PHYS 1443 – Section 001
Lecture #3

Thursday, June 1, 2006
Dr. Jaehoon Yu

- One Dimensional Motion:
  - Motion under constant acceleration
  - Free Fall
- Motion in Two Dimensions
  - Vector Properties and Operations
  - Motion under constant acceleration
  - Projectile Motion

Today's homework is HW #2, due 7pm, Monday, June 5!!

Announcements

- Reading assignment #2: Read section 2.8
- Ten of you have registered for the homework
  - Seven of you submitted #1
  - You MUST register and submit the homework to obtain 100% of the credit…
- The class e-mail distribution list is available as of yesterday
  - Please go ahead and subscribe to the list
    - Phys1443-001-summer06
  - Extra credit
    - 5 points if done by Tuesday, June 6
    - 3 points if done by Thursday, June 8

Acceleration

Change of velocity in time (what kind of quantity is this?)

- Average acceleration:

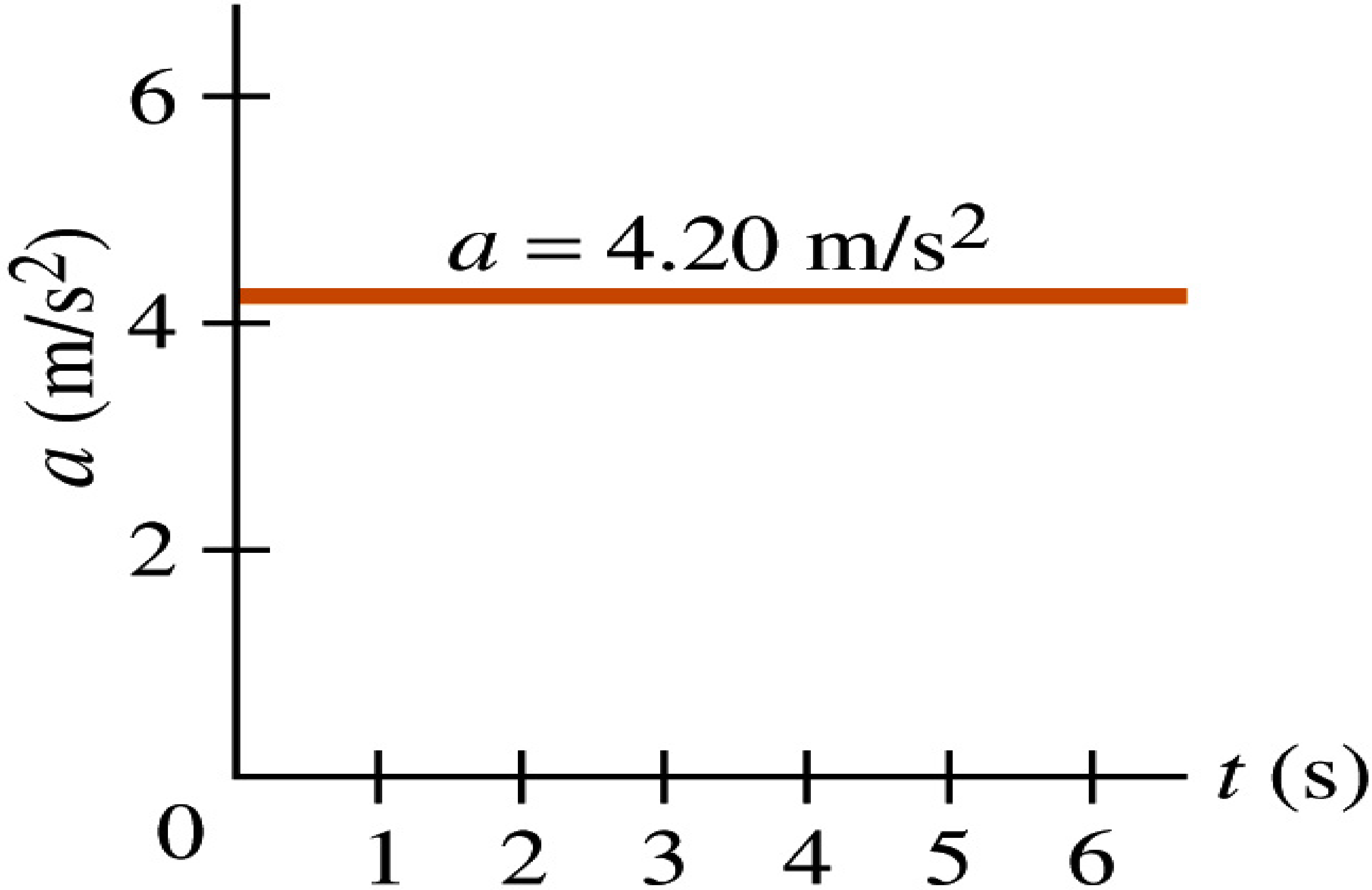
$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} \quad \text{analogs to} \quad v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

