PHYS 1443 – Section 003
Lecture #16

Wednesday, April 7, 2021
Dr. Jaehoon Yu

- CH7: Linear Momentum
  - Linear Momentum and Force
  - Conservative of Linear Momentum
  - Impulse
  - Collisions – Elastic and Inelastic

Announcements

- 2^{nd} non-comprehensive exam in class next Wednesday, Apr. 14
  - Covers CH5.5 to what we finish coming Monday, Apr. 12
  - Do NOT miss the exam. You will get an F!
  - BYOF: You may bring one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the test
  - No derivations, word definitions, setups or solutions of any problems, figures, pictures, diagrams or arrows, etc!
  - Must email me the photos of front and back of the formula sheet, including the blank at jaehoonyu@uta.edu no later than **12:00pm the day of the test**
    - The subject of the email should be the same as your file name
    - File name must be FS-E3-LastName-FirstName-SP21.pdf
    - Once submitted, you cannot change, unless I ask you to delete part of the sheet!

Special Project #6: Electric Power Usage

- Make a list of the power consumption of all electric and electronic devices at your home and compile them in a table. (10 points total for the first 10 items and 0.5 points each additional item.)
  - Similar electric/electronic devices count as one item.
    - All light bulbs make up one item, computers another, refrigerators, TVs, dryers (hair and clothes), electric cooktops, heaters, microwave ovens, electric ovens, dishwashers, etc.
    - All you have to do is to count add all wattages of the light bulbs together as the power of the item
- Estimate the cost of electricity for each of the items on the table using your own electric cost per kWh (if you don't find your own, use \$0.12/kWh) and put them in the relevant column. (5 points total for the first 10 items and 0.2 points each additional items)
- Estimate the the total amount of energy in Joules and the total electricity cost per day, per month and per year for your home. (8 points)
- Due: Beginning of the class, 2:30pm, Monday, Apr. 19
  - Scan all pages of your special project into the pdf format
  - Save all pages into one file with the filename SP6-YourLastName-YourFirstName.pdf
  - Submit on CANVAS

Your Name					Electricity Rate:					$/kWh		
Item Name	Rated power (W)	Number of devices	Number of Hours per day	Daily Power Consumption (kWh)	Energy Cost per kWh (cents)	Daily Energy Consumption (J).	Daily Energy Cost ($)	Monthly Energy Consumption (J)	Monthly Energy Cost (S)	Yearly Energy Consumption (J)	Yearly Energy Cost ($)	
Light Bulbs	30	4										
	40	6										
	60	15										
Heaters	1000	2										
	1500	1										
	2000	1										
Home Appliances (Fans, vacuum cleaners, hair dryers, pool pumps, etc)												
Air Conditioners												
Kitchen Appliances (Fridges, freeezers, cook tops, microwave ovens, toaster ovens, etc)												
Computing devices (desktop, laptop, ipad, mobile phones, printers, chargers, etc))												
Tools (power tools, electric mower, electric cutter, etc)												
Medical Devices (blood pressure machine, thermometer, etc)												
Transporations (electric cars, electric bicycles, electric motor cycles, etc												
Total												

More on Conservation of Linear Momentum in a Two Body System

In the previous class, we've learned that the total momentum of the system is conserved if no external forces are exerted on the system.

$$\sum \vec{p} = \vec{p}_2 + \vec{p}_1 = const$$

Kind/unit? (poll 6, 3)

What does this mean?

As in the case of energy conservation, this means that the total vector sum of all momenta in the system is the same before and after any interactions

