PHYS 1443 – Section 003
Lecture #13

Monday, March 29, 2021
Dr. Jaehoon Yu

- CH6: Work and Energy
  - Work-Kinetic Energy Theorem
  - Work under friction
  - Potential Energy and the Conservative Force
    - Gravitational Potential Energy
    - Elastic Potential Energy
  - Conservation of Energy

Today's homework is homework #8, due 11pm, Tuesday, April 13!!

Announcements

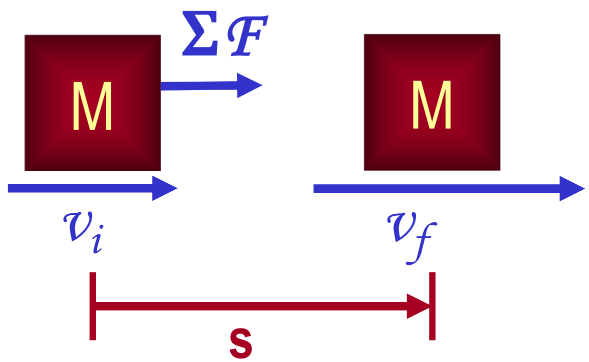
- Reading Assignments: CH7.9 and CH7.10

Reminder - SP #5: Comparing Fundamental Forces

- Two protons are separated by 1m.
  - Compute the gravitational force (F_G) between the two protons (10 points)
  - Compute the electric force (F_E) between the two protons (10 points)
  - Compute the ratio of FG/FE (5 points) and explain what this tells you (5 point)
- You must specify the formulae for each of the forces and the values of the necessary quantities, such as mass, charge, constants, etc, in your report!
- Maximum score: 30 points
- Please be sure to show details of your OWN, handwritten work!
- Due 2:30pm, this Wednesday, March 31
- Submit one pdf file SP5-YourLastName-YourFirstName.pdf on canvas assignment #5

Kinetic Energy and Work-Kinetic Energy Theorem

- Some problems are hard to solve using Newton's second law
  - If forces exerting on an object during the motion are complicated
  - Relate the work done on the object by the net force to the change of the speed of the object



Suppose net force **$\Sigma\mathcal{F}$** was exerted on an object for displacement **d** to increase its speed from v_i to v_f

The work on the object by the net force $\Sigma\mathcal{F}$ is

$$W = \left(\sum \vec{F}\right)\cdot\vec{s} = \left(ma\cos 0\right)s = \left(ma\right)s$$

Using the kinematic equation of motion

$$2as = v_f^2 - v_0^2 \quad\Rightarrow\quad as = \frac{v_f^2 - v_0^2}{2}$$

Kinetic Energy $\boxed{KE \equiv \frac{1}{2}mv^2}$

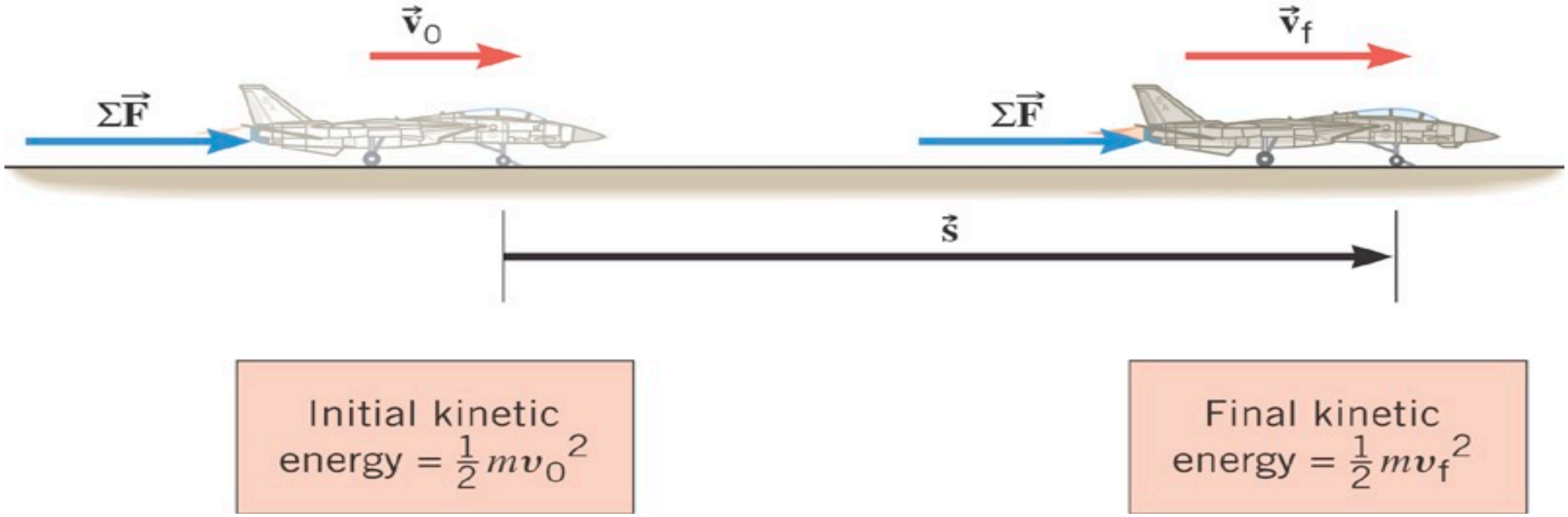
Work $W = \left(ma\right)s = \frac{1}{2}m\left(v_f^2 - v_0^2\right) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

