PHYS 1443 – Section 003
Lecture #17

Monday, April 12, 2021
Dr. Jaehoon Yu

- CH7: Linear Momentum
  - Collisions – Elastic and Inelastic
  - Collisions in Two Dimensions
  - Center of Mass
  - Center of Mass of a Rigid Body
  - Motion of a Group of Objects
- CH8: Rotational Motion

Announcements

- 2nd non-comprehensive exam in class this Wednesday, Apr. 14
  - Roll call starts at 2:20pm!
  - Covers CH5.5 to CH7.10
  - **Do NOT miss the exam**. You will get an F!
  - BYOF: You may bring one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the test
  - No derivations, word definitions, setups or solutions of any problems, figures, pictures, diagrams or arrows, etc!
  - Must email me the photos of front and back of the formula sheet, including the blank at jaehoonyu@uta.edu no later than **12:00pm the day of the test**
    - The subject of the email should be the same as your file name
    - File name must be FS-E3-LastName-FirstName-SP21.pdf
    - Once submitted, you cannot change, unless I ask you to delete part of the sheet!

Reminder SP#6: Electric Power Usage

- Make a list of the power consumption of all electric and electronic devices at your home and compile them in a table. (10 points total for the first 10 items and 0.5 points each additional item.)
  - Similar electric/electronic devices count as one item.
    - All light bulbs make up one item, computers another, refrigerators, TVs, dryers (hair and clothes), electric cooktops, heaters, microwave ovens, electric ovens, dishwashers, etc.
    - All you have to do is to count add all wattages of the light bulbs together as the power of the item
- Estimate the cost of electricity for each of the items on the table using your own electric cost per kWh (if you don't find your own, use $0.12/kWh) and put them in the relevant column. (5 points total for the first 10 items and 0.2 points each additional items)
- Estimate the the total amount of energy in Joules and the total electricity cost per day, per month and per year for your home. (8 points)
- Due: Beginning of the class, 2:30pm, Monday, Apr. 19
  - Scan all pages of your special project into the pdf format
  - Save all pages into one file with the filename SP6-YourLastName-YourFirstName.pdf
  - Submit on CANVAS

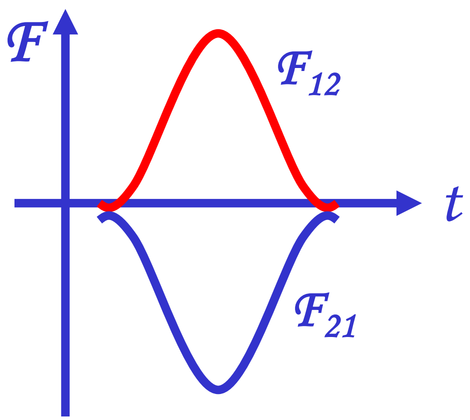
Your Name						Electricity Rate:				$/kWh		
Item Name	Rated power (W)	Number of devices	Number of Hours per day	Daily Power Consumption (kWh)	Energy Cost per kWh (cents)	Daily Energy Consumption (J).	Daily Energy Cost ($)	Monthly Energy Consumption (J)	Monthly Energy Cost ($)	Yearly Energy Consumption (J)	Yearly Energy Cost ($)	
Light Bulbs	30	4										
	40	6										
	60	15										
Heaters	1000	2										
	1500	1										
	2000	1										
Home Appliances (Fans, vacuum cleaners, hair dryers, pool pumps, etc)												
Air Conditioners												
Kitchen Appliances (Fridges, freeezers, cook tops, microwave ovens, toaster ovens, etc)												
Computing devices (desktop, laptop, ipad, mobile phones, printers, chargers, etc))												
Tools (power tools, electric mower, electric cutter, etc)												
Medical Devices (blood pressure machine, thermometer, etc)												
Transporations (electric cars, electric bicycles, electric motor cycles, etc												
Total												

Collisions

Generalized collisions must cover not only the physical contact but also the collisions without a physical contact such as that of electromagnetic ones on a microscopic scale.

Consider a case of a collision between a proton on a helium ion.

The collisions of these ions never involve physical contact because the electromagnetic repulsive force between these two become great as they get closer causing a collision.



Assuming no external forces, the force exerted on particle 1 by particle 2, $\mathcal{F}_{21}$, changes the momentum of particle 1 by

$$\boxed{d\vec{p}_1 = \vec{F}_{21}dt}$$

Likewise for particle 2 by particle 1

$$\boxed{d\vec{p}_2 = \vec{F}_{12}dt}$$

Using Newton's 3rd law we obtain

$$d\vec{p}_2 = \vec{F}_{12}dt = -\vec{F}_{21}dt = -d\vec{p}_1$$

So the **momentum change of the system** in a collision is **0**, and the momentum is conserved

$$d\vec{p} = d\vec{p}_1 + d\vec{p}_2 = 0$$

$$\vec{p}_{system} = \vec{p}_1 + \vec{p}_2 = \text{constant}$$

Elastic and Inelastic Collisions

Momentum is conserved in any collisions *as long as external forces are negligible.*

Collisions are classified as elastic or inelastic based on whether the ***kinetic energy is conserved, meaning whether KE is the same*** *before and after the collision.*

Elastic Collision	*A collision in which* ***the total kinetic energy and momentum*** *are the same before and after the collision.*
Inelastic Collision	*A collision in which* ***the momentum*** *is the same before and after the collision but not the total kinetic energy .*

Two types of inelastic collisions:Perfectly inelastic and inelastic

Perfectly Inelastic: *Two objects stick together after the collision, moving together with the same velocity.*

Inelastic: *Colliding objects do not stick together after the collision but some kinetic energy is lost.*

Note: Momentum is conserved in all collisions but kinetic energy is only in elastic collisions.