Mathematically this statement can be written as

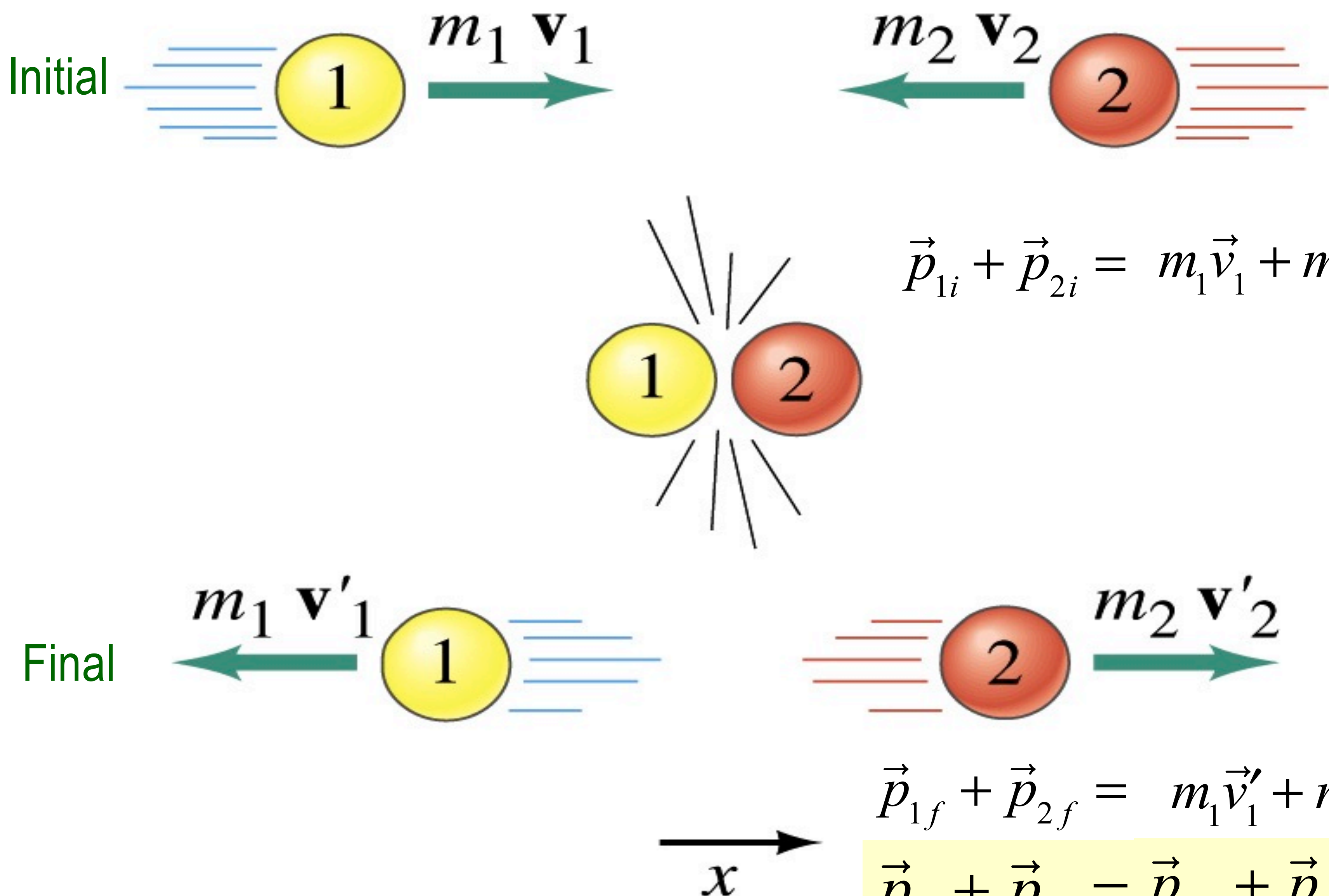
$$\vec{p}_{2i} + \vec{p}_{1i} = \vec{p}_{2f} + \vec{p}_{1f}$$

$$\sum_{system} P_{xi} = \sum_{system} P_{xf} \qquad \sum_{system} P_{yi} = \sum_{system} P_{yf} \qquad \sum_{system} P_{zi} = \sum_{system} P_{zf}$$

This can be generalized into conservation of linear momentum in many particle systems.