Work $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$

Work done by the net force causes the change in the object's kinetic energy.

Work-Kinetic Energy Theorem

Work-Kinetic Energy Theorem



When a net external force by the jet engine does work on an object, the kinetic energy of the object changes according to

$$W = \mathrm{KE_f} - \mathrm{KE_o} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

Work and Kinetic Energy

A meaningful work in physics is done only when the sum of the forces exerted on an object made a change in the object's motion.

What does this mean?

However much tired your arms feel, if you were just holding an object without moving it you have not done any meaningful work on the object.

Mathematically, the work is written as the product of magnitudes of the net force vector, the magnitude of the displacement vector and the angle between them.

$$W = \sum\left(\vec{F}_i\right)\bullet\vec{d} = \left|\sum\left(\vec{F}_i\right)\right|\left|\vec{d}\right|\cos\theta$$

Kinetic Energy is the energy associated with the motion and capacity to perform work. Work causes change of energy after the completion ← **Work-Kinetic energy theorem**

$$K = \frac{1}{2}mv^2$$

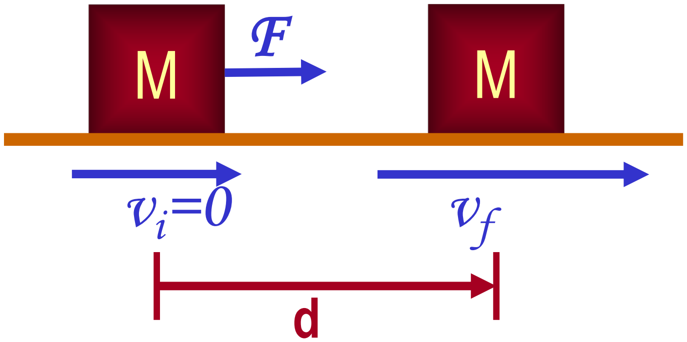
$$\sum W = K_f - K_i = \Delta K$$

Unit (Poll 6, 3, 1)?

Nm=Joule

Example for Work-KE Theorem

A 6.0kg block initially at rest is pulled to East along a horizontal, frictionless surface by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W = \vec{F}\cdot\vec{d} = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36\left(J\right)$$

From the work-kinetic energy theorem, we know $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

