PHYS 1443 – Section 001
Lecture #4

Monday, June 5, 2006
Dr. Jaehoon Yu

- Motion in Two Dimensions
  - Motion under constant acceleration
  - Projectile Motion
  - Maximum ranges and heights
- Reference Frames and relative motion
- Newton's Laws of Motion
  - Force
  - Newton's Law of Inertia & Mass
  - Newton's second law of motion
  - Newton's third law of motion

Announcements

- All of you have registered for the homework
  - Good job!!!
- Quiz result
  - Class average: 10.5/14
    - Equivalent to: 75/100
  - Top score: 14/14
- Mail distribution list
  - Problem has been fixed. Please go ahead and subscribe to the list
    - Phys1443-001-summer06
  - Extra credit
    - 5 points if done by Tomorrow, June 6
    - 3 points if done by Thursday, June 8

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
- <u>Dimensionless</u>
- <u>Magnitudes are exactly 1</u>
- Unit vectors are usually expressed in **i**, **j**, **k** or

$\vec{i},\ \vec{j},\ \vec{k}$

So the vector **A** can be re-written as

$$\vec{A} = A_x \vec{i} + A_y \vec{j} = \left|\vec{A}\right| \cos\theta\, \vec{i} + \left|\vec{A}\right| \sin\theta\, \vec{j}$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\boxed{\Delta\vec{r} = \vec{r}_f - \vec{r}_i}$$
- Average Velocity: $$\boxed{\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}}$$
- Instantaneous Velocity: $$\boxed{\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}}$$
- Average Acceleration $$\boxed{\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}}$$
- Instantaneous Acceleration: $$\boxed{\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}}$$

How is each of these quantities defined in 1-D?

Kinematic Quantities in 1d and 2d

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{r}}{dt^2}$

What is the difference between 1D and 2D quantities?

2-dim Motion Under Constant Acceleration

- Position vectors in x-y plane:

$$\vec{r}_i = x_i\vec{i} + y_i\vec{j} \qquad \vec{r}_f = x_f\vec{i} + y_f\vec{j}$$

- Velocity vectors in x-y plane:

$$\vec{v}_i = v_{xi}\vec{i} + v_{yi}\vec{j} \qquad \vec{v}_f = v_{xf}\vec{i} + v_{yf}\vec{j}$$

Velocity vectors in terms of acceleration vector

X-comp $v_{xf} = v_{xi} + a_x t$ Y-comp $v_{yf} = v_{yi} + a_y t$

$$\vec{v}_f = \left(v_{xi} + a_x t\right)\vec{i} + \left(v_{yi} + a_y t\right)\vec{j} = \left(v_{xi}\vec{i} + v_{yi}\vec{j}\right) + \left(a_x\vec{i} + a_y\vec{j}\right)t =$$

$$= \vec{v}_i + \vec{a}t$$

2-dim Motion Under Constant Acceleration

- How are the position vectors written in acceleration vectors?

Position vector components

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Putting them together in a vector form

$$\vec{r}_f = x_f \vec{i} + y_f \vec{j} =$$

$$= \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2 \right)\vec{i} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2 \right)\vec{j}$$

Regrouping the above

$$= \left(x_i\vec{i} + y_i\vec{j} \right) + \left(v_{xi}\vec{i} + v_{yi}\vec{j} \right) t + \frac{1}{2}\left(a_x\vec{i} + a_y\vec{j} \right) t^2$$

$$= \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

Example for 2-D Kinematic Equations

A particle starts at origin when t=0 with an initial velocity **v**=(20**i**-15**j**)m/s. The particle moves in the xy plane with $a_x=4.0m/s^2$. Determine the components of velocity vector at any time, t.

$$v_{xf}=v_{xi}+a_x t=20+4.0t\,(m/s) \quad v_{yf}=v_{yi}+a_y t=-15+0t=-15\,(m/s)$$