Whenever two or more particles in an ***isolated system*** *interact, the total momentum of the system remains constant.*

Linear Momentum Conservation



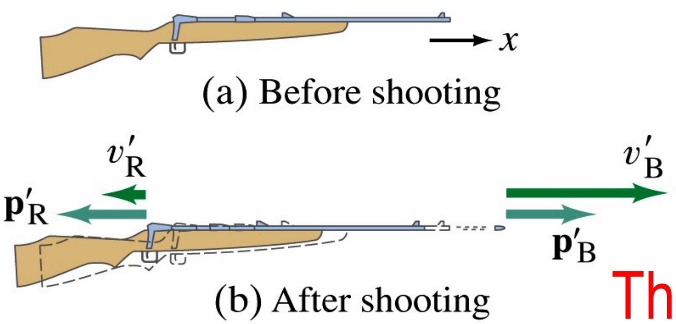
$$\vec{p}_{1i} + \vec{p}_{2i} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\vec{p}_{1f} + \vec{p}_{2f} = m_1\vec{v}'_1 + m_2\vec{v}'_2$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

Example: Rifle Recoil

Calculate the recoil velocity of a 5.0kg rifle that shoots a 0.020kg bullet at a speed of 620m/s.



From momentum conservation, we can write

$$\vec{p}_i = 0 = \vec{p}_f = \vec{P}_R + \vec{P}_B = m_R \vec{v}_R + m_B \vec{v}_B$$

The x-comp $\quad P_R + P_B = m_R v_R + m_B v_B = 0$

Solving the above for v_R and using the rifle's mass and the bullet's mass, we obtain

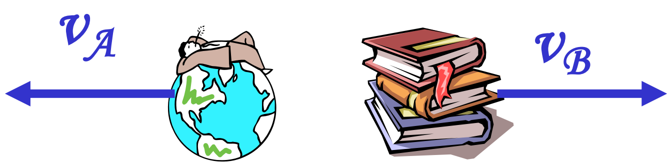
$$v_R = \frac{m_B}{m_R} v_B = \frac{0.020}{5.0} \cdot 620 = -2.5\, m/s$$

$$\vec{v}_R = -2.5\vec{i}\,(m/s)$$

Which law does this remind you of? (poll 14)

Example for Linear Momentum Conservation

Estimate an astronaut's (M=70kg) resulting velocity after he throws his book (m=1kg) to a direction in the space to move to another direction.



From momentum conservation, we can write

$$\vec{p}_i = 0 = \vec{p}_f = m_A\vec{v}_A + m_B\vec{v}_B$$

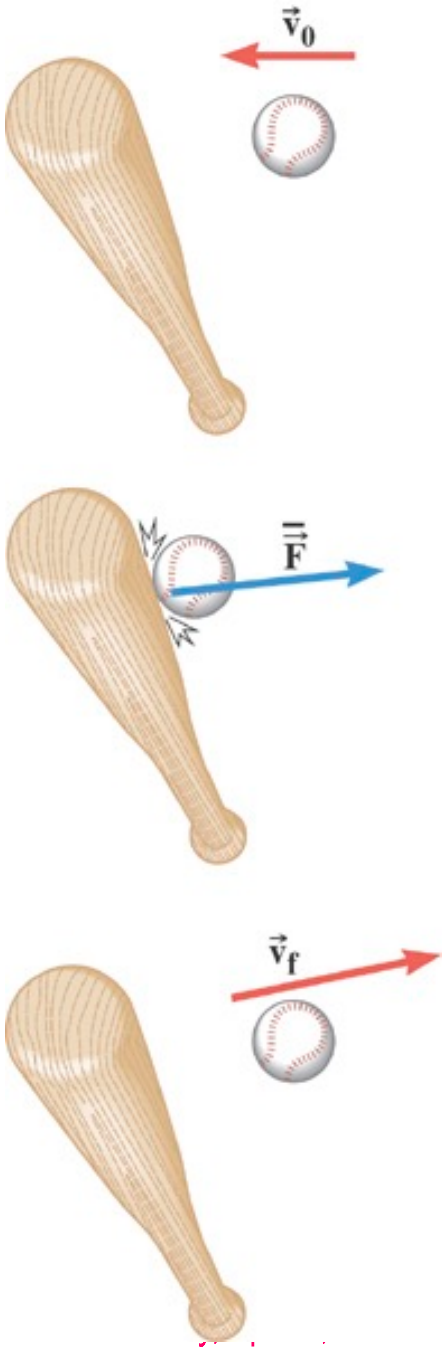
Assuming the astronaut's mass is 70kg, and the book's mass is 1kg and using linear momentum conservation