Elastic and Perfectly Inelastic Collisions

In a perfectly inelastic collision, the objects stick together after the collision, moving together. Momentum is conserved in this collision, so the final velocity of the stuck system is

$$m_1\vec{v}_{1i}+m_2\vec{v}_{2i}=(m_1+m_2)\vec{v}_f$$

$$\boxed{\vec{v}_f=\frac{m_1\vec{v}_{1i}+m_2\vec{v}_{2i}}{(m_1+m_2)}}$$

How about an elastic collision?

$$m_1\vec{v}_{1i}+m_2\vec{v}_{2i}=m_1\vec{v}_{1f}+m_2\vec{v}_{2f}$$

$$\frac{1}{2}m_1v_{1i}^2+\frac{1}{2}m_2v_{2i}^2=\frac{1}{2}m_1v_{1f}^2+\frac{1}{2}m_2v_{2f}^2$$

In an elastic collision, both the momentum and the kinetic energy are conserved. Therefore, the final speeds in an elastic collision can be obtained in terms of initial speeds as

$$m_1\left(v_{1i}^2-v_{1f}^2\right)=m_2\left(v_{2i}^2-v_{2f}^2\right)$$

$$m_1\left(v_{1i}-v_{1f}\right)\left(v_{1i}+v_{1f}\right)=m_2\left(v_{2i}-v_{2f}\right)\left(v_{2i}+v_{2f}\right)$$

From momentum conservation above

$$m_1\left(v_{1i}-v_{1f}\right)=m_2\left(v_{2i}-v_{2f}\right)$$

$$\boxed{v_{1f}=\left(\frac{m_1-m_2}{m_1+m_2}\right)v_{1i}+\left(\frac{2m_2}{m_1+m_2}\right)v_{2i}}$$

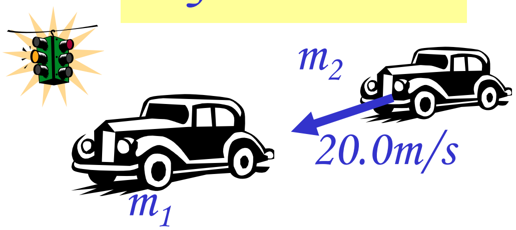
$$\boxed{v_{2f}=\left(\frac{2m_1}{m_1+m_2}\right)v_{1i}+\left(\frac{m_1-m_2}{m_1+m_2}\right)v_{2i}}$$

What happens when the two masses are the same? – See the video

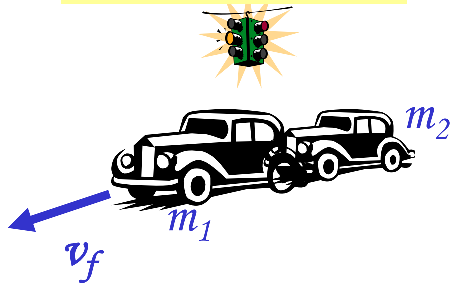
Example for a Collision in 1D

A car of mass 1800kg stopped at a traffic light is rear-ended by a 900kg car, and the two become entangled. If the lighter car was moving at 20.0m/s before the collision what is the velocity of the entangled cars after the collision?

Before collision



After collision



The momenta before and after the collision are

$$p_i = m_1 v_{1i} + m_2 v_{2i} = 0 + m_2 v_{2i}$$

$$p_f = m_1 v_{1f} + m_2 v_{2f} = (m_1 + m_2) v_f$$

Since momentum of the system must be conserved

$$p_i = p_f \quad \Rightarrow \quad (m_1 + m_2) v_f = m_2 v_{2i}$$

$$v_f = \frac{m_2 v_{2i}}{(m_1 + m_2)} = \frac{900 \times 20.0}{900 + 1800} = 6.67 m/s$$

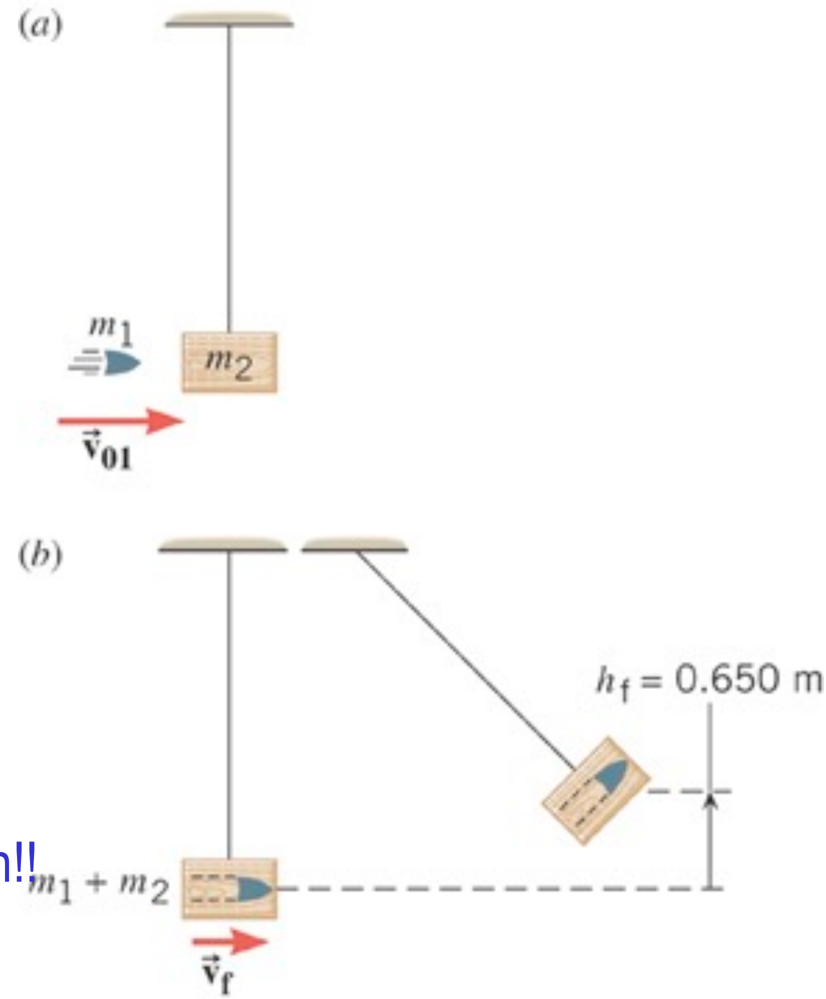
What can we learn from these equations on the direction and magnitude of the velocity before and after the collision?

