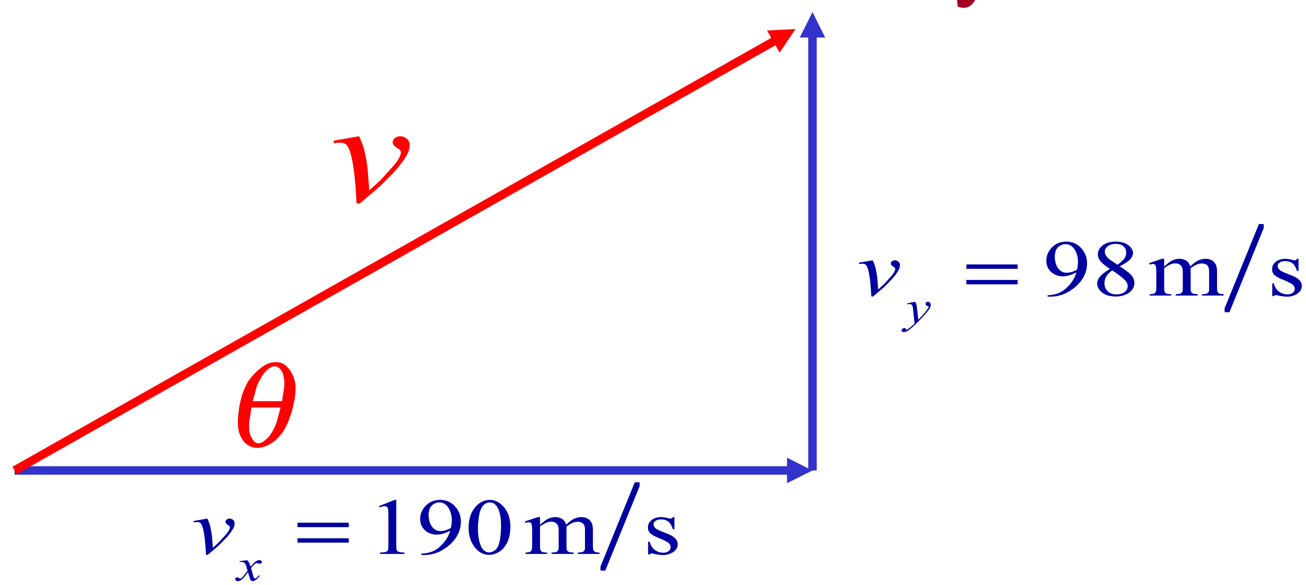
Now, the motion in y-direction…

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

$$\Delta y = v_{oy}t + \tfrac{1}{2}a_y t^2$$
$$= (14\,\mathrm{m/s})(7.0\ \mathrm{s}) + \tfrac{1}{2}(12\,\mathrm{m/s^2})(7.0\ \mathrm{s})^2 = +390\ \mathrm{m}$$

$$v_y = v_{oy} + a_y t$$
$$= (14\,\mathrm{m/s}) + (12\,\mathrm{m/s^2})(7.0\ \mathrm{s}) = +98\,\mathrm{m/s}$$

The final velocity…



$$v = \sqrt{(190\,\text{m/s})^2 + (98\,\text{m/s})^2} = 210\,\text{m/s}$$

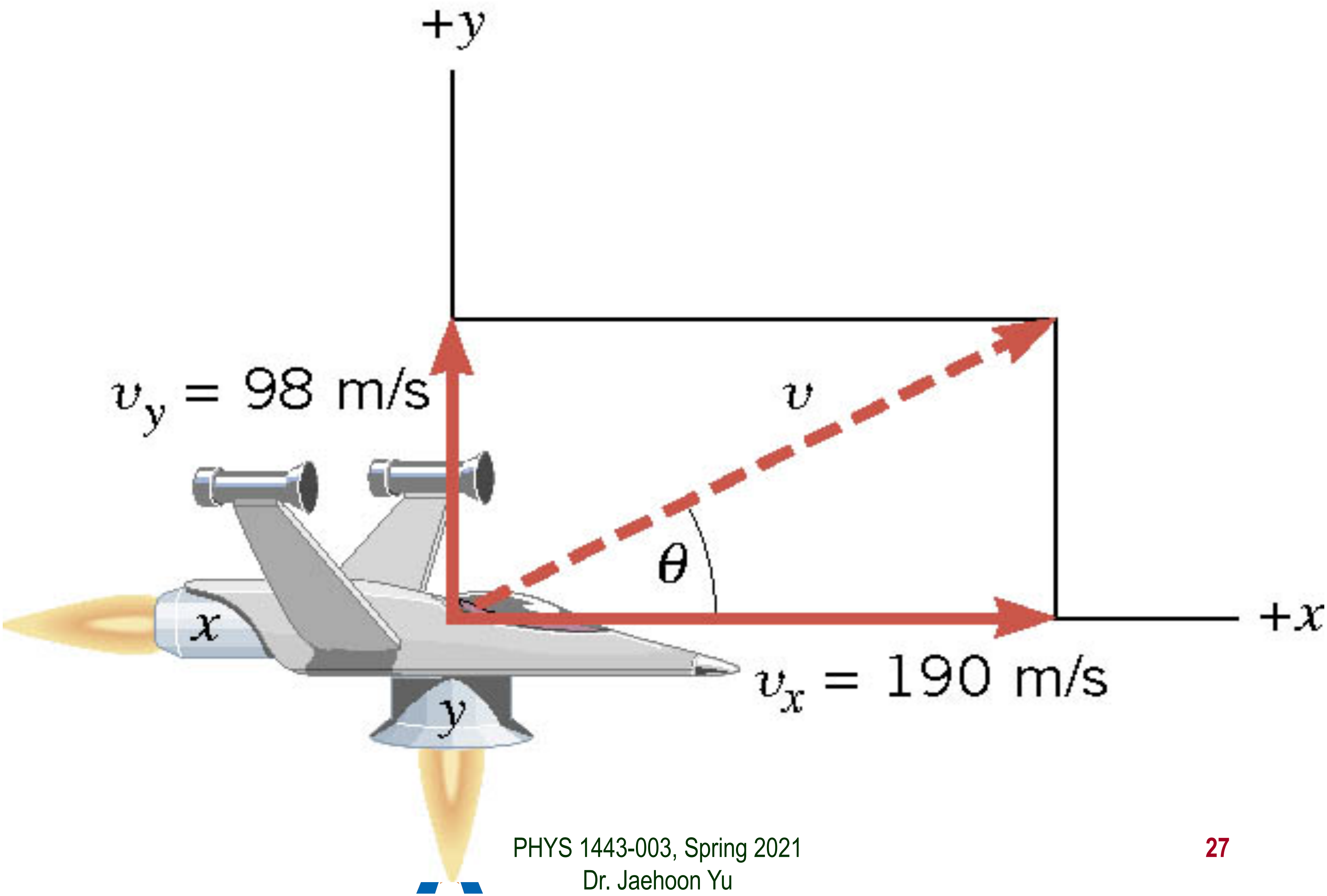
$$\theta = \tan^{-1}(98/190) = 27^\circ$$