Velocity vector

$$\vec{v}(t)=v_x(t)\vec{i}+v_y(t)\vec{j}=(20+4.0t)\vec{i}-15\vec{j}\,(m/s)$$

Compute the velocity and speed of the particle at t=5.0 s.

$$\vec{v}_{t=5}=v_{x,t=5}\vec{i}+v_{y,t=5}\vec{j}=(20+4.0\times5.0)\vec{i}-15\vec{j}=\left(40\vec{i}-15\vec{j}\right)\,m/s$$

$$speed=\left|\vec{v}\right|=\sqrt{(v_x)^2+(v_y)^2}=\sqrt{(40)^2+(-15)^2}=43m/s$$

Example for 2-D Kinematic Eq. Cnt'd

Angle of the Velocity vector

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = \tan^{-1}\left(\frac{-3}{8}\right) = -21^{\circ}$$

Determine the x and y components of the particle at t=5.0 s.

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = 20\times5+\frac{1}{2}\times4\times5^2 = 150(m)$$

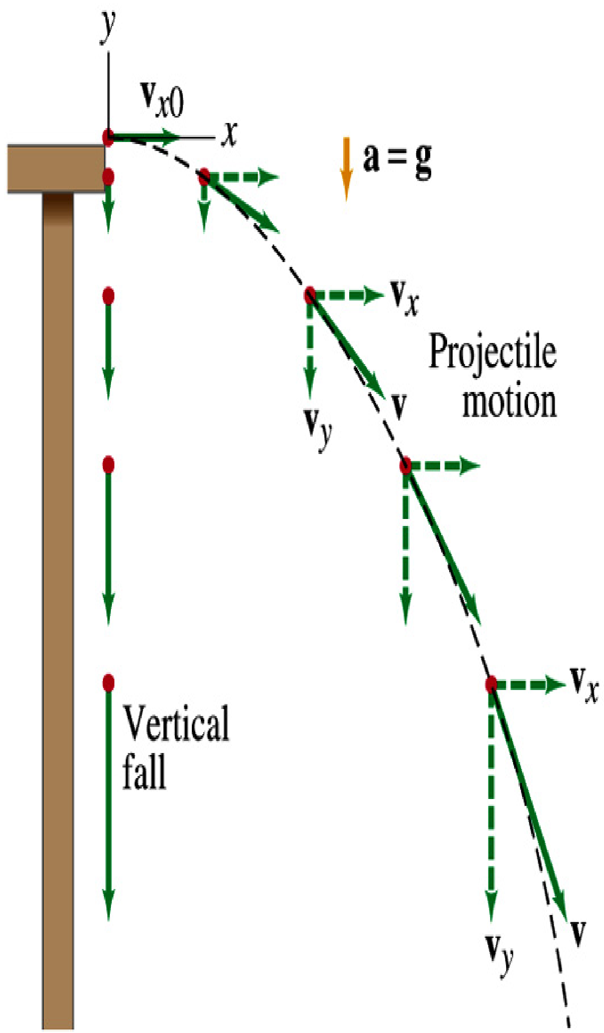
$$y_f = v_{yi}t = -15\times5 = -75\,(m)$$

Can you write down the position vector at t=5.0s?

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} = 150\vec{i} - 75\vec{j}\,(m)$$

Projectile Motion

- A 2-dim motion of an object under the gravitational acceleration with the following assumptions
  - Free fall acceleration, $-g$, is constant over the range of the motion
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity (no acceleration)
  - Vertical motion under constant acceleration (g)



Show that a projectile motion is a parabola!!!

x-component $v_{xi} = v_i \cos\theta_i$

y-component $v_{yi} = v_i \sin\theta_i$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = -g\vec{j}$$

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

$a_x=0$

$$x_f = v_{xi}t = v_i\cos\theta_i t$$

$$\boxed{t = \frac{x_f}{v_i\cos\theta_i}}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)\,t^2 = v_i\sin\theta_i t - \frac{1}{2}gt^2$$