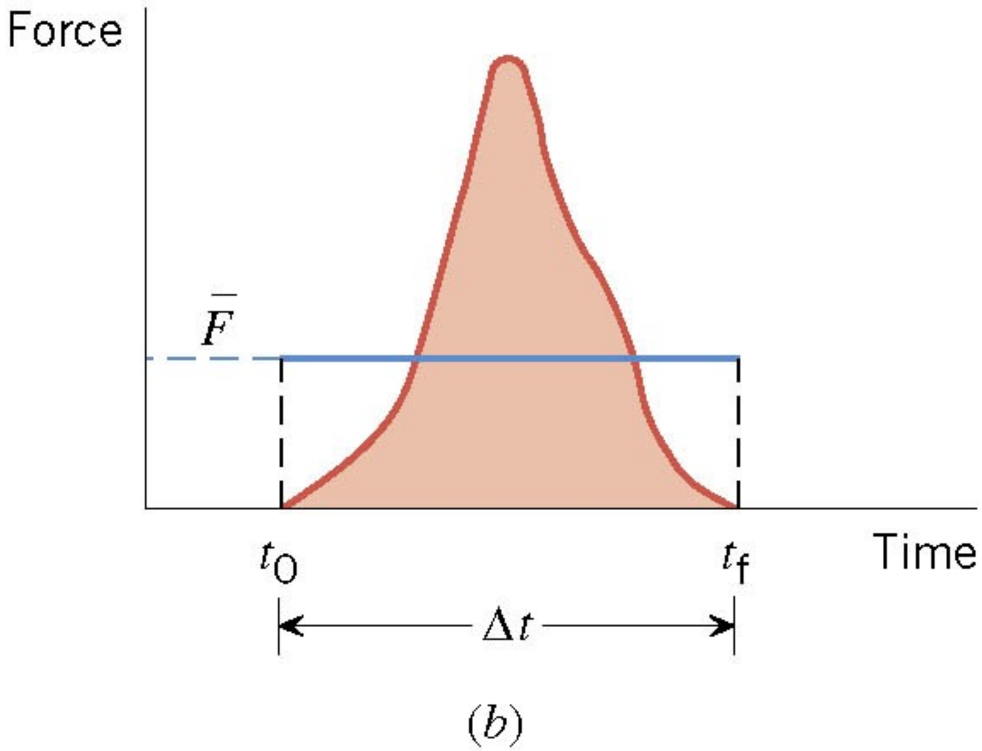
$$v_A = -\frac{m_B v_B}{m_A} = -\frac{1}{70}v_{\mathbf{B}}$$

Now if the book gained a velocity of 20 m/s in +x-direction, the Astronaut's velocity is

$$\vec{v}_A = -\frac{1}{70}\left(20\vec{i}\right) = -0.3\vec{i}\ \left(m/s\right)$$