The cars are moving in the same direction as the lighter car's original direction to conserve momentum.

The magnitude is inversely proportional to its own mass.

Example: A Ballistic Pendulum

The mass of a block of wood is 2.50-kg and the mass of the bullet is 0.0100-kg. The block swings to a maximum height of 0.650 m above the initial position. Find the initial speed of the bullet. Ignore the resistance from the air and the pivot point.



What kind of collision? Perfectly inelastic collision

No net external force ➔ momentum conserved

$$m_1 v_{f1} + m_2 v_{f2} = m_1 v_{01} + m_2 v_{02}$$

$$(m_1 + m_2)\, v_f = m_1 v_{01}$$

Solve for V_{01}

$$v_{01} = \frac{(m_1 + m_2) v_f}{m_1}$$

What do we not know? Final speed after collision!!

How can we get it? Using the mechanical energy conservation!

Ex. A Ballistic Pendulum, cnt'd

Now using the mechanical energy conservation

$$\tfrac{1}{2}mv^2 = mgh$$

$$\cancel{(m_1+m_2)}gh_f = \tfrac{1}{2}\cancel{(m_1+m_2)}v_f^2$$

$$gh_f = \tfrac{1}{2}v_f^2$$

Solve for V_f

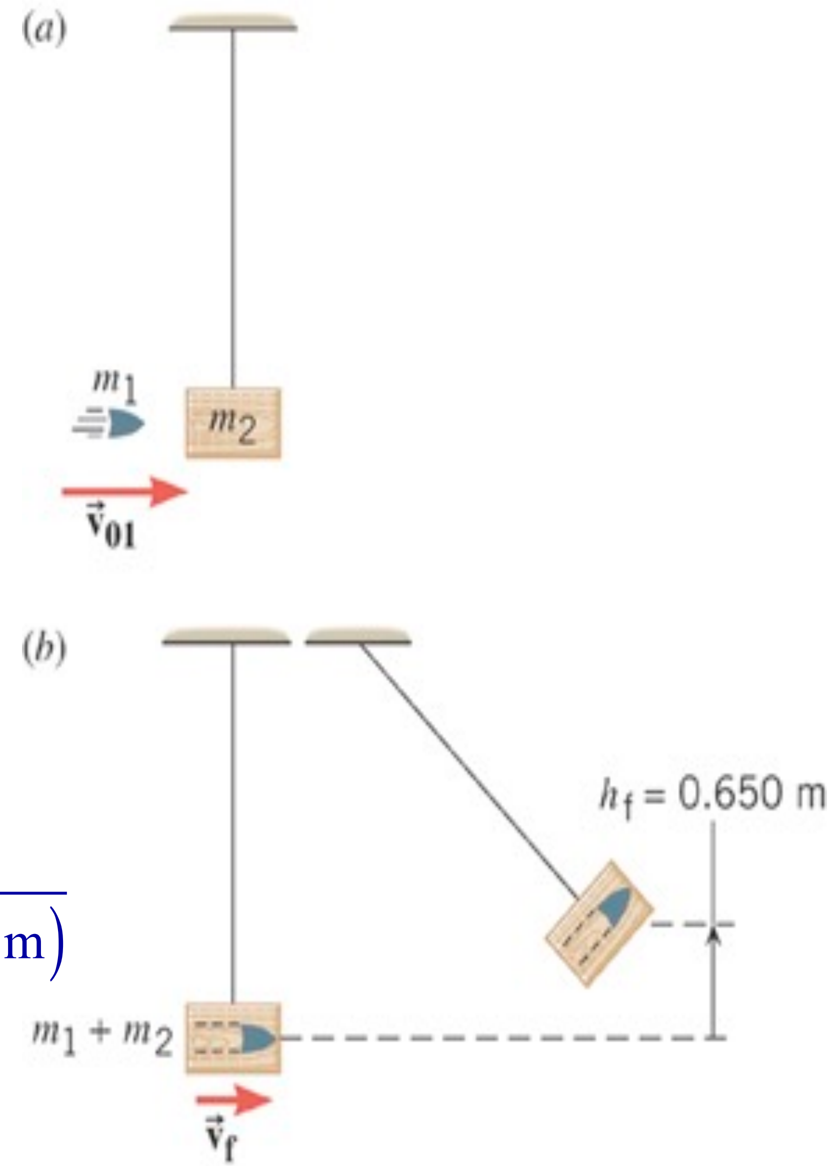
$$v_f = \sqrt{2gh_f} = \sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(0.650\ \mathrm{m}\right)}$$

