PHYS 1443 – Section 001
Lecture #5

Tuesday, June 6, 2006

Dr. Jaehoon Yu

- Newton's Laws of Motion
  - Force
  - Newton's Law of Inertia & Mass
  - Newton's second law of motion
  - Gravitational Force and Weight
  - Newton's third law of motion
- Application of Newton's Laws
  - Free-body diagrams
  - Application of Newton's Laws
  - Motion without friction

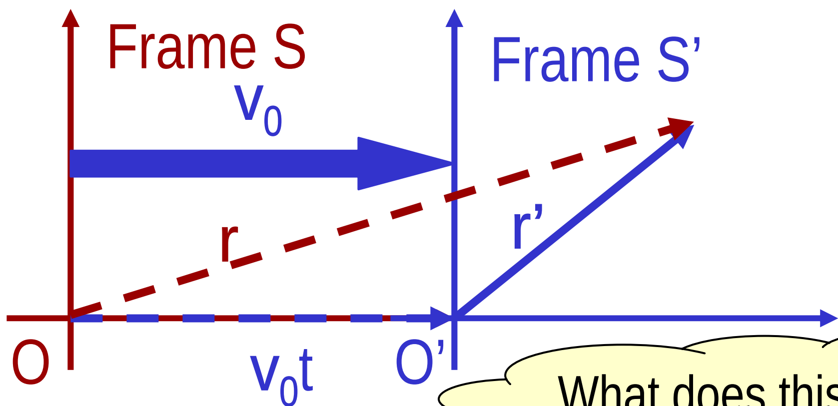
Today's homework is HW #3, due 7pm, Friday, June 9!!

Announcements

- Quiz this Thursday, June 8
  - At the beginning of the class
  - CH 1 – what we cover tomorrow
- Mail distribution list
  - 8 of you have been added to the list
  - Extra credit
    - 5 points if done by Today, June 6
    - 3 points if done by Thursday, June 8

Relative Velocity and Acceleration

The velocity and acceleration in two different frames of references can be denoted, using the formula in the previous slide:



Galilean transformation equation

$$\vec{r}\,' = \vec{r} - \vec{v}_0 t$$

$$\frac{d\vec{r}\,'}{dt} = \frac{d\vec{r}}{dt} - \vec{v}_0$$

$$\vec{v}\,' = \vec{v} - \vec{v}_0$$

What does this tell you?

$$\frac{d\vec{v}\,'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{v}_0}{dt}$$

$\vec{a}\,' = \vec{a}$, when $\vec{v}_0$ is constant

The accelerations measured in two frames are the same when the frames move at a constant velocity with respect to each other!!!

The earth's gravitational acceleration is the same in a frame moving at a constant velocity wrt the earth.

The reference frame that moves at a constant velocity is called the Inertial Frame of Reference!!

Force

We've been learning kinematics; describing motion without understanding what the cause of the motion is. Now we are going to learn dynamics!!

Can someone tell me what FORCE is?

FORCE is what causes an object to move.

The above statement is not entirely correct. Why?

Because when an object is moving with a constant velocity no force is exerted on the object!!!

FORCEs are what cause any change in the velocity of an object!!

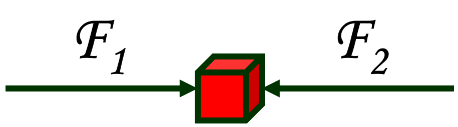
What does this statement mean?

When there is force, there is change of velocity!!

What does force cause? It causes an acceleration.!!

What happens if there are several forces being exerted on an object?

Forces are vector quantities, so vector sum of all forces, the NET FORCE, determines the direction of the acceleration of the object.



NET FORCE, $F = F_1 + F_2$

When the net force on an object is 0, it has constant velocity and is at its equilibrium!!

More Force

There are various classes of forces

Contact Forces: Forces exerted by physical contact of objects

Examples of Contact Forces: Baseball hit by a bat, Car collisions

Field Forces: Forces exerted without physical contact of objects

Examples of Field Forces: Gravitational Force, Electro-magnetic force

What are possible ways to measure strength of Force?

A calibrated spring whose length changes linearly with the force exerted .

Forces are vector quantities, so addition of multiple forces must be done following the rules of vector additions.

Newton's First Law and Inertial Frames

Aristotle (384-322BC): *A natural state of a body is rest. Thus force is required to move an object. To move faster, ones needs higher force.*

Galileo's statement on natural states of matter: *Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed!!*

Galileo's statement is formulated by Newton into the 1st **law of motion (Law of Inertia)**: *In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.*

What does this statement tell us?