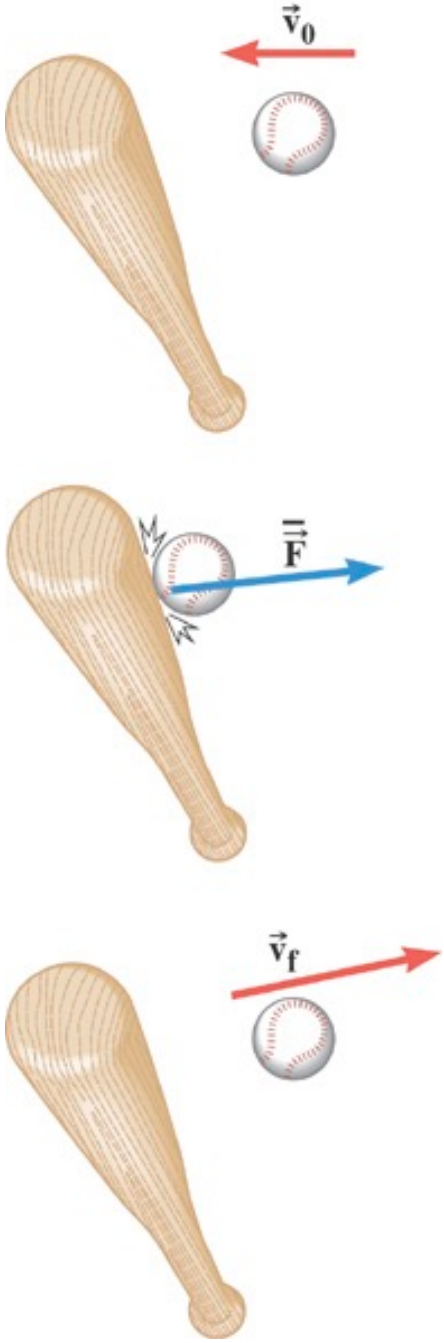
Impulse





There are many situations when the force on an object is not constant and in fact quite complicated!!

Ball Hit by a Bat



$$\bar{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}}_{\mathbf{f}} - \vec{\mathbf{v}}_{\mathbf{o}}}{\Delta t}$$

$$\sum \bar{\vec{\mathbf{F}}} = m\bar{\vec{\mathbf{a}}}$$

$$\sum \bar{\vec{\mathbf{F}}} = \frac{m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{o}}}{\Delta t}$$

Multiply either side by Δt

$$\left(\sum \bar{\vec{\mathbf{F}}}\right)\Delta t = m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{o}} = \vec{J}$$

Impulse and Linear Momentum

Net force causes change of momentum → Newton's second law

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = \vec{F}dt$$

By integrating the above equation in a time interval t_i to t_f, one can obtain impulse $\mathcal{J}$.

$$\int_{t_i}^{t_f} d\vec{p} = \vec{p}_f - \vec{p}_i = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}\,dt = \vec{J}$$

So what do you think an impulse is?

Effect of the force F acting on an object over the time interval $\Delta t = t_f - t_i$ is equal to the change of the momentum of the object caused by that force. Impulse is the degree of which an external force changes an object's momentum.

The above statement is called the __impulse-momentum theorem__ and is equivalent to __Newton's second law__.

What are the dimension and unit of Impulse? (poll 6,3) What is the direction of an impulse vector?

Defining a time-averaged force

$$\bar{\vec{F}} \equiv \frac{1}{\Delta t}\sum_i \vec{F}_i \Delta t$$

Impulse can be rewritten

$$\vec{J} \equiv \bar{\vec{F}}\Delta t$$

If force is constant

$$\vec{J} \equiv \vec{F}\Delta t$$

It is generally assumed that the impulse force acts on a short time but much greater than any other forces present.

Example for Impulse

In a crash test, an automobile of mass 1500kg collides with a wall. The initial and final velocities of the automobile are v_i= -15.0$\mathbf{i}$ m/s and v_f=2.60$\mathbf{i}$ m/s. If the collision lasts for 0.150 seconds, what would be the impulse caused by the collision and the average force exerted on the automobile?

Let's assume that the force involved in the collision is a lot larger than any other forces in the system during the collision. From the problem, the initial and final momentum of the automobile before and after the collision is

$$\vec{p}_i = m\vec{v}_i = 1500 \times (-15.0)\vec{i} = -22500\vec{i}\ kg \cdot m/s$$

$$\vec{p}_f = m\vec{v}_f = 1500 \times (2.60)\vec{i} = 3900\vec{i}\ kg \cdot m/s$$

