PHYS 1443 – Section 003
Lecture #18

Wednesday, April 21, 2021

Dr. Jaehoon Yu

- CH8: Rotational Motion
  - Relationship Between Rotational and Translational Quantities
  - Torque & Vector Product
  - Moment of Inertia
  - Rotational Kinetic Energy

Announcements

- **The skipped homework #9 is due 11pm, Tuesday, April 27!!**
- Quiz #4: next Wednesday, April 28, beginning of the class
  - Roll call begins at 2:20pm
  - Covers: CH8.1 to what we finish Monday, Apr. 26
  - BYOF: You may bring one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the test
  - No derivations, word definitions, setups or solutions of any problems, figures, pictures, diagrams or arrows, etc!
  - Must email me the photos of front and back of the formula sheet, including the blank at jaehoonyu@uta.edu no later than **12:00pm the day of the test**
    - The subject of the email should be the same as your file name
    - File name must be FS-Q4-LastName-FirstName-SP21.pdf
    - Once submitted, you cannot change, unless I ask you to delete part of the sheet!

Rotational Kinematics and the Relationships between Rotational and Translational Quantities

Angular velocity under constant angular acceleration: $\omega_f = \omega_0 + \alpha t$

Translational kinematics $v = v_o + at$

Angular displacement under constant angular acceleration: $\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

Translational kinematics $x_f = x_0 + v_o t + \frac{1}{2}at^2$

One can also obtain $\omega_f^2 = \omega_0^2 + 2\alpha\left(\theta_f - \theta_0\right)$

Translational kinematics $v_f^2 = v_o^2 + 2a\left(x_f - x_i\right)$

The arc-length is $l = r\theta$ So, the tangential speed v is $v = \frac{dl}{dt} = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt} = r\omega$

The magnitude of tangential acceleration a_t is $a_t = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha$

The radial or centripetal acceleration a_r is $a_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$

Total linear acceleration is $a = \sqrt{a_t^2 + a_r^2} = \sqrt{(r\alpha)^2 + \left(r\omega^2\right)^2} = r\sqrt{\alpha^2 + \omega^4}$

Example

(a) What is the linear speed of a child seated 1.2m from the center of a steadily rotating merry-go-around that makes one complete revolution in 4.0s? (b) What is her total linear acceleration?

First, figure out what the angular speed of the merry-go-around is.

$$\varpi = \frac{1rev}{4.0s} = \frac{2\pi}{4.0s} = 1.6rad/s$$

Using the formula for linear speed

$$v = r\omega = 1.2m \times 1.6rad/s = 1.9m/s$$

Since the angular speed is constant, there is no angular acceleration.

Tangential acceleration is

$$a_t = r\alpha = 1.2m \times 0rad/s^2 = 0m/s^2$$

Radial acceleration is

$$a_r = r\varpi^2 = 1.2m \times (1.6rad/s)^2 = 3.1m/s^2$$

Thus the total acceleration is

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{0 + (3.1)^2} = 3.1m/s^2$$

Example for a Rotational Motion

Audio information on compact discs are transmitted digitally through the readout system consisting of laser and lenses. The digital information on the disc are stored by the pits and flat areas on the track. Since the speed of readout system is constant, it reads out the same number of pits and flats in the same time interval. In other words, the linear speed is the same no matter which track is played. a) Assuming the linear speed is 1.3 m/s, find the angular speed of the disc in revolutions per minute when the inner most (r=23mm) and outer most tracks (r=58mm) are read.