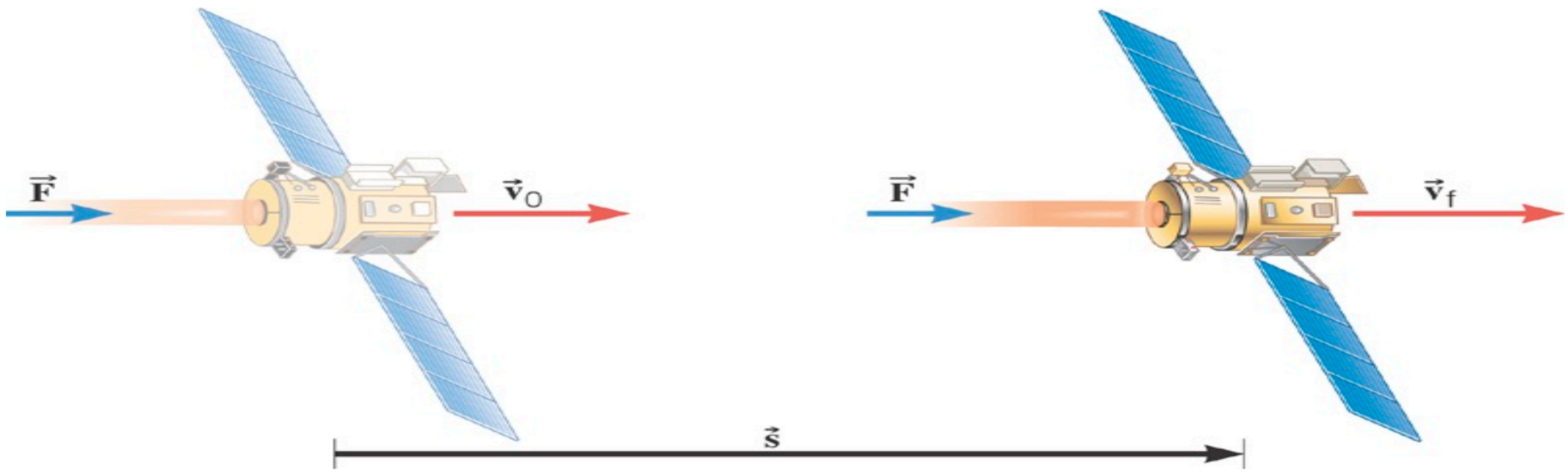
Since initial speed is 0, the above equation becomes $W = \frac{1}{2}mv_f^2$

Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 36}{6.0}} = 3.5m/s$$

Ex. Deep Space 1

The mass of the space probe is 474-kg, and its initial speed is 275 m/s. If the 56.0-mN force acts on the probe parallel through a displacement of 2.42×10^9m, what is its final speed?



$$\left[\left(\sum F\right)\cos\theta\right]s = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

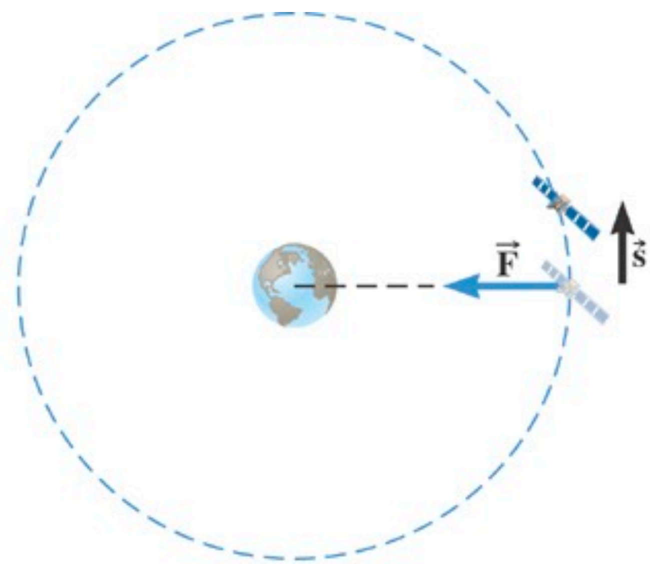
Solve for v_f

$$v_f = \sqrt{v_o^2 + 2\left(\sum F\cos\theta\right)s/m} = \sqrt{(275\,\mathrm{m/s})^2 + 2\left(5.60\times10^{-2}\mathrm{N}\right)\cos 0^\circ\left(2.42\times10^9\,\mathrm{m}\right)/474}$$

$$v_f = 805\,\mathrm{m/s}$$

Ex. Satellite Motion and Work By the Gravity

A satellite is moving about the earth in a circular orbit and an elliptical orbit. For these two orbits, determine whether the kinetic energy of the satellite changes during the motion.