- Instantaneous acceleration:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2} \quad \text{analogs to} \quad \boxed{v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

- In calculus terms: The slope (derivative) of the velocity vector with respect to time or the change of slopes of position as a function of time

Acceleration vs Time Plot



Meanings of Acceleration

- When an object is moving in a constant velocity ($v=v_0$), there is no acceleration ($a=0$)
  - Is there any acceleration when an object is not moving?
- When an object speeds up as time goes on, ($v=v(t)$), acceleration has the same sign as v.
- When an object slows down as time goes on, ($v=v(t)$), acceleration has the opposite sign as v.
- Is there acceleration if an object moves in a constant speed but changes direction? YES!!

One Dimensional Motion

- Let's start with the simplest case: acceleration is constant ($a=a_0$)
- Using definitions of average acceleration and velocity, we can draw equation of motion (description of motion, position *wrt* time)

$$a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

If $t_f=t$ and $t_i=0$

$$a_x = \frac{v_{xf} - v_{xi}}{t}$$

Solve for v_{xf}

$$v_{xf} = v_{xi} + a_x t$$

For constant acceleration, simple numeric average

$$\overline{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2}a_x t$$

$$\overline{v}_x = \frac{x_f - x_i}{t_f - t_i}$$

If $t_f=t$ and $t_i=0$

$$\overline{v}_x = \frac{x_f - x_i}{t}$$

Solve for x_f

$$x_f = x_i + \overline{v}_x t$$

Resulting Equation of Motion becomes

$$x_f = x_i + \overline{v}_x t = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v_{xf}(t) = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}(v_{xf} + v_{xi})t$$

Displacement as a function of velocities and time

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!

How do we solve a problem using a kinematic formula for constant acceleration?

- Identify what information is given in the problem.
  - Initial and final velocity?
  - Acceleration?
  - Distance?
  - Time?
- Convert the units of all quantities to SI units to be consistent.
- Identify what the problem wants.
- Identify which kinematic formula is **appropriate and easiest** to solve for what the problem wants.
  - Frequently multiple formulae can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that can be easiest to solve.
- Solve the equation for the quantity wanted.

Example 2.11

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? ⇨ As long as it takes for it to crumple.

The initial speed of the car is $v_{xi} = 100km/h = \frac{100000m}{3600s} = 28m/s$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinematic formula $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$

The acceleration is $a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{0-(28m/s)^2}{2\times 1m} = -390m/s^2$

Thus the time for air-bag to deploy is $t = \frac{v_{xf} - v_{xi}}{a} = \frac{0-28m/s}{-390m/s^2} = 0.07s$

Free Fall

- Free fall is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
  - A motion under constant acceleration
  - All kinematic formulae we learned can be used to solve for falling motions.

Down to the center of the Earth!

- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The magnitude of the gravitational acceleration is $|g|=9.80m/s^2$ on the surface of the Earth, most of the time.
- The direction of gravitational acceleration is <u>ALWAYS</u> toward the center of the earth, which we normally call (-y); where up and down directions are indicated with the variable "y"
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80m/s^2$

Note the negative sign!!

Example for Using 1D Kinematic Equations on a Falling object

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

What is the acceleration in this motion? $g=-9.80m/s^2$

(a) Find the time the stone reaches at the maximum height.