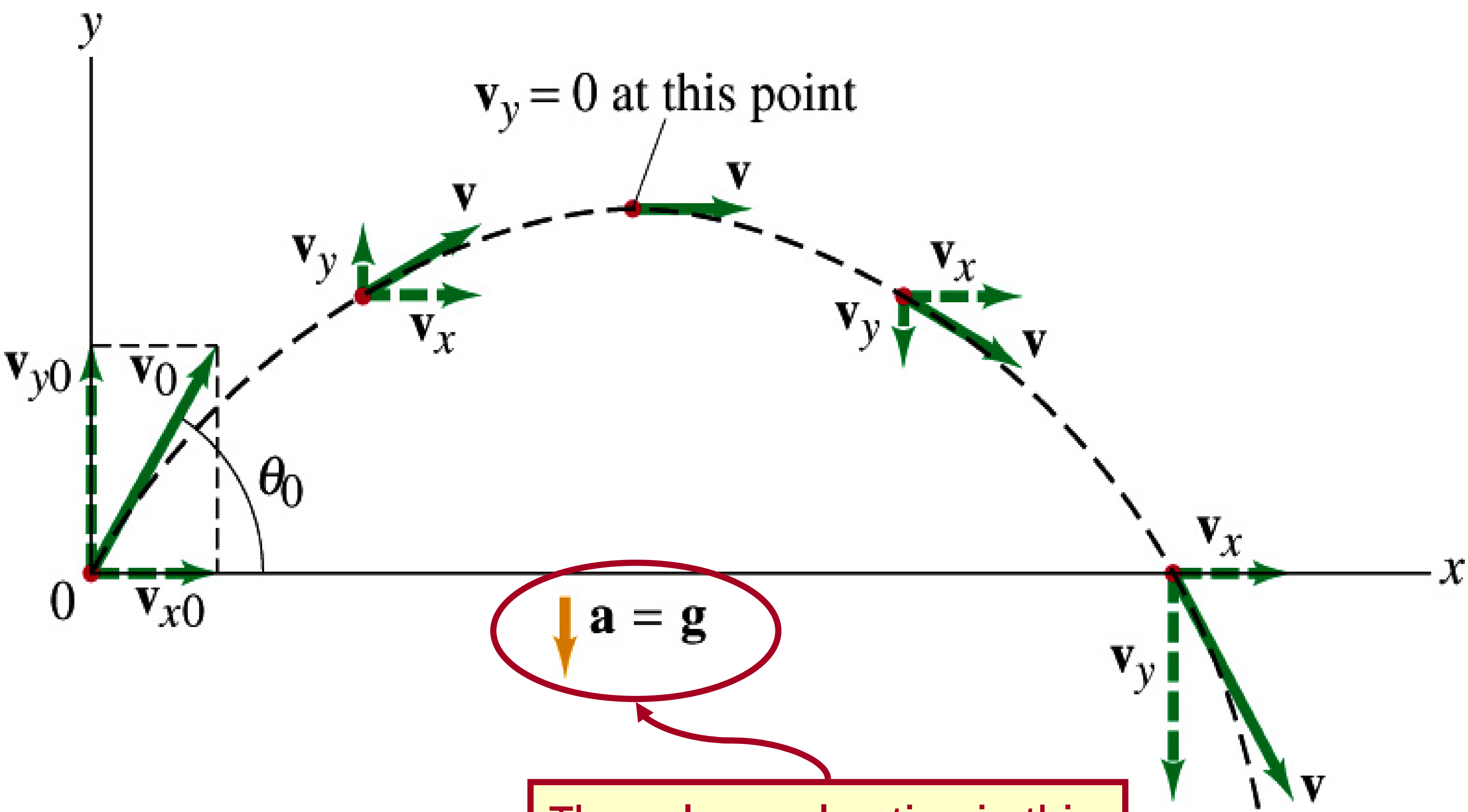
Plug t into the above

$$y_f = v_i\sin\theta_i\left(\frac{x_f}{v_i\cos\theta_i}\right) - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta_i}\right)^2$$

$$\boxed{y_f = x_f\tan\theta_i - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2}$$

What kind of parabola is this?