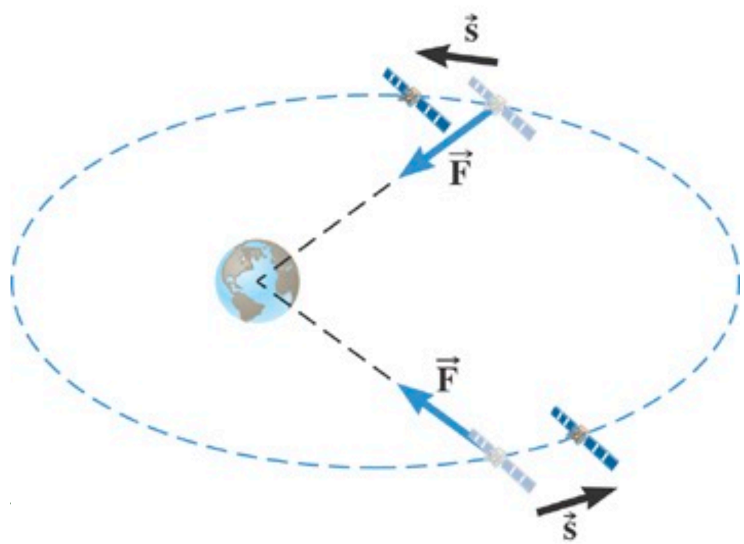
(a)

For a circular orbit No change! Why not?

Gravitational force is the only external force but it is perpendicular to the displacement. So no work.

For an elliptical orbit Changes! Why?

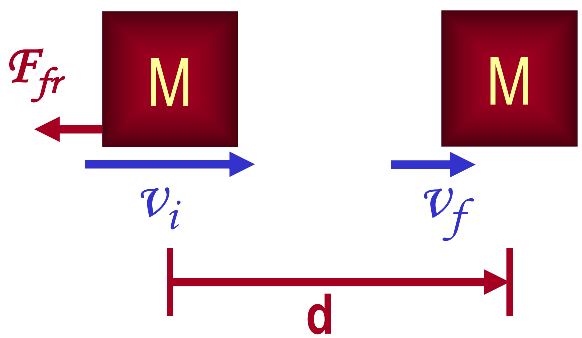
Gravitational force is the only external force but its angle with respect to the displacement varies. So it performs work.



(b)

Work and Energy Involving Kinetic Friction

- What do you think the work looks like if there is friction?
  - Static friction does not matter! Why? **It isn't there when the object is moving.**
  - Then which friction matters? **Kinetic Friction**



Friction force $\mathcal{F}_{fr}$ works on the object to slow down

The work on the object by the friction $\mathcal{F}_{fr}$ is

$$W_{fr} = \vec{F}_{fr} \bullet \vec{d} = F_{fr} d \cos(180) = -F_{fr} d \quad \Delta KE = -F_{fr} d$$

The negative sign means that the work is done on the friction!

The final kinetic energy of an object, taking into account its initial kinetic energy, friction force and all other sources of work, is

$$KE_f = KE_i + \sum W - F_{fr} d$$

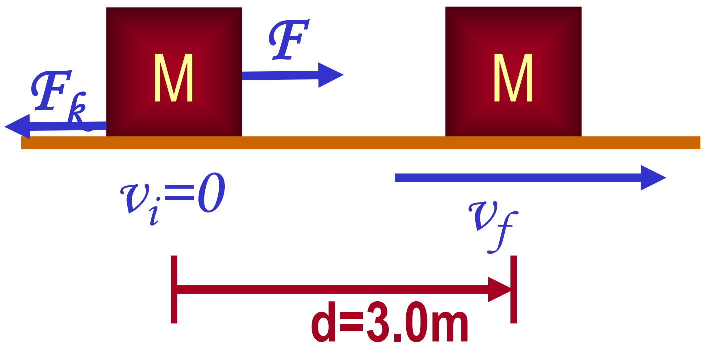
t=0, KE_i

Friction, Engine work

t=T, KE_f

Example of Work Under Friction

A 6.0kg block initially at rest is pulled to East along a horizontal surface with coefficient of kinetic friction μ_k=0.15 by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W_F = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36(J)$$

Work done by friction $\mathcal{F}_k$ is

$$W_k = \vec{F}_k \cdot \vec{d} = \left|\vec{F}_k\right|\left|\vec{d}\right|\cos\theta = \left|\mu_k mg\right|\left|\vec{d}\right|\cos\theta$$

$$= 0.15\times 6.0\times 9.8\times 3.0\cos 180 = -26(J)$$

Thus the total work is

$$W = W_F + W_k = 36 - 26 = 10(J)$$

Do this problem by first computing the net force. Is the answer the same?