What is so special about the maximum height? V=0

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00 m/s$$

Solve for t

$$t = \frac{20.0}{9.80} = 2.04s$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

(e) Find the velocity and position of the stone at t=5.00s.

Velocity

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0(m/s)$$

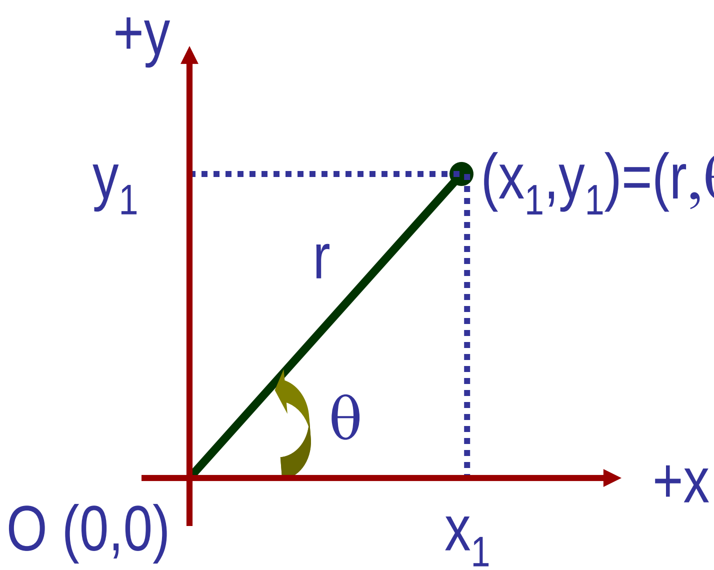
Position

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$

Coordinate Systems

- They make it easy to express locations or positions
- Two commonly used systems, depending on convenience, are
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in (r,θ)
- Vectors become a lot easier to express and compute



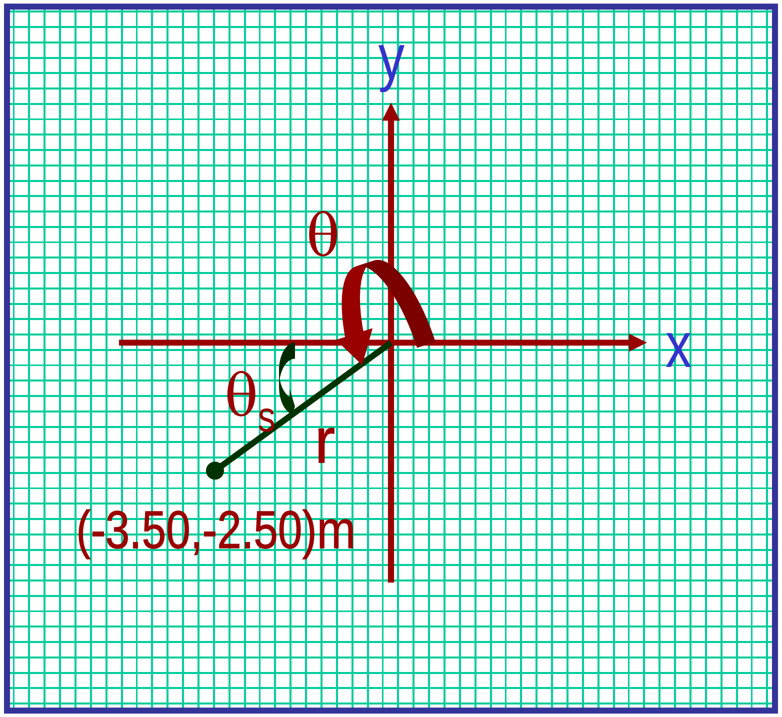
How are Cartesian and Polar coordinates related?

$$x_1 = r\cos\theta \qquad r = \sqrt{\left(x_1^2 + y_1^2\right)}$$

$$y_1 = r\sin\theta \qquad \tan\theta = \frac{y_1}{x_1}$$

Example

Cartesian Coordinate of a point in the xy plane are (x,y)= (-3.50,-2.50)m. Find the equivalent polar coordinates of this point.