Using the relationship between angular and tangential speed $v = r\omega$

$$r = 23mm \qquad \omega = \frac{v}{r} = \frac{1.3m/s}{23mm} = \frac{1.3}{23\times10^{-3}} = 56.5rad/s$$

$$= 9.00rev/s = 5.4\times10^{2} rev/\min$$

$$r = 58mm \qquad \omega = \frac{1.3m/s}{58mm} = \frac{1.3}{58\times10^{-3}} = 22.4rad/s$$

$$= 2.1\times10^{2} rev/\min$$

b) The maximum playing time of a standard music CD is 74 minutes and 33 seconds. How many revolutions does the disk make during that time?

$$\overline{\omega} = \frac{(\omega_i + \omega_f)}{2} = \frac{(540 + 210) rev/\min}{2} = 375 rev/\min$$

$$\theta_f = \theta_i + \overline{\omega} t = 0 + \frac{375}{60} rev/s \times 4473 s = 2.8 \times 10^4 rev$$

c) What is the total length of the track past through the readout mechanism?

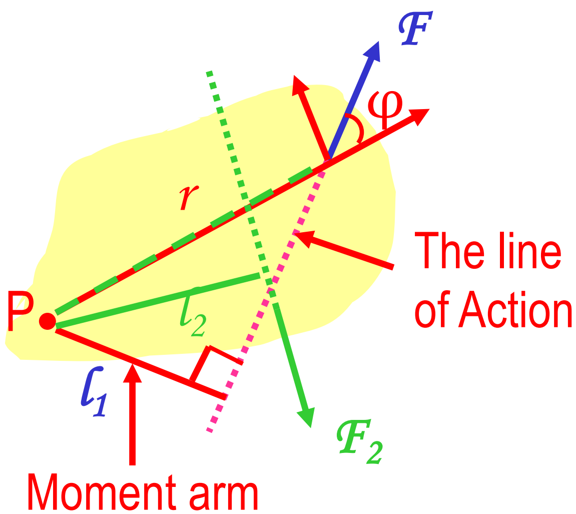
$$l = v_t \Delta t = 1.3 m/s \times 4473 s = 5.8 \times 10^3 m$$

d) What is the angular acceleration of the CD over the 4473s time interval, assuming constant α?

$$\alpha = \frac{(\omega_f - \omega_i)}{\Delta t} = \frac{(22.4 - 56.5) rad/s}{4473 s} = 7.6 \times 10^{-3} rad/s^2$$

Torque

Torque is the tendency of a force to rotate an object about an axis. Torque, τ, is a vector quantity.



Consider an object pivoting about the point P by the force *F* being exerted at a distance r from P.

The line that extends out of the tail of the force vector is called the line of action.

The perpendicular distance from the pivoting point P to the line of action is called the moment arm.

$$\left|\vec{\tau}\right| \equiv (\text{Magnitude of the Force}) \times (\text{Moment Arm or Lever Arm}) = (F)(r\sin\phi) = Fl_1$$

Magnitude of the torque is defined as the product of the force exerted on the object to rotate it and the moment arm.

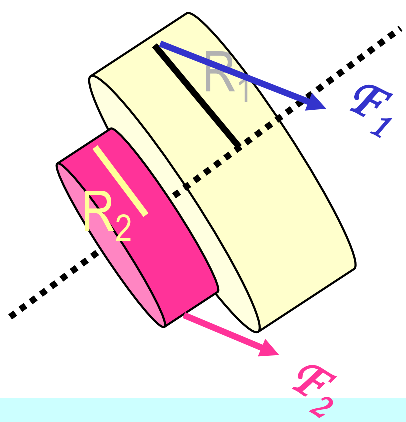
When there are more than one force exerting on certain points of an object, one can sum up the torque generated by each force vectorially. The convention for the sign of the torque is **positive if rotation is in counter-clockwise** and **negative if clockwise**.

$$\sum\tau = \tau_1 + \tau_2 = F_1 l_1 - F_2 l_2$$

Unit? $N \cdot m$

Ex. Torque

A one-piece cylinder is shaped as in the figure with core section protruding from the larger drum. The cylinder is free to rotate around the central axis shown in the picture. A rope wrapped around the drum whose radius is R_1 exerts force F_1 to the right on the cylinder, and another force exerts F_2 on the core whose radius is R_2 downward on the cylinder. A) What is the net torque acting on the cylinder about the rotation axis?