Therefore the impulse on the automobile due to the collision is

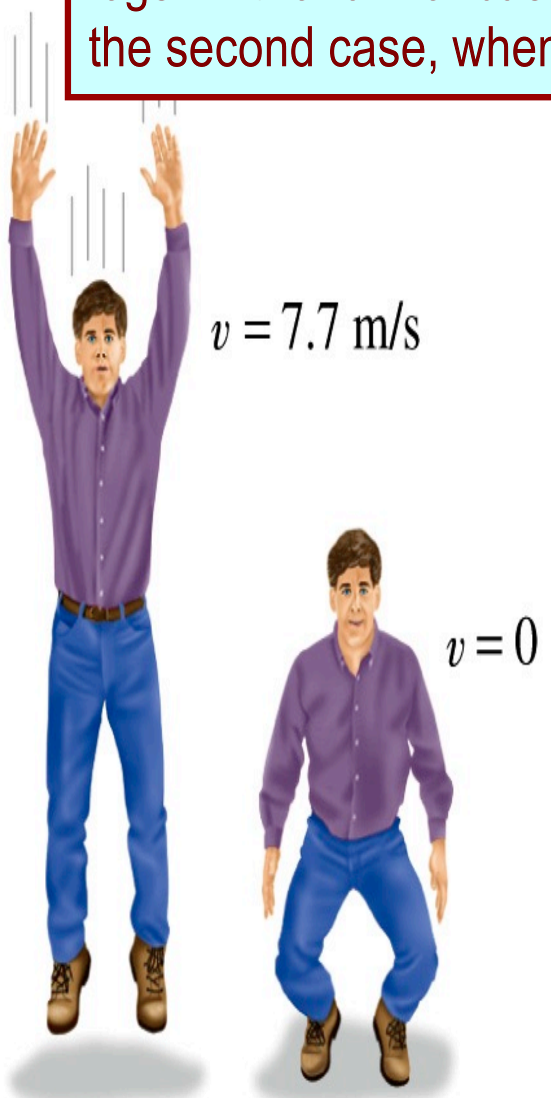
$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (3900 + 22500)\vec{i}\ kg \cdot m/s$$
$$= 26400\vec{i}\ kg \cdot m/s = 2.64 \times 10^4 \vec{i}\ kg \cdot m/s$$

The average force exerted on the automobile during the collision is

$$\vec{\overline{F}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{2.64 \times 10^4}{0.150}\vec{i}$$
$$= 1.76 \times 10^5 \vec{i}\ kg \cdot m/s^2 = 1.76 \times 10^5 \vec{i}\ \mathrm{N}$$

Another Example for Impulse

(a) Calculate the impulse experienced when a 70 kg person lands on firm ground after jumping from a height of 3.0 m. Then estimate the average force exerted on the person's feet by the ground, if the landing is (b) stiff-legged and (c) with bent legs. In the former case, assume the body moves 1.0cm during the impact, and in the second case, when the legs are bent, about 50 cm.



We don't know the force. How do we do this?

Obtain velocity of the person before striking the ground.

$$KE = -\Delta PE \qquad \frac{1}{2}mv^2 = -mg\left(y - y_i\right) = mgy_i$$

Solving the above for velocity v, we obtain

$$v = \sqrt{2gy_i} = \sqrt{2 \cdot 9.8 \cdot 3} = 7.7m/s$$

Then as the person strikes the ground, the momentum becomes 0 quickly giving the impulse

$$\vec{J} = \vec{\overline{F}}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0 - m\vec{v} =$$

$$= -70kg \cdot 7.7m/s\,\vec{j} = -540\vec{j}N \cdot s$$

Example cont'd

In coming to rest, the body decelerates from 7.7m/s to 0m/s in a distance d=1.0cm=0.01m.

The average speed during this period is $$\bar{v} = \frac{0+v_i}{2} = \frac{7.7}{2} = 3.8m/s$$

The time period the collision lasts is $$\Delta t = \frac{d}{\bar{v}} = \frac{0.01m}{3.8m/s} = 2.6\times 10^{-3}s$$

Since the magnitude of the impulse is $$\left|\vec{J}\right| = \left|\vec{\bar{F}}\Delta t\right| = 540N\cdot s$$

The average force on the feet during this landing is $$\bar{F} = \frac{J}{\Delta t} = \frac{540}{2.6\times 10^{-3}} = 2.1\times 10^5 N$$

How large is this average force? $$Weight = 70kg\cdot 9.8m/s^2 = 6.9\times 10^2 N$$

$$\bar{F} = 2.1\times 10^5 N = 304\times 6.9\times 10^2 N = 304\times Weight$$

If landed in stiff legged, the feet must sustain 300 times the body weight. The person will likely break his leg.

For bent legged landing: $$\Delta t = \frac{d}{\bar{v}} = \frac{0.50m}{3.8m/s} = 0.13s$$

$$\bar{F} = \frac{540}{0.13} = 4.1\times 10^3 N = 5.9Weight$$