$$r = \sqrt{\left(x^2 + y^2\right)}$$

$$= \sqrt{\left((-3.50)^2 + (-2.50)^2\right)}$$

$$= \sqrt{18.5} = 4.30(m)$$

$$\theta = 180 + \theta_s$$

$$\tan\theta_s = \frac{-2.50}{-3.50} = \frac{5}{7}$$

$$\theta_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$$

$$\therefore \theta = 180 + \theta_s = 180^\circ + 35.5^\circ = 216^\circ$$

Vector and Scalar

Vector quantities have both magnitudes (sizes) and directions

Force, gravitational acceleration, momentum

Normally denoted in **BOLD** letters, $\mathbf{F}$, or a letter with arrow on top $\vec{F}$

Their sizes or magnitudes are denoted with normal letters, F, or absolute values: $\left|\vec{F}\right|$ or $|F|$

Scalar quantities have magnitudes only

Can be completely specified with a value and its unit

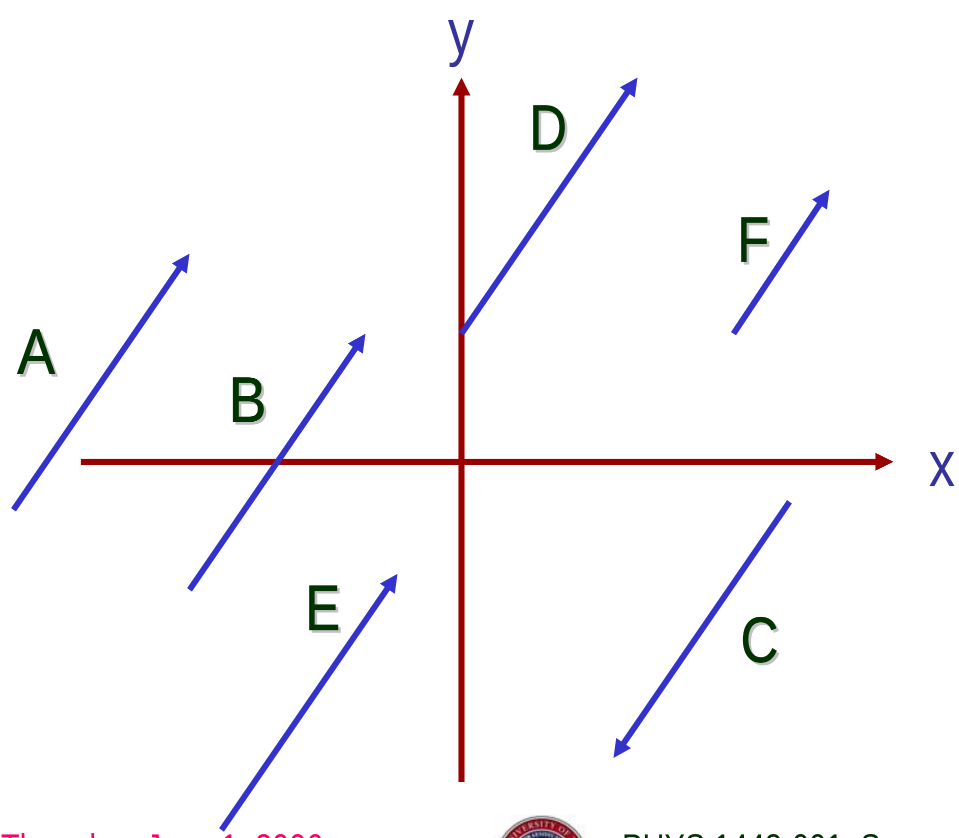
Energy, heat, mass, weight

Normally denoted in normal letters, E

Both have units!!!

Properties of Vectors

- Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system!!



Which ones are the same vectors?