A vector can be fully described when the magnitude and the direction are given. Any other way to describe it?

Yes, you are right! Using components and unit vectors!!

$$\vec{v} = v_x\vec{i} + v_y\vec{j} = (190\vec{i} + 98\vec{j})\,m/s$$

If we visualize the motion…



2-dim Motion Under Constant Acceleration

- Position vectors in x-y plane:

$$\vec{r}_i = x_i\vec{i} + y_i\vec{j} \qquad \vec{r}_f = x_f\vec{i} + y_f\vec{j}$$

- Velocity vectors in x-y plane:

$$\vec{v}_i = v_{xi}\vec{i} + v_{yi}\vec{j} \qquad \vec{v}_f = v_{xf}\vec{i} + v_{yf}\vec{j}$$

Velocity vectors in terms of the acceleration vector

X-comp $v_{xf} = v_{xi} + a_x t$ Y-comp $v_{yf} = v_{yi} + a_y t$

$$\vec{v}_f = \left(v_{xi} + a_x t\right)\vec{i} + \left(v_{yi} + a_y t\right)\vec{j} = \left(v_{xi}\vec{i} + v_{yi}\vec{j}\right) + \left(a_x\vec{i} + a_y\vec{j}\right)t =$$

$$= \vec{v}_i + \vec{a}t$$

2-dim Motion Under Constant Acceleration

- How are the 2D position vectors written in acceleration vectors?

Position vector components

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Putting them together in a vector form

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} =$$

$$= \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right)\vec{i} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2\right)\vec{j}$$

Regrouping the above

$$= \left(x_i\vec{i} + y_i\vec{j}\right) + \left(v_{xi}\vec{i} + v_{yi}\vec{j}\right)t + \frac{1}{2}\left(a_x\vec{i} + a_y\vec{j}\right)t^2$$

$$= \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

2D problems can be interpreted as two 1D problems in x and y

Example for 2-D Kinematic Equations

A particle starts at origin when t=0 with an initial velocity **v**=(20**i**-15**j**)m/s. The particle moves in the xy plane with $a_x=4.0m/s^2$. Determine the components of the velocity vector at any time t.

$$v_{xf} = v_{xi} + a_x t = 20 + 4.0t\,(m/s) \quad v_{yf} = v_{yi} + a_y t = -15 + 0t = -15\,(m/s)$$

Velocity vector $\vec{v}(t) = v_x(t)\vec{i} + v_y(t)\vec{j} = (20 + 4.0t)\vec{i} - 15\vec{j}\,(m/s)$

Compute the velocity and the speed of the particle at t=5.0 s.

$$\vec{v}_{t=5} = v_{x,t=5}\vec{i} + v_{y,t=5}\vec{j} = (20 + 4.0 \times 5.0)\vec{i} - 15\vec{j} = (40\vec{i} - 15\vec{j})\ m/s$$

$$speed = \left|\vec{v}\right| = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{(40)^2 + (-15)^2} = 43m/s$$

Example for 2-D Kinematic Eq. Cnt'd

Angle of the Velocity vector

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = \tan^{-1}\left(\frac{-3}{8}\right) = -21^\circ$$

Determine the x and y components of the particle at t=5.0 s.

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = 20\times 5 + \frac{1}{2}\times 4\times 5^2 = 150(m)$$

$$y_f = v_{yi}t = -15\times 5 = -75\ (m)$$

Can you write down the position vector at t=5.0s?

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} = 150\vec{i} - 75\vec{j}\,(m)$$