- When no force is exerted on an object, the acceleration of the object is 0.
- Any isolated object, the object that do not interact with its surrounding, is either at rest or moving at a constant velocity.
- Objects would like to keep its current state of motion, as long as there is no force that interferes with the motion. This tendency is called the <u>Inertia.</u>

A frame of reference that is moving at constant velocity is called an *Inertial Frame*

Mass

Mass: *A measure of the inertia of a body* or *quantity of matter*

- Independent of the object's surroundings: The same no matter where you go.
- Independent of method of measurement: The same no matter how you measure it.

The heavier an object the bigger the inertia gets!!

It is harder to make changes of motion of a heavier object than the lighter ones.

The same forces applied to two different masses result in different acceleration depending on the mass.

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1}$$

Note that mass and weight of an object are two different quantities!!

Weight of an object is the magnitude of the gravitational force exerted on the object.

Not an inherent property of an object!!!

Weight will change if you measure on the Earth or on the moon.

Newton's Second Law of Motion

The acceleration of an object is directly proportional to the net force exerted on it and is inversely proportional to the object's mass.

How do we write the above statement in a mathematical expression?

$$\boxed{\sum_i \vec{F}_i = m\vec{a}}$$

Newton's 2nd Law of Motion

Since it's a vector expression, each component should also satisfy:

$$\sum_i F_{ix} = ma_x \qquad \sum_i F_{iy} = ma_y \qquad \sum_i F_{iz} = ma_z$$

From the above vector expression, what do you conclude the dimension and unit of force are?

The dimension of force is $[m][a] = [M][LT^{-2}]$

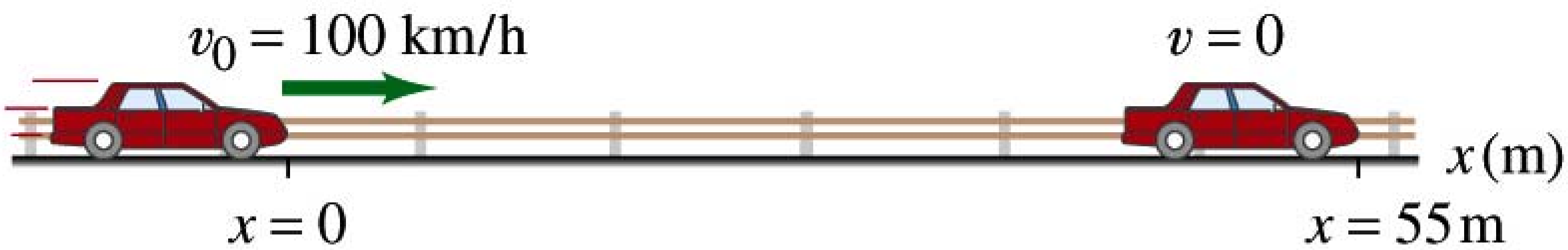
The unit of force in SI is $[Force] = [m][a] = [M][LT^{-2}] = kg \cdot m/s^2$

For ease of use, we define a new derived unit called, Newton (N)

$$1N \equiv 1kg \cdot m/s^2 \approx \frac{1}{4} lbs$$

Example 4.2

What constant net force is required to bring a 1500kg car to rest from a speed of 100km/h within a distance of 55m?



What do we need to know to figure out the force? **Acceleration!!**

What are given? Initial speed: $v_{xi} = 100\ km/h = 28\ m/s$ Final speed: $v_{xf} = 0\ m/s$

Displacement: $\Delta x = x_f - x_i = 55\ m$

This is a one dimensional motion. Which kinetic formula do we use to find acceleration?

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Acceleration

$$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{-(28m/s)^2}{2(55m)} = -7.1m/s^2$$

Thus, the force needed to stop the car is

$$F_x = ma_x = 1500kg \times (-7.1m/s^2) = -1.1 \times 10^4 N$$

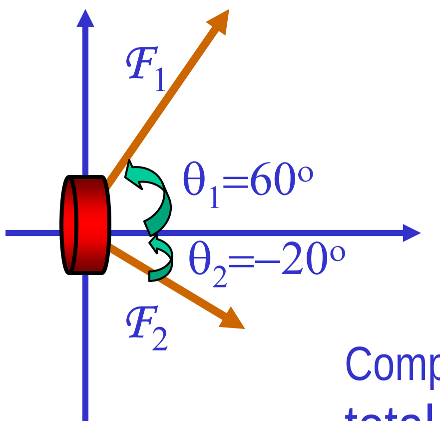
Given the force how far does the car move till it stops?

$$\Delta x = x_f - x_i = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{m(v_{xf}^2 - v_{xi}^2)}{2ma_x} = \frac{m(v_{xf}^2 - v_{xi}^2)}{2F_x}$$

- Linearly proportional to the mass of the car
- Squarely proportional to the speed of the car
- Inversely proportional to the force by the brake

Example for Newton's 2nd Law of Motion

Determine the magnitude and direction of acceleration of the puck whose mass is 0.30kg and is being pulled by two forces, F1 and F2, as shown in the picture, whose magnitudes of the forces are 8.0 N and 5.0 N, respectively.