Using work-kinetic energy theorem and the fact that the initial speed is 0, we obtain

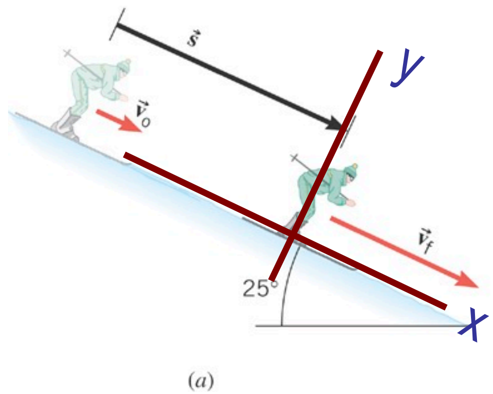
$$W = W_F + W_k = \frac{1}{2}mv_f^2$$

Solving the equation for v_f, we obtain

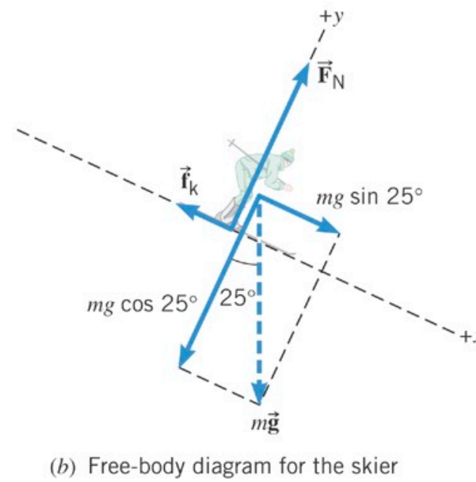
$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 10}{6.0}} = 1.8m/s$$

Ex. Downhill Skiing

A 58kg skier is coasting down a 25° slope. A kinetic frictional force of magnitude f_k=70N opposes her motion. At the top of the slope, the skier's speed is v_0=3.6m/s. Ignoring air resistance, determine the speed $\mathbf{v_f}$ at the point that is displaced 57m downhill.



(a)



(b) Free-body diagram for the skier

What are the forces in this motion?

Gravitational force: F_g Normal force: F_N Kinetic frictional force: f_k

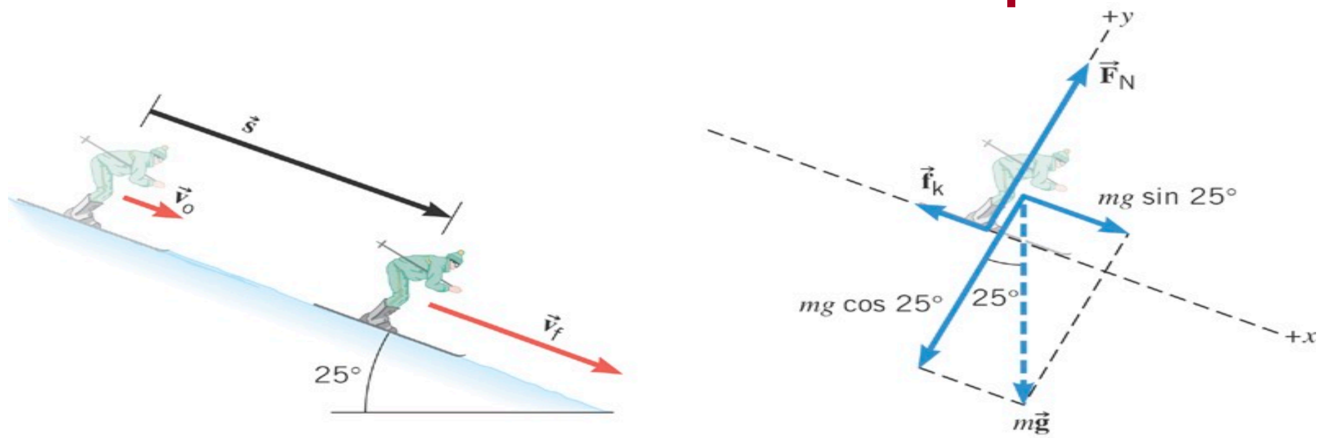
What are the X and Y component of the net force in this motion?