A=B=E=D

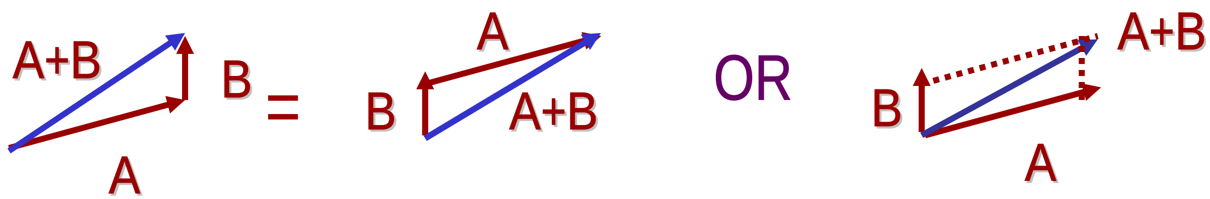
Why aren't the others?

C: The same magnitude but opposite direction: **C=-A:** A negative vector

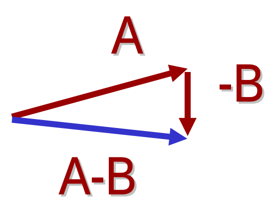
F: The same direction but different magnitude

Vector Operations

- Addition:
  - Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  - Parallelogram method: Connect the tails of the two vectors and extend
  - Addition is commutative: Changing order of operation does not affect the results A+B=B+A, A+B+C+D+E=E+C+A+B+D



- Subtraction:
  - The same as adding a negative vector: A - B = A + (-B)



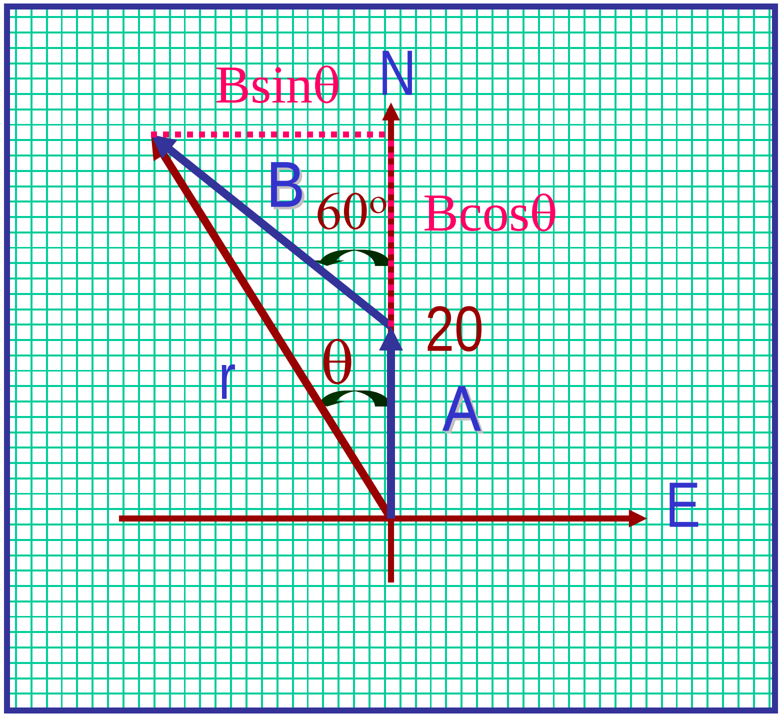
Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude A, B=2A

$$|\mathcal{B}| = 2|\mathcal{A}|$$

Example for Vector Addition

A car travels 20.0km due north followed by 35.0km in a direction 60.0° west of north. Find the magnitude and direction of resultant displacement.