Components of $\mathcal{F}_1$

$$F_{1x} = \left|\vec{F}_1\right|\cos\theta_1 = 8.0\times\cos\left(60^\circ\right) = 4.0N$$

$$F_{1y} = \left|\vec{F}_1\right|\sin\theta_1 = 8.0\times\sin\left(60^\circ\right) = 6.9N$$

Components of $\mathcal{F}_2$

$$F_{2x} = \left|\vec{F}_2\right|\cos\theta_2 = 5.0\times\cos\left(-20^\circ\right) = 4.7N$$

$$F_{2y} = \left|\vec{F}_2\right|\sin\theta_2 = 5.0\times\sin\left(-20^\circ\right) = -1.7N$$

Components of total force $\mathcal{F}$

$$F_x = F_{1x} + F_{2x} = 4.0 + 4.7 = 8.7N = ma_x$$

$$F_y = F_{1y} + F_{2y} = 6.9 - 1.7 = 5.2N = ma_y$$

Magnitude and direction of acceleration a

$$a_x = \frac{F_x}{m} = \frac{8.7}{0.3} = 29m/s^2 \quad a_y = \frac{F_y}{m} = \frac{5.2}{0.3} = 17m/s^2 \quad \left|\vec{a}\right| = \sqrt{(a_x)^2 + (a_y)^2} = \sqrt{(29)^2 + (17)^2} = 34m/s^2$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^\circ$$

Acceleration Vector a

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \left(29\hat{i} + 17\hat{j}\right)m/s^2$$

Gravitational Force and Weight

Gravitational Force, $\mathcal{F}_g$

The attractive force exerted on an object by the Earth

$$\vec{F}_G = m\vec{a} = m\vec{g}$$

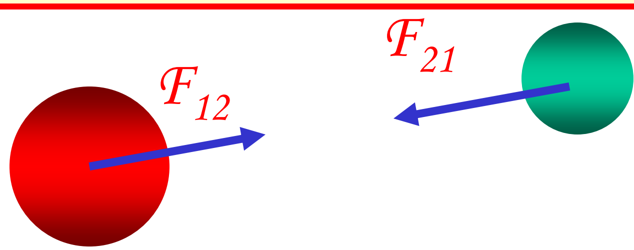
Weight of an object with mass M is $W = \left|\vec{F}_G\right| = M\left|\vec{g}\right| = Mg$

Since weight depends on the magnitude of gravitational acceleration, g, it varies depending on geographical location.

By measuring the forces one can determine masses. This is why you can measure mass using the spring scale.

Newton's Third Law (Law of Action and Reaction)

If two objects interact, the force F_{21} exerted on object 1 by object 2 is equal in magnitude and opposite in direction to the force F_{12} exerted on object 2 by object 1.



$$\vec{F}_{12} = -\vec{F}_{21}$$

The action force is equal in magnitude to the reaction force but in opposite direction. These two forces always act ***on different objects***.

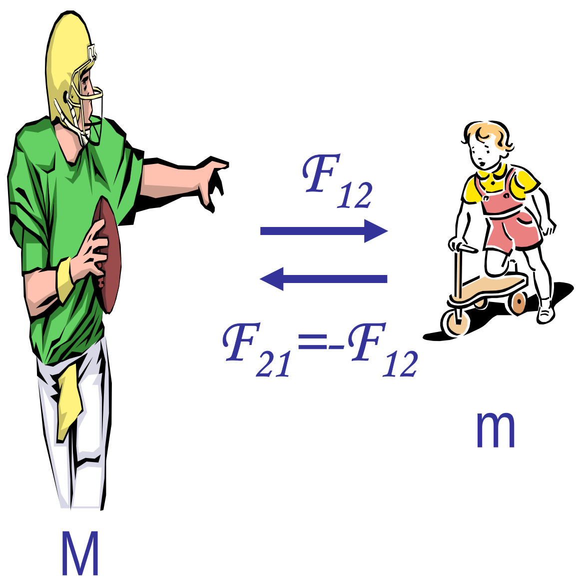
What is the reaction force to the force of a free fall object?

The gravitational force exerted by the object to the Earth!

Stationary objects on top of a table has a reaction force (normal force) from table to balance the action force, the gravitational force.