Using the solution obtained previously, we obtain

$$v_{01} = \frac{(m_1+m_2)v_f}{m_1} = \frac{(m_1+m_2)\sqrt{2gh_f}}{m_1}$$

$$= \left(\frac{0.0100\ \mathrm{kg} + 2.50\ \mathrm{kg}}{0.0100\ \mathrm{kg}}\right)\sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(0.650\ \mathrm{m}\right)}$$

$$= +896\,\mathrm{m/s}$$



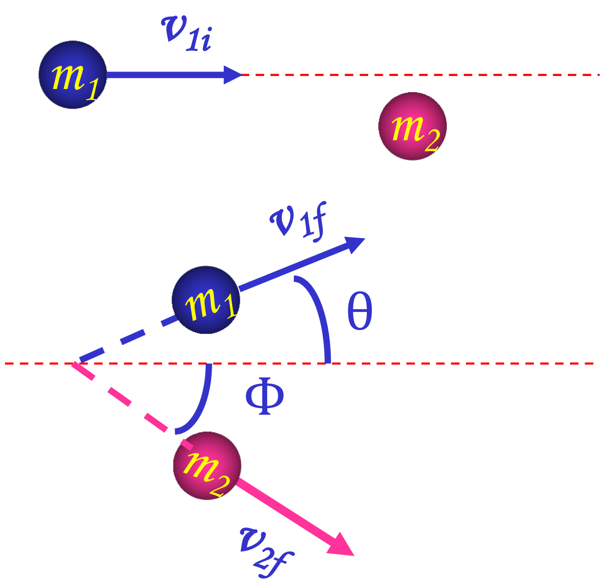
Two dimensional Collisions

In two dimension, one needs to use the components of the momenta and apply the momentum conservation to solve physical problems.

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

x-comp. $$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

y-comp. $$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$



Consider a system of two particle collisions and scatters in two dimension as shown in the picture. (This is the case at a fixed target accelerator experiment.) The momentum conservation tells us:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1i}$$

$$m_1 v_{1ix} = m_1 v_{1fx} + m_2 v_{2fx} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\phi$$

$$m_1 v_{1iy} = 0 = m_1 v_{1fy} + m_2 v_{2fy} = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\phi$$

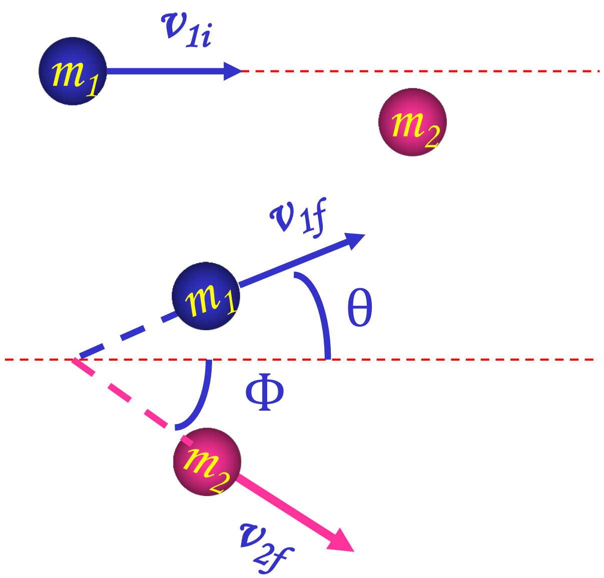
And for the elastic collisions, the kinetic energy is conserved:

$$\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

What do you think we can learn from these relationships?

Example for Two Dimensional Collisions

Proton #1 with a speed 3.50×10^5 m/s collides elastically with proton #2 initially at rest. After the collision, proton #1 moves at an angle of 37° to the horizontal axis and proton #2 deflects at an angle ϕ to the same axis. Find the final speeds of the two protons and the scattering angle of proton #2, Φ.



Since both the particles are protons $m_1=m_2=m_p$.

Using momentum conservation, one obtains

x-comp. $m_p v_{1i} = m_p v_{1f}\cos\theta + m_p v_{2f}\cos\phi$

y-comp. $m_p v_{1f}\sin\theta - m_p v_{2f}\sin\phi = 0$

Canceling m_p and putting in all known quantities, one obtains

$$v_{1f}\cos 37^\circ + v_{2f}\cos\phi = 3.50\times10^5 \quad (1)$$

$$v_{1f}\sin 37^\circ = v_{2f}\sin\phi \quad (2)$$

From kinetic energy conservation:

$$\left(3.50\times10^5\right)^2 = v_{1f}^2 + v_{2f}^2 \quad (3)$$

Solving Eqs. 1-3, one gets

$$v_{1f} = 2.80\times10^5\, m/s$$
$$v_{2f} = 2.11\times10^5\, m/s$$
$$\phi = 53.0^\circ$$

Do this at home☺

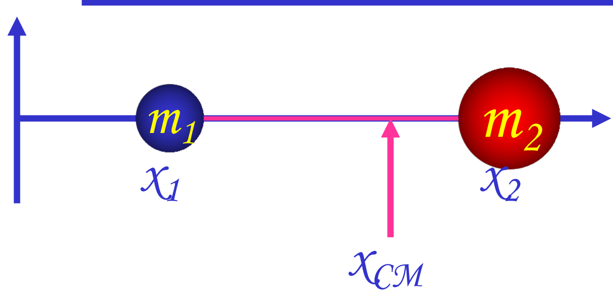
Center of Mass

We've been solving physical problems treating objects as sizeless points with masses but in reality, objects have shapes with masses distributed throughout the body.