Projectile Motion



The only acceleration in this motion. It is a constant!!

Example for Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance the ball is from the original position when landed.

Which component determines the flight time and the distance?

Flight time is determined by y component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

So the possible solutions are…

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\sec$$

$$\therefore \ t \approx 8\sec$$

Why isn't 0 the solution?

Distance is determined by x component in 2-dim, because the ball is at $y=0$ position when it completed it's flight.

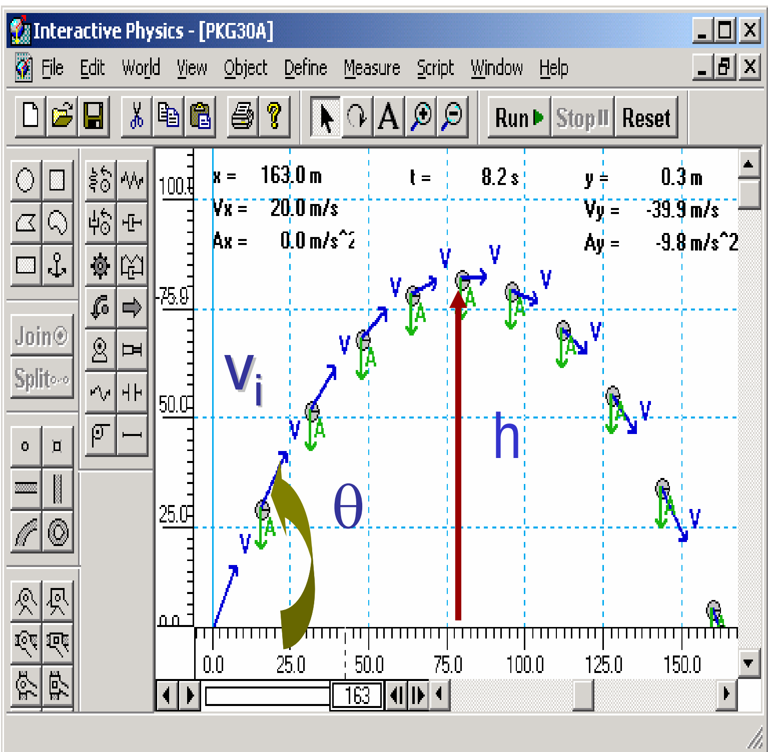
$$x_f = v_{xi}t = 20 \times 8 = 160(m)$$

Horizontal Range and Max Height

- Based on what we have learned in the previous pages, one can analyze a projectile motion in more detail
  - Maximum height an object can reach
  - Maximum range

What happens at the maximum height?

At the maximum height the object's vertical motion stops to turn around!!



$$v_{yf} = v_{yi} + a_y t$$

$$= v_i \sin\theta_i - g t_A = 0$$

Solve for t_A

$$\therefore t_A = \frac{v_i \sin\theta_i}{g}$$

Horizontal Range and Max Height

Since no acceleration is in x direction, it still flies even if v_y=0.

$$R = v_{xi}t = v_{xi}(2t_A) = 2v_i\cos\theta_i\left(\frac{v_i\sin\theta_i}{g}\right)$$

Range

$$\boxed{R = \left(\frac{v_i^2\sin 2\theta_i}{g}\right)}$$

$$y_f = h = v_{yi}t + \frac{1}{2}(-g)t^2 = v_i\sin\theta_i\left(\frac{v_i\sin\theta_i}{g}\right) - \frac{1}{2}g\left(\frac{v_i\sin\theta_i}{g}\right)^2$$

Height

$$\boxed{y_f = h = \left(\frac{v_i^2\sin^2\theta_i}{2g}\right)}$$

Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left(\frac{v_i^2 \sin^2 \theta_i}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_i = 90^o$!!!