Example of Newton's 3rd Law

A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push against each other so that they move apart. a) Who moves away with the higher speed and by how much?



$$\vec{F}_{12} = -\vec{F}_{21} \quad\Rightarrow\quad \left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = F$$

$$\vec{F}_{12} = m\vec{a}_b \quad\Rightarrow\quad F_{12x} = ma_{bx}, \quad F_{12y} = ma_{by} = 0$$

$$\vec{F}_{21} = M\vec{a}_M \quad\Rightarrow\quad F_{21x} = Ma_{Mx}, \quad F_{21y} = Ma_{My} = 0$$

Since $\vec{F}_{12} = -\vec{F}_{21}$ and $\left|\vec{F}_{12}\right| = \left|-\vec{F}_{21}\right| = F$

Establish the equation

$$ma_{bx} = F = Ma_{Mx}$$

Divide by m

$$a_{bx} = \frac{F}{m} = \frac{M}{m}a_{Mx}$$

Example of Newton's 3rd Law

Man's velocity

$$v_{Mxf} = v_{Mxi} + a_{Mx}t = a_{Mx}t$$

Boy's velocity

$$v_{bxf} = v_{bxi} + a_{bx}t = a_{bx}t = \frac{M}{m}a_{Mx}t = \frac{M}{m}v_{Mxf}$$

So boy's velocity if higher than man's, if M>m, by the ratio of the masses.

b) Who moves farther while their hands are in contact?

Boy's displacement

$$x_b = v_{bxi}t + \frac{1}{2}a_{bx}t^2 = \frac{M}{2m}a_{Mx}t^2$$

$$x_b = \frac{M}{m}\left(\frac{1}{2}a_{Mx}t^2\right) = \frac{M}{m}x_M$$

Man's displacement

Given in the same time interval, since the boy has higher acceleration and thereby higher speed, he moves farther than the man.

Some Basic Information

When Newton's laws are applied, *external forces* are only of interest!!

Why?

Because, as described in Newton's first law, an object will keep its current motion unless non-zero net external force is applied.

Normal Force, n:

Reaction force that reacts to gravitational force due to the surface structure of an object. Its direction is perpendicular to the surface.

Tension, T:

The reactionary force by a stringy object against an external force exerted on it.

Free-body diagram

*A graphical tool which is a **diagram of external forces on an object** and is extremely useful analyzing forces and motion!! Drawn only on an object.*

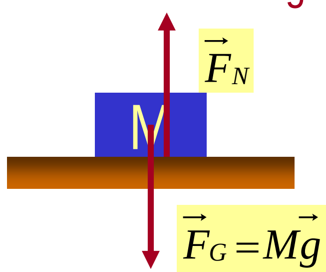
Free Body Diagrams and Solving Problems

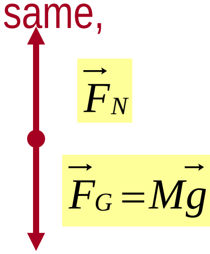
- Free-body diagram: A diagram of vector forces acting on an object

⇒ A great tool to solve a problem using forces or using dynamics

1. Select a point on an object in the problem
2. Identify all the forces acting only on the selected object
3. Define a reference frame with positive and negative axes specified
4. Draw arrows to represent the force vectors on the selected point
5. Write down net force vector equation
6. Write down the forces in components to solve the problems

⇒ No matter which one we choose to draw the diagram on, the results should be the same, as long as they are from the same motion



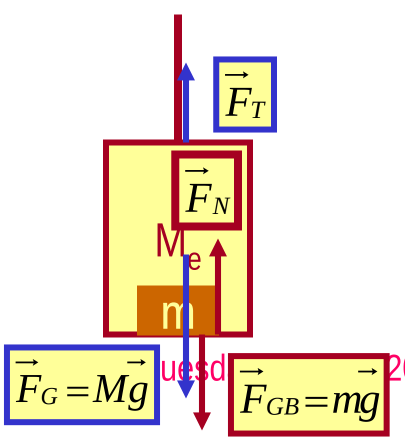


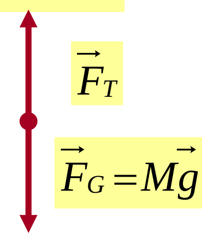
Which one would you like to select to draw FBD?

What do you think are the forces acting on this object?

Gravitational force

A force supporting the object exerted by the floor





Which one would you like to select to draw FBD?

What do you think are the forces acting on this elevator?

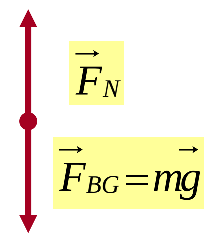
Gravitational force

The force pulling the elevator (Tension)

What about the box in the elevator?