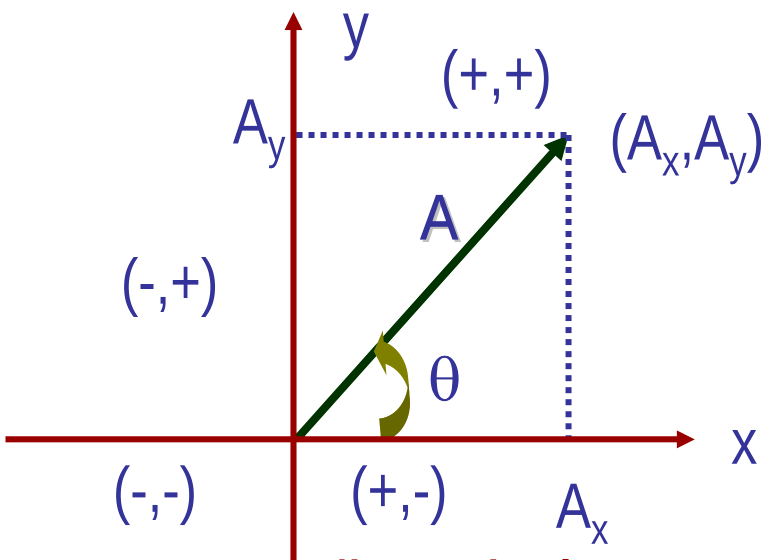
$$r = \sqrt{(A + B\cos\theta)^2 + (B\sin\theta)^2}$$
$$= \sqrt{A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta}$$
$$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$$
$$= \sqrt{(20.0)^2 + (35.0)^2 + 2\times 20.0\times 35.0\cos 60}$$
$$= \sqrt{2325} = 48.2(km)$$

$$\theta = \tan^{-1}\frac{|\vec{B}|\sin 60}{|\vec{A}| + |\vec{B}|\cos 60}$$
$$= \tan^{-1}\frac{35.0\sin 60}{20.0 + 35.0\cos 60}$$
$$= \tan^{-1}\frac{30.3}{37.5} = 38.9^\circ \text{ to W wrt N}$$

Find other ways to solve this problem…

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$A_x = \left|\vec{A}\right| \cos\theta$$

$$A_y = \left|\vec{A}\right| \sin\theta$$

} Components

$$\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2}$$

} Magnitude

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right|\cos\theta\right)^2 + \left(\left|\vec{A}\right|\sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2 \left(\cos^2\theta + \sin^2\theta\right)} = \left|\vec{A}\right|$$

- Unit vectors are **dimensionless** vectors whose **magnitude are exactly 1**
  - Unit vectors are usually expressed in **i, j, k** or $\vec{i}, \vec{j}, \vec{k}$
  - Vectors can be expressed using components and unit vectors

So the above vector **A** can be written as

$$\vec{A} = A_x \vec{i} + A_y \vec{j} = \left|\vec{A}\right| \cos\theta \, \vec{i} + \left|\vec{A}\right| \sin\theta \, \vec{j}$$

Examples of Vector Operations

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C}=\vec{A}+\vec{B}=\left(2.0\vec{i}+2.0\vec{j}\right)+\left(2.0\vec{i}-4.0\vec{j}\right)$$

$$=\left(2.0+2.0\right)\vec{i}+\left(2.0-4.0\right)\vec{j}=4.0\vec{i}-2.0\vec{j}\left(m\right)$$

$$\left|\vec{C}\right|=\sqrt{\left(4.0\right)^2+\left(-2.0\right)^2}=\sqrt{16+4.0}=\sqrt{20}=4.5(m)$$

$$\theta=\tan^{-1}\frac{C_y}{C_x}=\tan^{-1}\frac{-2.0}{4.0}=-27^\circ$$

Find the resultant displacement of three consecutive displacements: **d**$_1$=(15**i**+30**j** +12**k**)cm, **d**$_2$=(23**i**+14**j** -5.0**k**)cm, and **d**$_3$=(-13**i**+15**j**)cm

$$\vec{D}=\vec{d}_1+\vec{d}_2+\vec{d}_3=\left(15\vec{i}+30\vec{j}+12\vec{k}\right)+\left(23\vec{i}+14\vec{j}-5.0\vec{k}\right)+\left(-13\vec{i}+15\vec{j}\right)$$

$$=\left(15+23-13\right)\vec{i}+\left(30+14+15\right)\vec{j}+\left(12-5.0\right)\vec{k}=25\vec{i}+59\vec{j}+7.0\vec{k}(cm)$$

Magnitude

$$\left|\vec{D}\right|=\sqrt{\left(25\right)^2+\left(59\right)^2+\left(7.0\right)^2}=65(cm)$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$
- Average Acceleration $$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

How is each of these quantities defined in 1-D?