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i = 90^o$, i.e., $\theta_i = 45^o$!!!

Example for a Projectile Motion

- A stone was thrown upward from the top of a cliff at an angle of 37º to horizontal with initial speed of 65.0m/s. If the height of the cliff is 125.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yi} = v_i \sin\theta_i = 65.0 \times \sin 37^\circ = 39.1 m/s$$

$$y_f = -125.0 = v_{yi}t - \frac{1}{2}gt^2$$

Becomes

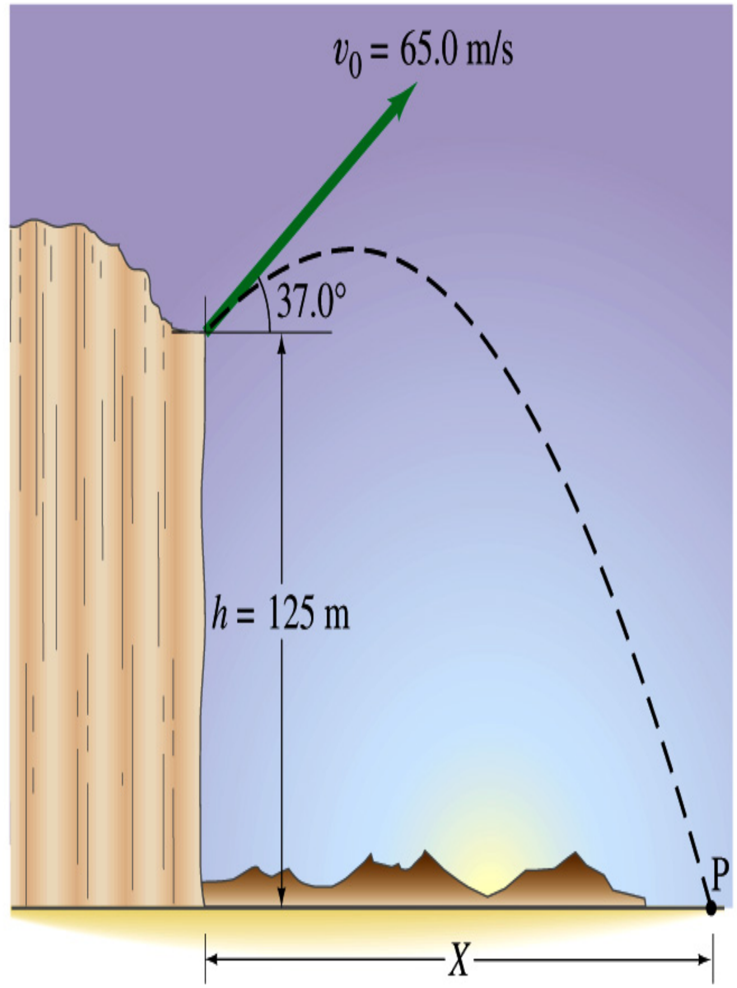
$$gt^2 - 78.2t - 250 = 9.80t^2 - 78.2t - 250 = 0$$

$$t = \frac{78.2 \pm \sqrt{(-78.2)^2 - 4 \times 9.80 \times (-250)}}{2 \times 9.80}$$

$$t = -2.43 s \quad \text{or} \quad t = 10.4 s$$

$$\boxed{t = 10.4 s}$$

Since negative time does not exist.



Example cont'd

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yf} = v_{yi} - gt = v_i \sin\theta_i - gt = 39.1 - 9.80 \times 10.4 = -62.8 m/s$$

$$|v| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{51.9^2 + (-62.8)^2} = 81.5 m/s$$

- What are the maximum height and the maximum range of the stone?