Y component $\sum F_y = F_{gy} + F_N = -mg\cos 25^\circ + F_N = 0$

From this we obtain $F_N = mg\cos 25^\circ = 58 \cdot 9.8 \cdot \cos 25^\circ = 515N$

What is the coefficient of kinetic friction? $f_k = \mu_k F_N \Rightarrow \mu_k = \frac{f_k}{F_N} = \frac{70}{515} = 0.14$

Ex. Now with the X component



(a) (b) Free-body diagram for the skier

X component $\sum F_x = F_{gx} - f_k = mg\sin 25^\circ - f_k = \left(58\cdot 9.8\cdot \sin 25^\circ - 70\right) = 170N = ma$

Total work by this force $W = \left(\sum F_x\right)\cdot s = \left(mg\sin 25^\circ - f_k\right)\cdot s = \left(58\cdot 9.8\cdot \sin 25^\circ - 70\right)\cdot 57 = 9700J$

From work-kinetic energy theorem $W = KE_f - KE_i \Rightarrow KE_f = \frac{1}{2}mv_f^2 = W + KE_i = W + \frac{1}{2}mv_0^2$

Solving for v_f $v_f^2 = \frac{2W + mv_0^2}{m} \Rightarrow v_f = \sqrt{\frac{2W + mv_0^2}{m}} = \sqrt{\frac{2\cdot 9700 + 58\cdot (3.6)^2}{58}} = 19\,m/s$

What is her acceleration? $\sum F_x = ma \Rightarrow a = \frac{\sum F_x}{m} = \frac{170}{58} = 2.93\,m/s^2$

Potential Energy & Conservation of Mechanical Energy

Energy associated with a system of objects ➔ *Stored energy which has the potential or the possibility to work or to convert to kinetic energy*

What does this mean?

In order to describe potential energy, U, a system must be defined.

The concept of potential energy can only be used under the special class of forces called the ***conservative force*** *which results in the principle of* ***conservation of mechanical energy.***

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

What are other forms of energies in the universe?

Mechanical Energy *Chemical Energy* *Biological Energy*

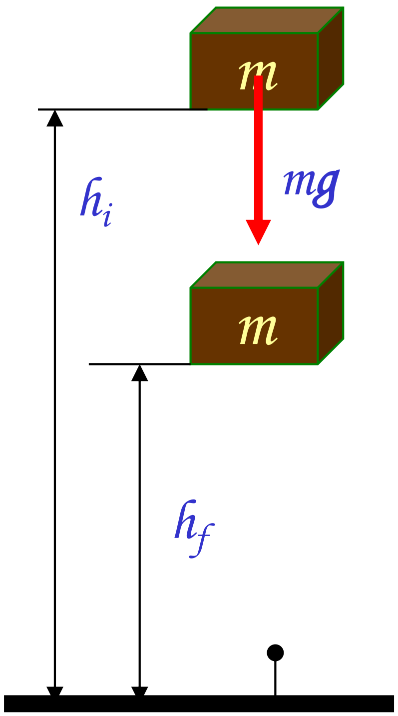
Electromagnetic Energy *Nuclear Energy*

These different types of energies are stored in the universe in many different forms!!!

If ALL forms of energy are accounted for, the total energy in the entire universe is conserved. It just transforms from one form to another.

Gravitational Potential Energy

This potential energy is given to an object by the gravitational field in the system of Earth by virtue of the object's height from an arbitrary zero level



When an object is falling, the gravitational force, mg, performs the work on the object, increasing the object's kinetic energy. So the potential energy of an object at height h, the potential to do work, is expressed as

$$PE = \vec{F}_g \cdot \vec{y} = \left|\vec{F}_g\right|\left|\vec{y}\right|\cos\theta = \left|\vec{F}_g\right|\left|\vec{y}\right| = mgh \qquad PE \equiv mgh$$

The work done on the object by the gravitational force as the brick drops from h_i to h_f is:

$$W_g = PE_i - PE_f$$
$$= mgh_i - mgh_f = -\Delta PE$$

$$(\text{since } \Delta PE = PE_f - PE_i)$$

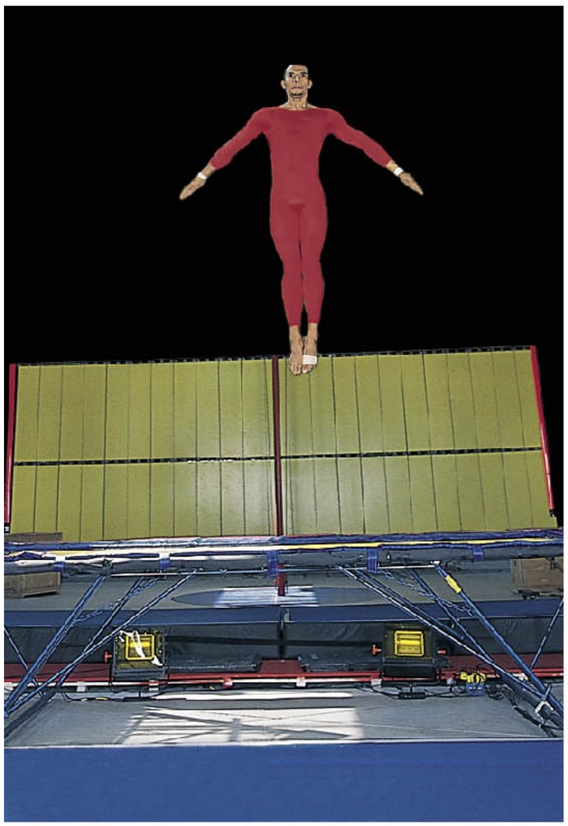
What does this mean?

Work by the gravitational force as the brick drops from h_i to h_f is the negative change of the system's potential energy

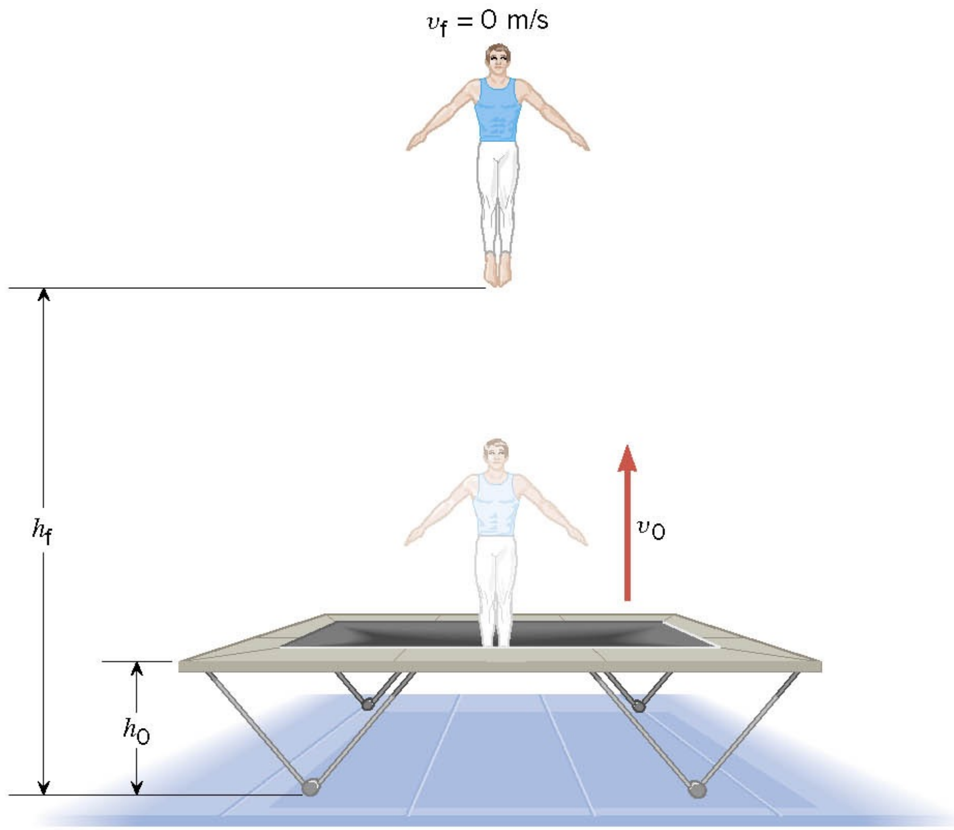
➔ Potential energy was spent in order for the gravitational force to increase the brick's kinetic energy.

Ex. A Gymnast on a Trampoline

The gymnast leaves the trampoline at an initial height of 1.20 m and reaches a maximum height of 4.80 m before falling back down. What was the initial speed of the gymnast?