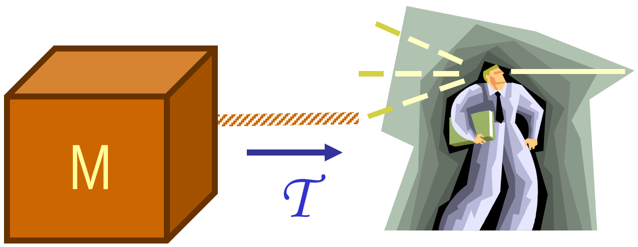
Gravitational force

Normal force



Applications of Newton's Laws

Suppose you are pulling a box on frictionless ice, using a rope.



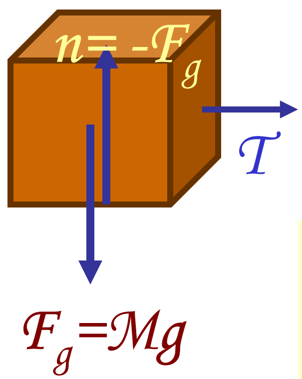
What are the forces being exerted on the box?

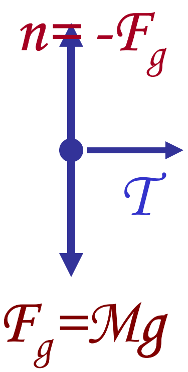
Gravitational force: F_g

Normal force: n

Tension force: T

Free-body diagram





Total force:
$F = F_g + n + T = T$

$$\sum F_x = T = Ma_x \qquad \boxed{a_x = \frac{T}{M}}$$

$$\sum F_y = -F_g + n = Ma_y = 0 \qquad \boxed{a_y = 0}$$

If T is a constant force, a_x, is constant

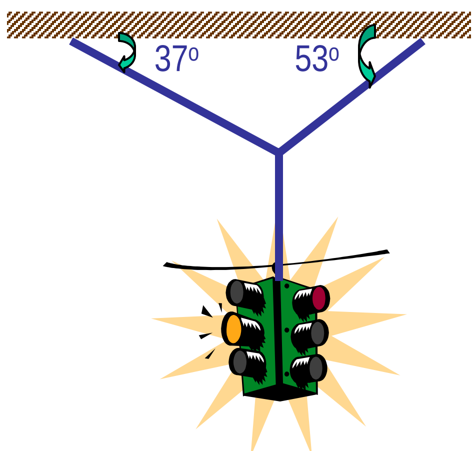
$$v_{xf} = v_{xi} + a_x t = v_{xi} + \left(\frac{T}{M}\right)t$$

$$\Delta x = x_f - x_i = v_{xi}t + \frac{1}{2}\left(\frac{T}{M}\right)t^2$$

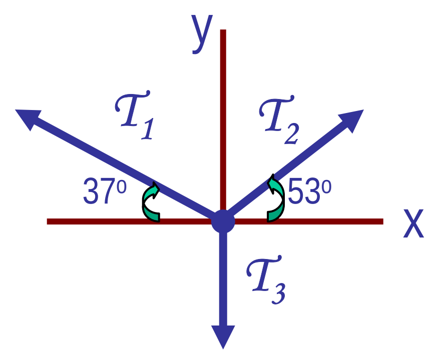
What happened to the motion in y-direction?

Example for Using Newton's Laws

A traffic light weighing 125 N hangs from a cable tied to two other cables fastened to a support. The upper cables make angles of 37.0° and 53.0° with the horizontal. Find the tension in the three cables.



Free-body Diagram



$$\vec{F} = \vec{T}_1 + \vec{T}_2 + \vec{T}_3 = m\vec{a} = 0$$ Newton's 2nd law

x-comp. of net force

$$F_x = \sum_{i=1}^{i=3} T_{ix} = 0 \quad -T_1\cos\left(37^\circ\right) + T_2\cos\left(53^\circ\right) = 0 \;\therefore T_1 = \frac{\cos\left(53^\circ\right)}{\cos\left(37^\circ\right)} T_2 = 0.754 T_2$$

y-comp. of net force

$$F_y = \sum_{i=1}^{i=3} T_{iy} = 0$$

$$T_1\sin\left(37^\circ\right) + T_2\sin\left(53^\circ\right) - mg = 0$$

$$T_2\left[\sin\left(53^\circ\right) + 0.754\times\sin\left(37^\circ\right)\right] = 1.25T_2 = 125N$$

$$T_2 = 100N; \quad T_1 = 0.754T_2 = 75.4N$$