The center of mass of a system is the average position of the system's mass and represents the motion of the system as if all the mass is on this point.

What does above statement tell you concerning the forces being exerted on the system?

The total external force exerted on the system of total mass $\mathcal{M}$ causes the center of mass to move at an acceleration given by $\vec{a} = \sum \vec{F} / M$ *as if all the mass of the system is concentrated on the center of mass.*



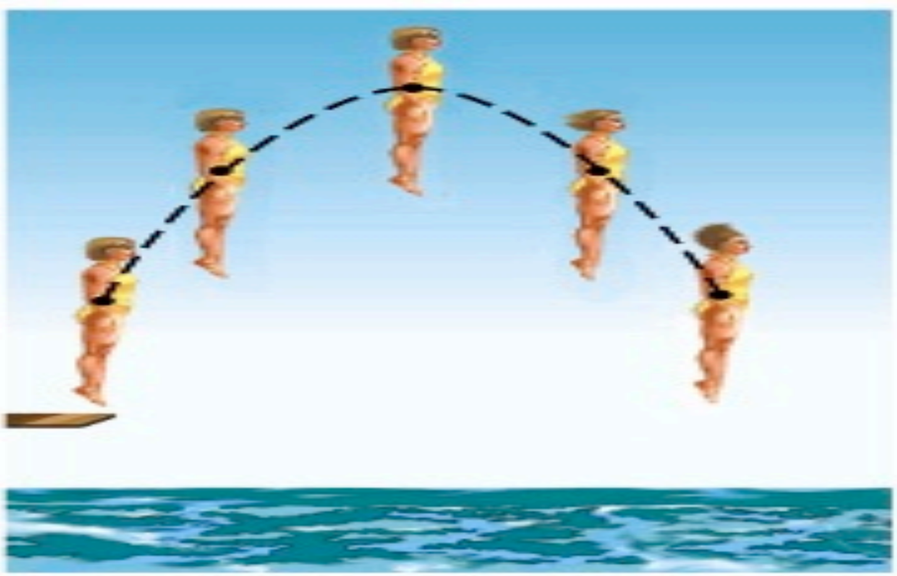
Consider a massless rod with two balls attached at either end.

The position of the center of mass of this system is the mass averaged position of the system

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

CM is closer to the heavier object

Motion of a Diver and the Center of Mass



(a)

A diver performs a simple dive. The motion of the center of mass follows a parabola since it is a projectile motion.

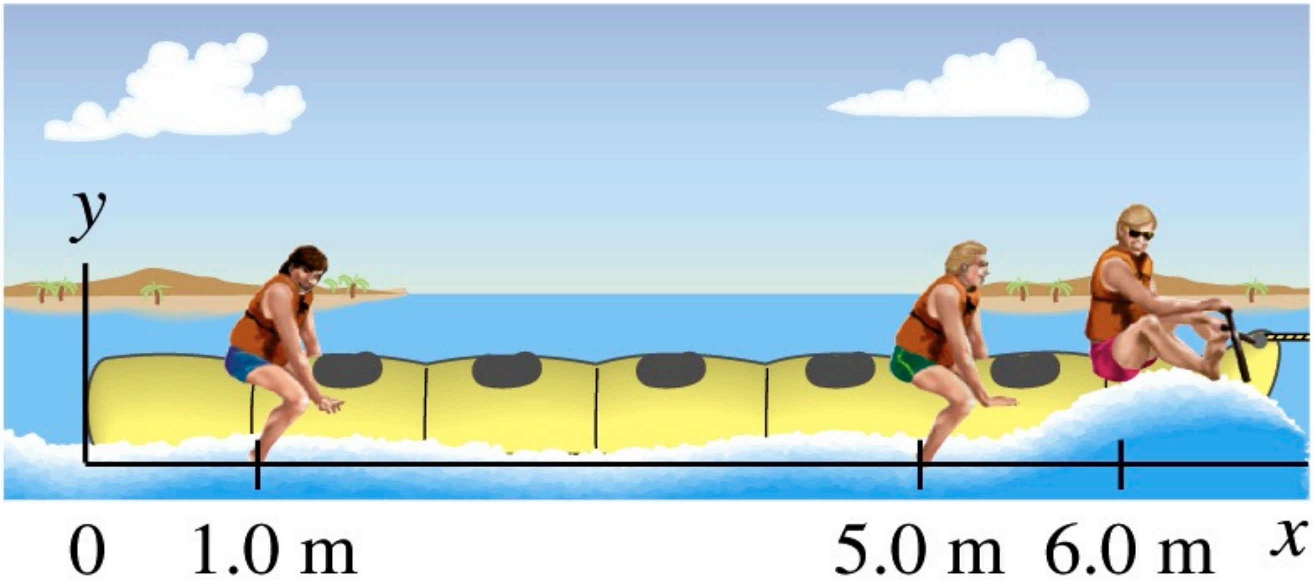
(b)

A diver performs a complicated dive. The motion of the center of mass still follows the same parabola since it still is a projectile motion.

The motion of the center of mass of the diver is always the same.

Example for CM

Thee people of roughly equivalent mass M on a lightweight (air-filled) banana boat sit along the x axis at positions x_1=1.0m, x_2=5.0m, and x_3=6.0m. Find the position of CM.