The torque due to F_1 $\tau_1 = -R_1 F_1$ and due to F_2 $\tau_2 = R_2 F_2$

So the total torque acting on the system by the forces is

$$\sum \tau = \tau_1 + \tau_2 = -R_1 F_1 + R_2 F_2$$

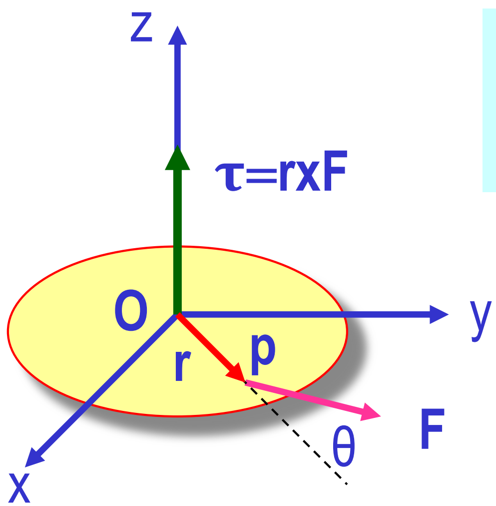
Suppose F_1=5.0 N, R_1=1.0 m, F_2= 15.0 N, and R_2=0.50 m. What is the net torque about the rotation axis and which way does the cylinder rotate from the rest?

Using the above result

$$\sum \tau = -R_1 F_1 + R_2 F_2 = -5.0 \times 1.0 + 15.0 \times 0.50 = 2.5\ N \cdot m$$

The cylinder rotates in counter-clockwise.

Torque and Vector Product



Let's consider a disk fixed onto the origin O and the force $\mathcal{F}$ exerts on the point p. What happens?

The disk will start rotating counterclockwise about the Z axis

The magnitude of the torque given to the disk by the force $\mathcal{F}$ is

$$\tau = Fr\sin\theta$$

But torque is a vector quantity, what is the direction?
How is torque expressed mathematically?

$$\vec{\tau} \equiv \vec{r} \times \vec{F}$$

What is the direction?

The direction of the torque follows the right-hand rule!!

The above operation is called the Vector product or Cross product

$$\vec{C} \equiv \vec{A} \times \vec{B}$$

$$\left|\vec{C}\right| = \left|\vec{A} \times \vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta$$

What is the result of a vector product?

Another vector

What is another vector operation we've learned?

Scalar product

$$C \equiv \vec{A} \cdot \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$$

Result? A scalar

Properties of the Vector Product

Vector Product is Non-commutative

What does this mean?

If the order of operation changes the result changes $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$

Following the right-hand rule, the direction changes

$$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$$

Vector Product of two parallel vectors is 0.

$$\left|\vec{C}\right|=\left|\vec{A}\times\vec{B}\right|=\left|\vec{A}\right|\left|\vec{B}\right|\sin\theta=\left|\vec{A}\right|\left|\vec{B}\right|\sin 0=0$$

Thus, $\vec{A}\times\vec{A}=0$

If two vectors are perpendicular to each other

$$\left|\vec{A}\times\vec{B}\right|=\left|\vec{A}\right|\left|\vec{B}\right|\sin\theta=\left|\vec{A}\right|\left|\vec{B}\right|\sin 90^\circ=\left|\vec{A}\right|\left|\vec{B}\right|=AB$$

Vector product follows distribution law

$$\vec{A}\times\left(\vec{B}+\vec{C}\right)=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$

The derivative of a Vector product with respect to a scalar variable is

$$\frac{d\left(\vec{A}\times\vec{B}\right)}{dt}=\frac{d\vec{A}}{dt}\times\vec{B}+\vec{A}\times\frac{d\vec{B}}{dt}$$

