• One Dimensional Motion:
  – Motion under constant acceleration
  – Free Fall
• Motion in Two Dimensions
  – Vector Properties and Operations
  – Motion under constant acceleration
  – Projectile Motion

Today’s homework is HW #2, due 7pm, Monday, June 5!!
Announcements

• Reading assignment #2: Read section 2.8
• Ten of you have registered for the homework
  – Seven of you submitted #1
  – You MUST register and submit the homework to obtain 100% of the credit…
• The class e-mail distribution list is available as of yesterday
  – Please go ahead and subscribe to the list
    • Phys1443-001-summer06
  – Extra credit
    • 5 points if done by Tuesday, June 6
    • 3 points if done by Thursday, June 8
Acceleration

Change of velocity in time (what kind of quantity is this?)

• Average acceleration:

\[ a_x \equiv \frac{v_{sf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} \quad \text{analogs to} \quad v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} \]

• Instantaneous acceleration:

\[ a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt} \left( \frac{dx}{dt} \right) = \frac{d^2 x}{dt^2} \quad \text{analogs to} \quad v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \]

• In calculus terms: The slope (derivative) of the velocity vector with respect to time or the change of slopes of position as a function of time
Acceleration vs Time Plot

\[ a = 4.20 \text{ m/s}^2 \]
Meanings of Acceleration

• When an object is moving in a constant velocity \( (v = v_0) \), there is no acceleration \( (a = 0) \)
  – Is there any acceleration when an object is not moving?

• When an object speeds up as time goes on, \( (v = v(t)) \), acceleration has the same sign as \( v \).

• When an object slows down as time goes on, \( (v = v(t)) \), acceleration has the opposite sign as \( v \).

• Is there acceleration if an object moves in a constant speed but changes direction? YES!!
One Dimensional Motion

- Let's start with the simplest case: acceleration is constant \((a=a_0)\)
- Using definitions of average acceleration and velocity, we can draw equation of motion (description of motion, position \(wrt\) time)

\[
a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}
\]

*If \(t_f=t\) and \(t_i=0\)*

\[
a_x = \frac{v_{xf} - v_{xi}}{t}
\]

Solve for \(v_{xf}\)

\[
v_{xf} = v_{xi} + a_x t
\]

For constant acceleration, simple numeric average

\[
\bar{v} = \frac{v_{xi} + v_{xf}}{2}
\]

\[
\bar{v} = \frac{2v_{xi} + a_xt}{2} = v_{xi} + \frac{1}{2}a_xt
\]

*If \(t_f=t\) and \(t_i=0\)*

\[
\bar{v} = \frac{x_f - x_i}{t_f - t_i}
\]

\[
\bar{v} = \frac{x_f - x_i}{t}
\]

Solve for \(x_f\)

\[
x_f = x_i + \bar{v} t
\]

Resulting Equation of Motion becomes

\[
x_f = x_i + \bar{v} t = x_i + v_{xi}t + \frac{1}{2}a_xt^2
\]
Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

\[ v_{xf}(t) = v_{xi} + at \]  
Velocity as a function of time

\[ x_f - x_i = \frac{1}{2} v_x t = \frac{1}{2} (v_{xf} + v_{xi})t \]  
Displacement as a function of velocities and time

\[ x_f = x_i + v_{xi}t + \frac{1}{2} at^2 \]  
Displacement as a function of time, velocity, and acceleration

\[ v_{xf}^2 = v_{xi}^2 + 2ax(x_f - x_i) \]  
Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!
How do we solve a problem using a kinematic formula for constant acceleration?

• Identify what information is given in the problem.
  – Initial and final velocity?
  – Acceleration?
  – Distance?
  – Time?

• Convert the units of all quantities to SI units to be consistent.

• Identify what the problem wants.

• Identify which kinematic formula is appropriate and easiest to solve for what the problem wants.
  – Frequently multiple formulae can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that can be easiest to solve.

• Solve the equation for the quantity wanted.
Example 2.11

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? As long as it takes for it to crumple.

The initial speed of the car is

\[ v_{xi} = 100 \text{km/h} = \frac{100000 \text{m}}{3600 \text{s}} = 28 \text{m/s} \]

We also know that

\[ v_{xf} = 0 \text{m/s} \quad \text{and} \quad x_f - x_i = 1 \text{m} \]

Using the kinematic formula

\[ v_{xf}^2 = v_{xi}^2 + 2a_x (x_f - x_i) \]

The acceleration is

\[ a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{0 - (28 \text{m/s})^2}{2 \times 1 \text{m}} = -390 \text{m/s}^2 \]

Thus the time for air-bag to deploy is

\[ t = \frac{v_{xf} - v_{xi}}{a} = \frac{0 - 28 \text{m/s}}{-390 \text{m/s}^2} = 0.07 \text{s} \]
Free Fall

- Free fall is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
  - A motion under constant acceleration
  - All kinematic formulae we learned can be used to solve for falling motions.

- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth

- The magnitude of the gravitational acceleration is \(|g|=9.80\text{m/s}^2\) on the surface of the Earth, most of the time.

- The direction of gravitational acceleration is **ALWAYS** toward the center of the earth, which we normally call \((-y)\); where up and down directions are indicated with the variable “y”

- Thus the correct denotation of gravitational acceleration on the surface of the earth is \(g=-9.80\text{m/s}^2\)

Note the negative sign!!
Example for Using 1D Kinematic Equations on a Falling object

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building.

What is the acceleration in this motion? \( g = -9.80 \text{m/s}^2 \)

(a) Find the time the stone reaches at the maximum height.

What is so special about the maximum height? \( V = 0 \)

\[
\begin{align*}
v_f &= v_i + at = 20.0 - 9.80t = 0.00 \text{m/s} \quad \text{Solve for } t \quad t = \frac{20.0}{9.80} = 2.04 \text{s}
\end{align*}
\]

(b) Find the maximum height.

\[
\begin{align*}
y_f &= y_i + v_it + \frac{1}{2}at^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2 \\
&= 50.0 + 20.4 = 70.4 \text{(m)}
\end{align*}
\]
Example of a Falling Object cnt’d

(c) Find the time the stone reaches back to its original height.
\[ t = 2.04 \times 2 = 4.08 \text{s} \]

(d) Find the velocity of the stone when it reaches its original height.
\[ v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0 (\text{m/s}) \]

(e) Find the velocity and position of the stone at \( t=5.00\text{s} \).
\[ v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0 (\text{m/s}) \]
\[ y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2 \]
\[ = 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5 (\text{m}) \]
Coordinate Systems

- They make it easy to express locations or positions
- Two commonly used systems, depending on convenience, are
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in (r,θ)
- Vectors become a lot easier to express and compute

How are Cartesian and Polar coordinates related?

\[
\begin{align*}
  x_1 &= r \cos \theta \\
  y_1 &= r \sin \theta \\
  r &= \sqrt{x_1^2 + y_1^2} \\
  \tan \theta &= \frac{y_1}{x_1}
\end{align*}
\]
Example

Cartesian Coordinate of a point in the xy plane are \((x,y)=(-3.50,-2.50)\text{m}\). Find the equivalent polar coordinates of this point.

\[
r = \sqrt{(x^2 + y^2)}
\]

\[
= \sqrt{((-3.50)^2 + (-2.50)^2)}
\]

\[
= \sqrt{18.5} = 4.30(\text{m})
\]

\[
\theta = 180 + \theta_s
\]

\[
\tan \theta_s = \frac{-2.50}{-3.50} = \frac{5}{7}
\]

\[
\theta_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ
\]

\[
\therefore \theta = 180 + \theta_s = 180^\circ + 35.5^\circ = 216^\circ
\]
Vector and Scalar

Vector quantities have both magnitudes (sizes) and directions

*Force, gravitational acceleration, momentum*

Normally denoted in **BOLD** letters, \( \mathbf{F} \), or a letter with arrow on top \( \vec{F} \)

Their sizes or magnitudes are denoted with normal letters, \( F \), or absolute values: \( |\mathbf{F}| \) or \( |F| \)

Scalar quantities have magnitudes only

Can be completely specified with a value and its unit

Normally denoted in normal letters, \( E \)

*Energy, heat, mass, weight*

Both have units!!!
Properties of Vectors

• Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system!!

Which ones are the same vectors?

A = B = E = D

Why aren’t the others?

C: The same magnitude but opposite direction: C = -A

F: The same direction but different magnitude
Vector Operations

• Addition:
  – Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  – Parallelogram method: Connect the tails of the two vectors and extend
  – Addition is commutative: Changing order of operation does not affect the results $A + B = B + A$, $A + B + C + D + E = E + C + A + B + D$

• Subtraction:
  – The same as adding a negative vector: $A - B = A + (-B)$

• Multiplication by a scalar is increasing the magnitude $A, B = 2A$

$|B| = 2|A|$
Example for Vector Addition

A car travels 20.0 km due north followed by 35.0 km in a direction 60.0° west of north. Find the magnitude and direction of resultant displacement.