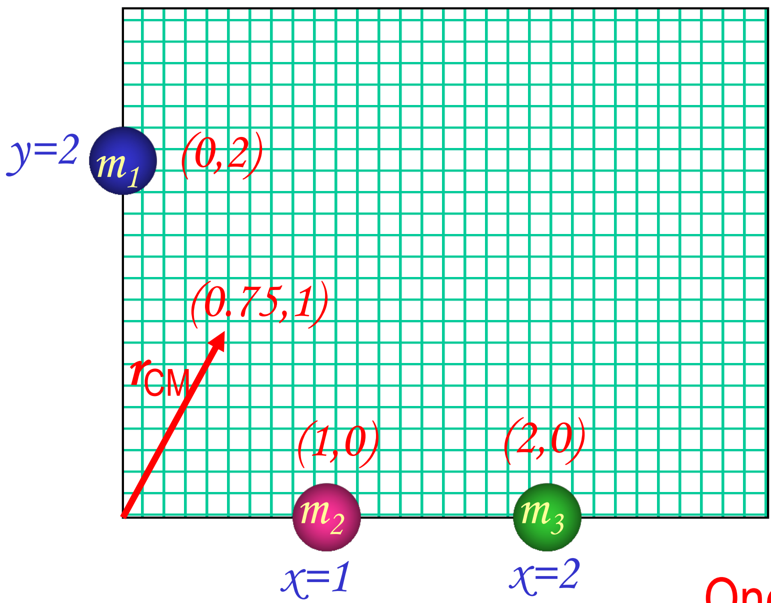
Using the formula for CM

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$= \frac{M \cdot 1.0 + M \cdot 5.0 + M \cdot 6.0}{M + M + M} = \frac{12.0M}{3M} = 4.0(m)$$

Example for Center of Mass in 2-D

A system consists of three particles as shown in the figure. Find the position of the center of mass of this system.



Using the formula for CM for each position vector component

$$x_{CM}=\frac{\sum_i m_i x_i}{\sum_i m_i} \qquad y_{CM}=\frac{\sum_i m_i y_i}{\sum_i m_i}$$

One obtains $\vec{r}_{CM}=x_{CM}\vec{i}+y_{CM}\vec{j}=\frac{(m_2+2m_3)\vec{i}+2m_1\vec{j}}{m_1+m_2+m_3}$

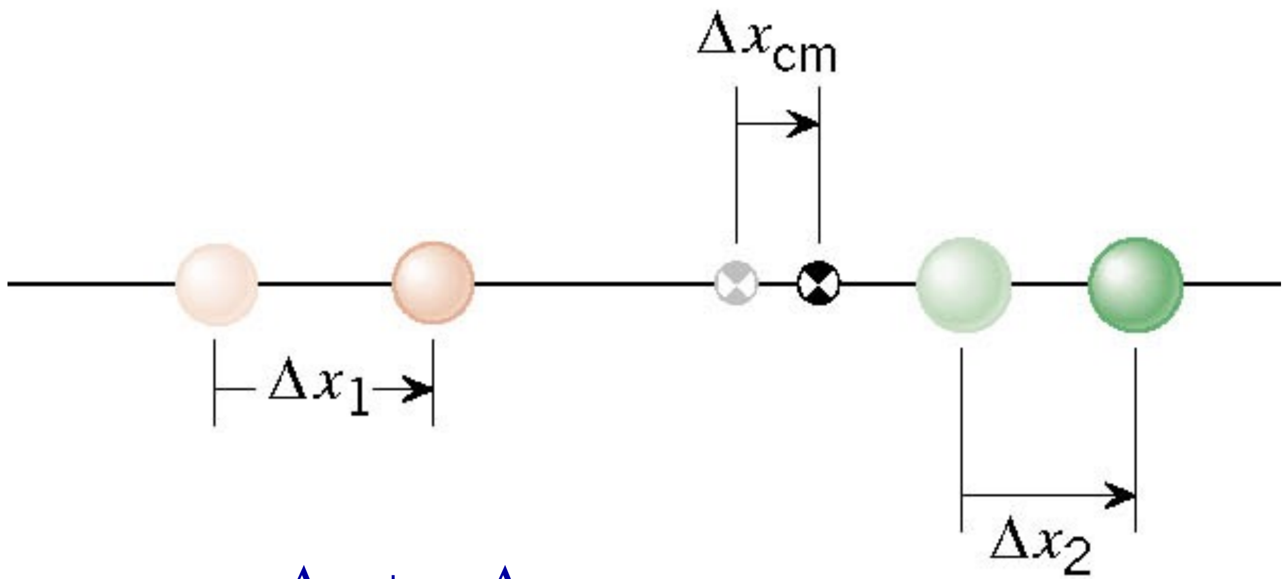
$$x_{CM}=\frac{\sum_i m_i x_i}{\sum_i m_i}=\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}=\frac{m_2+2m_3}{m_1+m_2+m_3}$$

$$y_{CM}=\frac{\sum_i m_i y_i}{\sum_i m_i}=\frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}=\frac{2m_1}{m_1+m_2+m_3}$$

If $m_1=2kg; m_2=m_3=1kg$

$$\vec{r}_{CM}=\frac{3\vec{i}+4\vec{j}}{4}=0.75\vec{i}+\vec{j}$$

Velocity of the Center of Mass



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

$$\Rightarrow \quad v_{cm} = \frac{\Delta x_{cm}}{\Delta t} = \frac{m_1 \Delta x_1 / \Delta t + m_2 \Delta x_2 / \Delta t}{m_1 + m_2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

The Ice Skater Problem

Starting from rest, two skaters push off against each other on ice where friction is negligible. One is a 54-kg woman and one is a 88-kg man. The woman moves away with a velocity of +2.5 m/s. Man's velocity?