More Properties of Vector Product

The relationship between unit vectors, $\vec{i}$, $\vec{j}$ and $\vec{k}$

$$\vec{i}\times\vec{i}=\vec{j}\times\vec{j}=\vec{k}\times\vec{k}=0$$

$$\vec{i}\times\vec{j}=-\vec{j}\times\vec{i}=\vec{k}$$

$$\vec{j}\times\vec{k}=-\vec{k}\times\vec{j}=\vec{i}$$

$$\vec{k}\times\vec{i}=-\vec{i}\times\vec{k}=\vec{j}$$

Vector product of two vectors can be expressed in the following determinant form

$$\vec{A}\times\vec{B}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}=\vec{i}\begin{vmatrix}A_y & A_z\\ B_y & B_z\end{vmatrix}-\vec{j}\begin{vmatrix}A_x & A_z\\ B_x & B_z\end{vmatrix}+\vec{k}\begin{vmatrix}A_x & A_y\\ B_x & B_y\end{vmatrix}$$

$$=\left(A_yB_z-A_zB_y\right)\vec{i}-\left(A_xB_z-A_zB_x\right)\vec{j}+\left(A_xB_y-A_yB_x\right)\vec{k}$$

Moment of Inertia

Rotational Inertia: The measure of resistance of an object against changes in its rotational motion. Equivalent to mass in translational motion.

For a group of particles $I \equiv \sum_i m_i r_i^2$

For a rigid body $I \equiv \int r^2 dm$

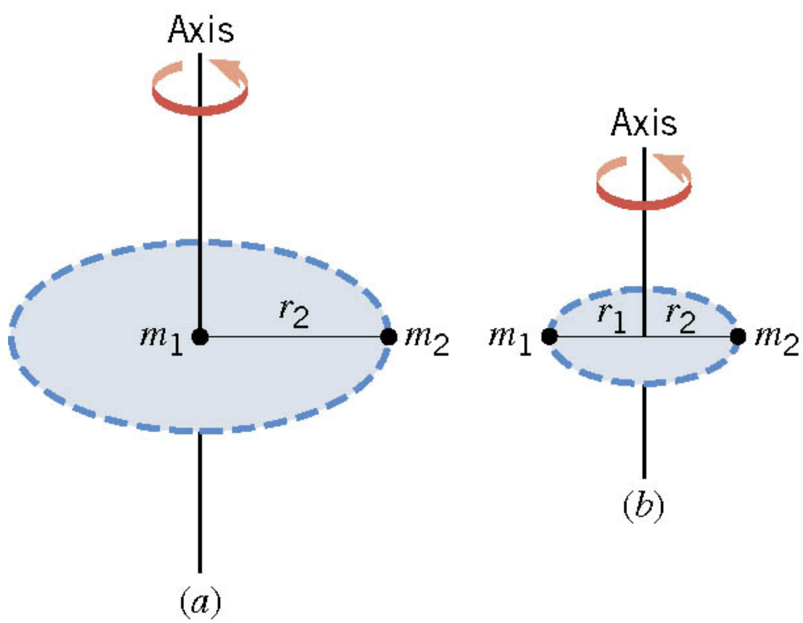
What are the dimension and unit of Moment of Inertia? $\left[ML^2 \right]$ $kg \cdot m^2$

Determining Moment of Inertia is extremely important for computing equilibrium of a rigid body, such as a building.

Dependent on the axis of rotation!!!

Ex. The Moment of Inertia Depends on Where the Axis Is.

Two particles each have mass m_1 and m_2 and are fixed at the ends of a thin rigid rod. The length of the rod is *L*. Find the moment of inertia when this object rotates relative to an axis that is perpendicular to the rod at (a) one end and (b) the center.



(a) $I = \sum\left(mr^2\right) = m_1 r_1^2 + m_2 r_2^2$

$m_1 = m_2 = m \qquad r_1 = 0 \quad r_2 = L$

$$I = m(0)^2 + m(L)^2 = mL^2$$

(b) $I = \sum\left(mr^2\right) = m_1 r_1^2 + m_2 r_2^2$

$m_1 = m_2 = m \qquad r_1 = L/2 \quad r_2 = L/2$

$$I = m(L/2)^2 + m(L/2)^2 = \tfrac{1}{2}mL^2$$

Which case is easier to spin?

Case (b)

Why? Because the moment of inertia is smaller