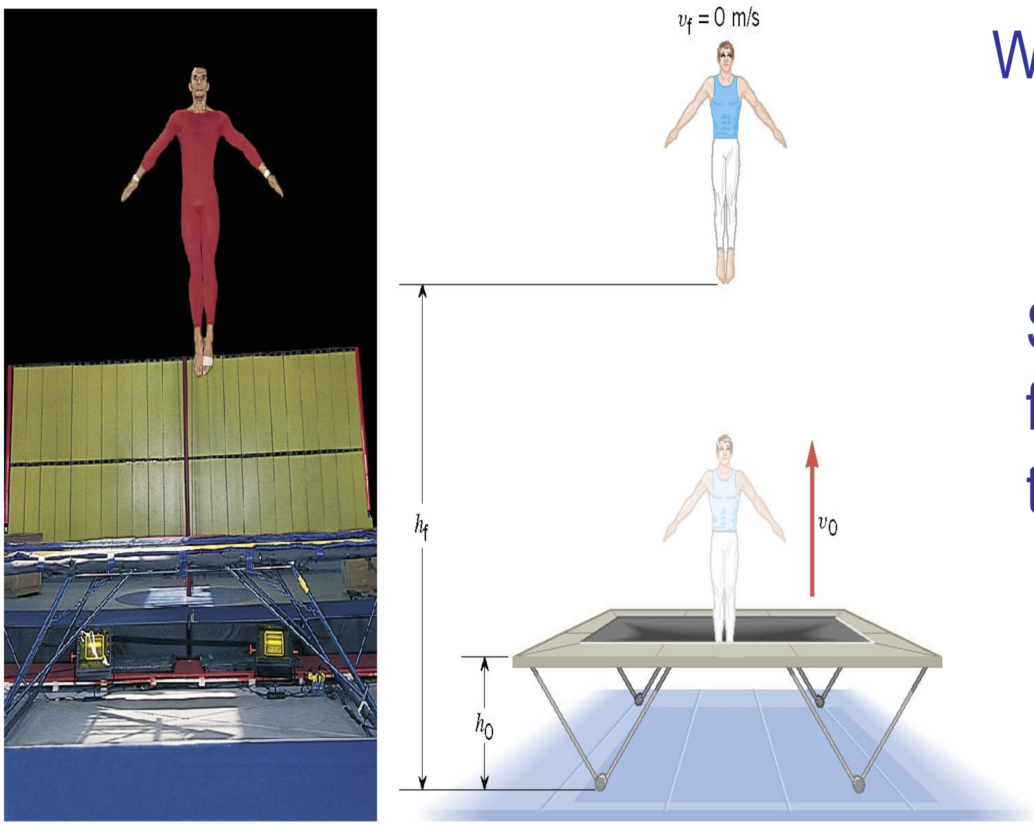
(a)



(b)

Ex. Continued

From the work-kinetic energy theorem $\mathrm{W} = \frac{1}{2}mv_{\mathrm{f}}^2 - \frac{1}{2}mv_o^2$



(a) (b)

Work done by the gravitational force

$$W_{\mathrm{gravity}} = mg\left(h_o - h_f\right)$$

Since at the maximum height, the final speed is 0. Using work-KE theorem, we obtain

$$\cancel{m}g\left(h_o - h_f\right) = -\frac{1}{2}\cancel{m}v_o^2$$

$$v_o = \sqrt{-2g\left(h_o - h_f\right)}$$

$$\therefore v_o = \sqrt{-2\left(9.80\,\mathrm{m/s^2}\right)\left(1.20\ \mathrm{m} - 4.80\ \mathrm{m}\right)} = 8.40\,\mathrm{m/s}$$

Conservative Forces and Potential Energy

The work done on an object by a conservative force is equal to the decrease in the potential energy of the system

$$W_c = \int_{x_i}^{x_f} F_x dx = -\Delta U$$

What does this statement tell you?

The work done by a conservative force is equal to the negative change of the potential energy associated with that force.

Only the changes in potential energy of a system is physically meaningful!!

We can rewrite the above equation in terms of the potential energy U

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

So the potential energy associated with a conservative force at any given position becomes

$$U_f(x) = -\int_{x_i}^{x_f} F_x dx + U_i$$

Potential energy function

What can you tell from the potential energy function above?

Since U_i is a constant, it only shifts the resulting $U_f(x)$ by a constant amount. One can always change the initial potential so that U_i can be 0.