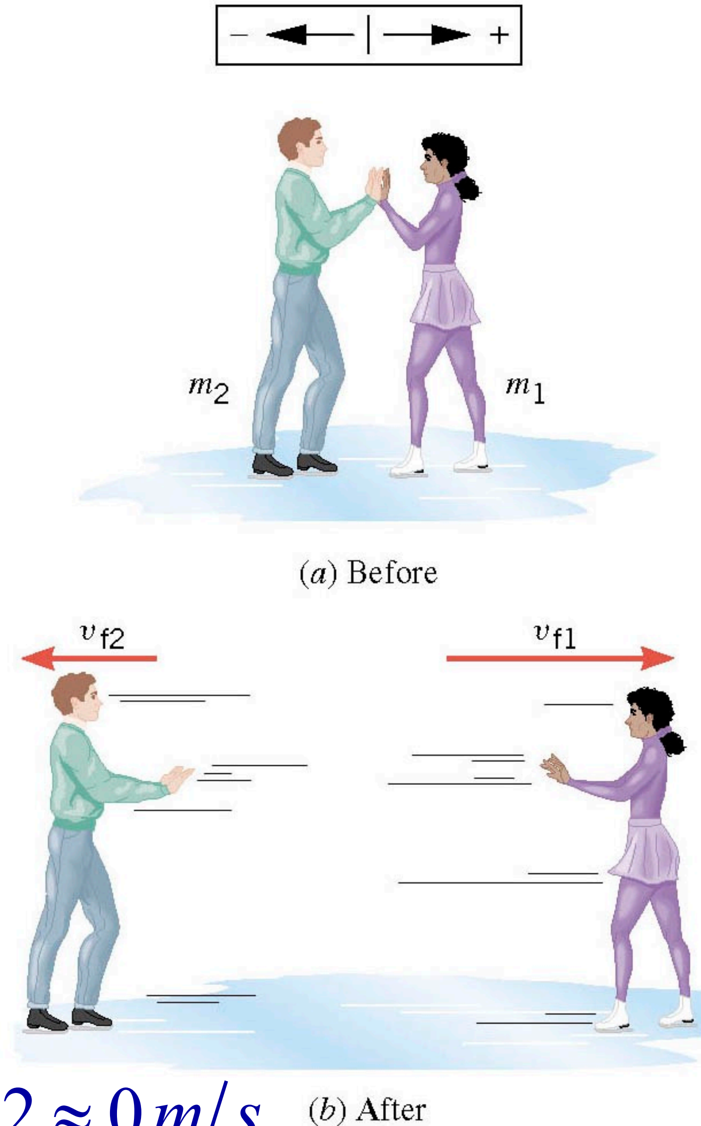
$$v_{10} = 0\, m/s \qquad v_{20} = 0\, m/s$$

$$v_{cm0} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$

$$v_{1f} = +2.5\, m/s \qquad v_{2f} = -1.5\, m/s$$

$$v_{cmf} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2}$$

$$= \frac{54\cdot(+2.5) + 88\cdot(-1.5)}{54+88} = \frac{3}{142} = 0.02 \approx 0\, m/s$$



Center of Mass of a Rigid Object

The formula for CM can be extended to a system of many particles or a Rigid Object

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{m_1 + m_2 + \cdots + m_n} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

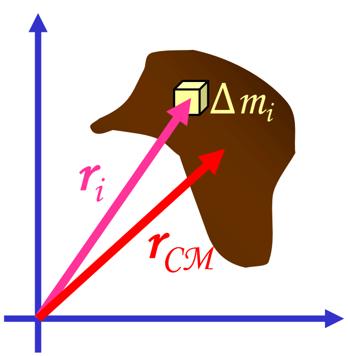
$$y_{CM} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

$$z_{CM} = \frac{\sum_i m_i z_i}{\sum_i m_i}$$

The position vector of the center of mass of a many particle system is

$$\vec{r}_{CM} = x_{CM}\vec{i} + y_{CM}\vec{j} + z_{CM}\vec{k} = \frac{\sum_i m_i x_i \vec{i} + \sum_i m_i y_i \vec{j} + \sum_i m_i z_i \vec{k}}{\sum_i m_i}$$

$$\vec{r}_{CM} = \frac{\sum_i m_i \vec{r}_i}{M}$$



A rigid body – an object with shape and size with mass spread throughout the body, ordinary objects – can be considered as a group of particles with mass m_i densely spread throughout the given shape of the object

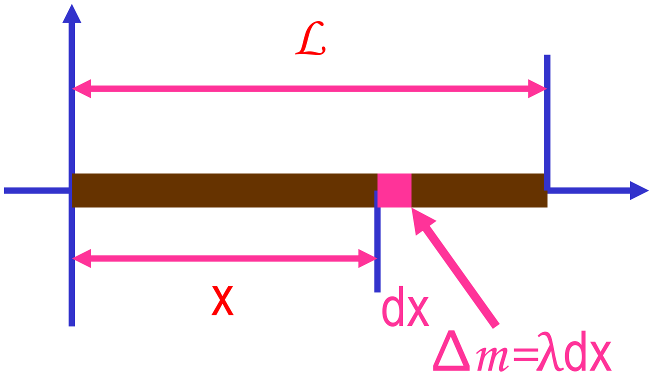
$$x_{CM} \approx \frac{\sum_i \Delta m_i x_i}{M}$$

$$x_{CM} = \lim_{\Delta m_i \to 0} \frac{\sum_i \Delta m_i x_i}{M} = \frac{1}{M}\int x\,dm$$

$$\vec{r}_{CM} = \frac{1}{M}\int \vec{r}\,dm$$

Example: CM of a thin rod

Show that the center of mass of a rod of mass $\mathcal{M}$ and length $\mathcal{L}$ lies in midway between its ends, assuming the rod has a uniform mass per unit length.