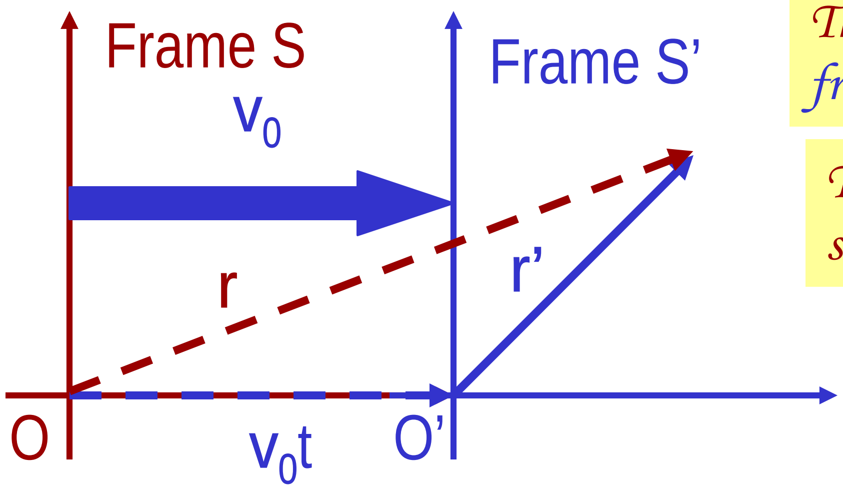
Do these yourselves at home for fun!!!

Observations in Different Reference Frames

Results of Physical measurements in different reference frames could be different

Observations of the same motion in a stationary frame would be different than the ones made in the frame moving together with the moving object.

Consider that you are driving a car. To you, the objects in the car do not move while to the person outside the car they are moving in the same speed and direction as your car is.



The position vector r' *is still* r' *in the moving frame S'.no matter how much time has passed!!*

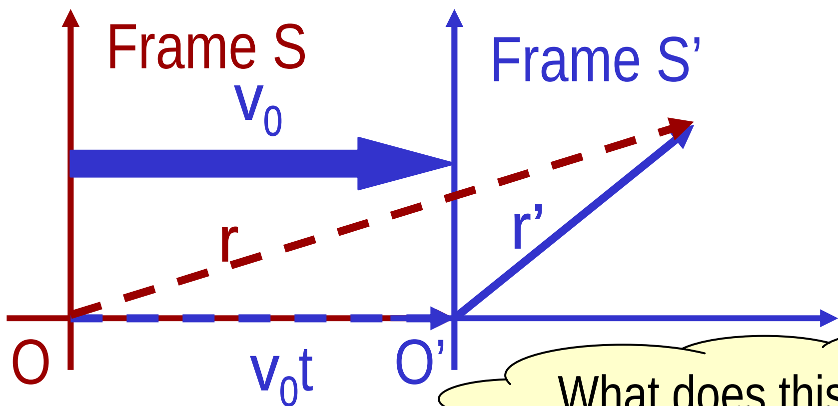
The position vector r *is no longer* r *in the stationary frame S when time t has passed.*

How are these position vectors related to each other?

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t$$

Relative Velocity and Acceleration

The velocity and acceleration in two different frames of references can be denoted, using the formula in the previous slide:



Galilean transformation equation

$$\vec{r}' = \vec{r} - \vec{v}_0 t$$

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{r}}{dt} - \vec{v}_0$$

$$\vec{v}' = \vec{v} - \vec{v}_0$$

What does this tell you?

$$\frac{d\vec{v}'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{v}_0}{dt}$$

$$\boxed{\vec{a}' = \vec{a}, \text{ when } \vec{v}_0 \text{ is constant}}$$

The accelerations measured in two frames are the same when the frames move at a constant velocity with respect to each other!!!

The earth's gravitational acceleration is the same in a frame moving at a constant velocity wrt the earth.