\[ r = \sqrt{(A + B \cos \theta)^2 + (B \sin \theta)^2} \]
\[ = \sqrt{A^2 + B^2 \left( \cos^2 \theta + \sin^2 \theta \right) + 2AB \cos \theta} \]
\[ = \sqrt{A^2 + B^2 + 2AB \cos \theta} \]
\[ = \sqrt{(20.0)^2 + (35.0)^2 + 2 \times 20.0 \times 35.0 \cos 60} \]
\[ = \sqrt{2325} = 48.2 \text{ (km)} \]

\[ \theta = \tan^{-1} \left( \frac{|B| \sin 60}{|A| + |B| \cos 60} \right) \]
\[ = \tan^{-1} \left( \frac{35.0 \sin 60}{20.0 + 35.0 \cos 60} \right) \]
\[ = \tan^{-1} \left( \frac{35.0 \times 0.866}{20.0 + 35.0 \times 0.5} \right) \]
\[ = \tan^{-1} \left( \frac{30.3}{37.5} \right) = 38.9° \text{ to W wrt N} \]
Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components

\[
\begin{align*}
A_x &= |\vec{A}| \cos \theta \\
A_y &= |\vec{A}| \sin \theta \\
|\vec{A}| &= \sqrt{A_x^2 + A_y^2}
\end{align*}
\]

\[
|\vec{A}| = \sqrt{\left( |\vec{A}| \cos \theta \right)^2 + \left( |\vec{A}| \sin \theta \right)^2} = \sqrt{|\vec{A}|^2 \left( \cos^2 \theta + \sin^2 \theta \right)} = |\vec{A}|
\]

- Unit vectors are **dimensionless** vectors whose **magnitude are exactly 1**
  - Unit vectors are usually expressed in \( \mathbf{i}, \mathbf{j}, \mathbf{k} \) or \( \vec{i}, \vec{j}, \vec{k} \)
  - Vectors can be expressed using components and unit vectors

So the above vector \( \vec{A} \) can be written as

\[
\vec{A} = A_x \mathbf{i} + A_y \mathbf{j} = |\vec{A}| \cos \theta \mathbf{i} + |\vec{A}| \sin \theta \mathbf{j}
\]
Examples of Vector Operations

Find the resultant vector which is the sum of \( \mathbf{A} = (2.0 \mathbf{i} + 2.0 \mathbf{j}) \) and \( \mathbf{B} = (2.0 \mathbf{i} - 4.0 \mathbf{j}) \)

\[
\mathbf{C} = \mathbf{A} + \mathbf{B} = (2.0 \mathbf{i} + 2.0 \mathbf{j}) + (2.0 \mathbf{i} - 4.0 \mathbf{j}) = (2.0 + 2.0) \mathbf{i} + (2.0 - 4.0) \mathbf{j} = 4.0 \mathbf{i} - 2.0 \mathbf{j} \text{(m)}
\]

\[
|\mathbf{C}| = \sqrt{(4.0)^2 + (-2.0)^2} = \sqrt{16 + 4.0} = \sqrt{20} = 4.5 \text{(m)}
\]

\[
\theta = \tan^{-1} \left( \frac{C_y}{C_x} \right) = \tan^{-1} \left( \frac{-2.0}{4.0} \right) = -27^\circ
\]

Find the resultant displacement of three consecutive displacements: \( \mathbf{d}_1 = (15 \mathbf{i} + 30 \mathbf{j} + 12 \mathbf{k}) \text{cm}, \mathbf{d}_2 = (23 \mathbf{i} + 14 \mathbf{j} - 5.0 \mathbf{k}) \text{cm}, \text{and } \mathbf{d}_3 = (-13 \mathbf{i} + 15 \mathbf{j}) \text{cm}

\[
\mathbf{D} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 = (15 \mathbf{i} + 30 \mathbf{j} + 12 \mathbf{k}) + (23 \mathbf{i} + 14 \mathbf{j} - 5.0 \mathbf{k}) + (-13 \mathbf{i} + 15 \mathbf{j}) = (15 + 23 - 13) \mathbf{i} + (30 + 14 + 15) \mathbf{j} + (12 - 5.0) \mathbf{k} = 25 \mathbf{i} + 59 \mathbf{j} + 7.0 \mathbf{k} \text{(cm)}
\]

Magnitude \[
|\mathbf{D}| = \sqrt{(25)^2 + (59)^2 + (7.0)^2} = 65 \text{(cm)}
\]
Displacement, Velocity, and Acceleration in 2-dim

- **Displacement:**
  \[ \Delta \vec{r} = \vec{r}_f - \vec{r}_i \]

- **Average Velocity:**
  \[ \vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} \]

- **Instantaneous Velocity:**
  \[ \vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d \vec{r}}{dt} \]

- **Average Acceleration**
  \[ \vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} \]

- **Instantaneous Acceleration:**
  \[ \vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d \vec{v}}{dt} = \frac{d}{dt} \left( \frac{d \vec{r}}{dt} \right) = \frac{d^2 \vec{r}}{dt^2} \]

How is each of these quantities defined in 1-D?