The formula for CM of a continuous object is

$$\boxed{x_{CM} = \frac{1}{M}\int_{x=0}^{x=L} x\,dm}$$

Since the density of the rod (λ) is constant; $\lambda = M/L$

The mass of a small segment $dm = \lambda dx$

Therefore $$x_{CM} = \frac{1}{M}\int_{x=0}^{x=L} \lambda x dx = \frac{1}{M}\left[\frac{1}{2}\lambda x^2\right]_{x=0}^{x=L} = \frac{1}{M}\left(\frac{1}{2}\lambda L^2\right) = \frac{1}{M}\left(\frac{1}{2}ML\right) = \frac{L}{2}$$

Find the CM when the density of the rod non-uniform but varies linearly as a function of x, $\lambda = \alpha x$

$$M = \int_{x=0}^{x=L} \lambda dx = \int_{x=0}^{x=L} \alpha x dx$$

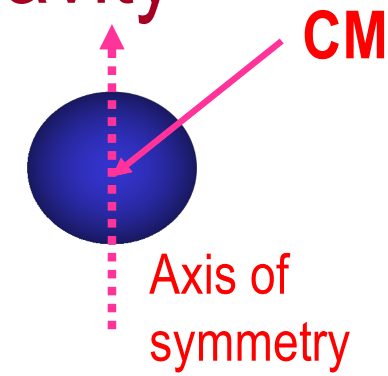
$$= \left[\frac{1}{2}\alpha x^2\right]_{x=0}^{x=L} = \frac{1}{2}\alpha L^2$$

$$x_{CM} = \frac{1}{M}\int_{x=0}^{x=L} \lambda x dx = \frac{1}{M}\int_{x=0}^{x=L} \alpha x^2 dx = \frac{1}{M}\left[\frac{1}{3}\alpha x^3\right]_{x=0}^{x=L}$$

$$x_{CM} = \frac{1}{M}\left(\frac{1}{3}\alpha L^3\right) = \frac{1}{M}\left(\frac{2}{3}ML\right) = \frac{2L}{3}$$

Center of Mass and Center of Gravity

The center of mass of any symmetric object lies on the axis of symmetry and on any plane of symmetry, if the object's mass is evenly distributed throughout the body.

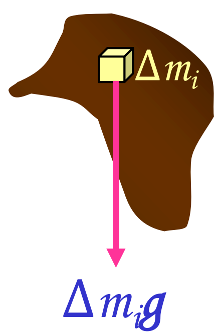


How do you think you can determine the CM of the objects that are not symmetric?

One can use gravity to locate CM.

1. Hang the object by one point and draw a vertical line following a plum-bob.
2. Hang the object by another point and do the same.
3. The point where the two lines meet is the CM.

Center of Gravity



Since a rigid object can be considered as a **collection of small masses**, one can see the total gravitational force exerted on the object as

$$\vec{F}_g = \sum_i \vec{F}_i = \sum_i \Delta m_i \vec{g} = M\vec{g}$$

What does this equation tell you?

The net effect of these small gravitational forces is equivalent to a single force acting on a point (Center of Gravity) with mass M.

The CoG is the point in an object as if all the gravitational force is acting on!

Motion of a Group of Particles

We've learned that the CM of a system can represent the motion of a system. Therefore, for an isolated system of many particles in which the total mass M is preserved, the velocity, total momentum, acceleration of the system are

Velocity of the system

$$\vec{v}_{CM} = \frac{d\vec{r}_{CM}}{dt} = \frac{d}{dt}\left(\frac{1}{M}\sum_i m_i\vec{r}_i\right) = \frac{1}{M}\sum_i \frac{m_i d\vec{r}_i}{dt} = \frac{\sum_i m_i\vec{v}_i}{M}$$

Total Momentum of the system

$$\vec{p}_{CM} = M\vec{v}_{CM} = M\frac{\sum_i m_i\vec{v}_i}{M} = \sum_i m_i\vec{v}_i = \sum_i \vec{p}_i = \vec{p}_{tot}$$

Acceleration of the system

$$\vec{a}_{CM} = \frac{d\vec{v}_{CM}}{dt} = \frac{d}{dt}\left(\frac{1}{M}\sum_i m_i\vec{v}_i\right) = \frac{1}{M}\left(\sum_i m_i\frac{d\vec{v}_i}{dt}\right) = \frac{\sum_i m_i\vec{a}_i}{M}$$

The external force acting on the system

$$\sum \vec{F}_{ext} = M\vec{a} = \left(\sum_i m_i\right)\vec{a} = \frac{d\vec{p}_{tot}}{dt}$$

What about the internal forces?

If net external force is 0

$$\sum \vec{F}_{ext} = 0 = \frac{d\vec{p}_{tot}}{dt}$$

$$\boxed{\vec{p}_{tot} = const}$$

System's momentum is conserved.