PHYS 1443 – Section 001
Lecture #6

Wednesday, June 7, 2006
Dr. Jaehoon Yu

- Application of Newton's Laws
  - Motion without friction
- Forces of Friction
- Uniform and Non-uniform Circular Motions
- Resistive Forces and Terminal Velocity
- Newton's Law of Universal Gravitation
- Kepler's Laws

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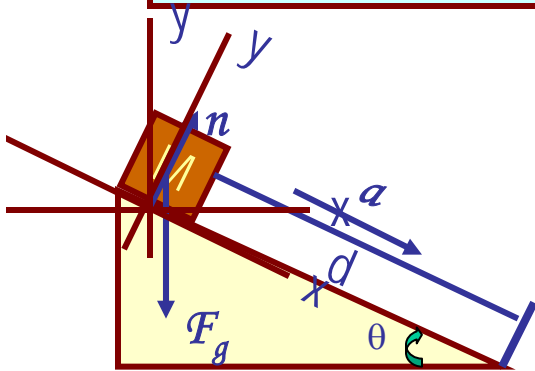   Dr. Jaehoon Yu

Announcements

- Quiz tomorrow, Thursday, June 8
  - At the beginning of the class
  - CH 1 – what we cover today (CH 6 – 4 or 5?)
- Reading assignments:
  - CH 4 – 7, 5 – 4, 5 – 5
- Mid-term exam
  - 8:00 – 10am, Thursday, June 15, in class
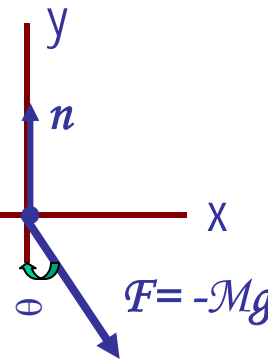 - CH 1 – 8 or 9?

Example w/o Friction

A crate of mass M is placed on a frictionless inclined plane of angle θ.

a) Determine the acceleration of the crate after it is released.

Free-body Diagram

$$\vec{F} = \vec{F}_g + \vec{n} = m\vec{a}$$

$$F_x = Ma_x = F_{gx} = Mg\sin\theta$$

$$\boxed{a_x = g\sin\theta}$$

$$F_y = Ma_y = n - F_{gy} = n - mg\cos\theta = 0$$

Supposed the crate was released at the top of the incline, and the length of the incline is d. How long does it take for the crate to reach the bottom and what is its speed at the bottom?

$$d = v_{ix}t + \frac{1}{2}a_x t^2 = \frac{1}{2}g\sin\theta\, t^2 \qquad \boxed{\therefore t = \sqrt{\frac{2d}{g\sin\theta}}}$$

$$v_{xf} = v_{ix} + a_x t = g\sin\theta\sqrt{\frac{2d}{g\sin\theta}} = \sqrt{2dg\sin\theta}$$

$$\boxed{\therefore v_{xf} = \sqrt{2dg\sin\theta}}$$

Forces of Friction

Resistive force exerted on a moving object due to viscosity or other types frictional property of the medium in or surface on which the object moves.

These forces are either proportional to the velocity or the normal force.

Force of static friction, f_s: *The resistive force exerted on the object until just before the beginning of its movement*

Empirical Formula

$$\left|\vec{f}_s\right| \le \mu_s \left|\vec{n}\right|$$

What does this formula tell you?

Frictional force increases till it reaches the limit!!

Beyond the limit, the object moves, and there is **NO MORE** static friction but kinetic friction takes it over.

Force of kinetic friction, f_k *The resistive force exerted on the object during its movement*

$$\left|\vec{f}_k\right| = \mu_k \left|\vec{n}\right|$$

Which direction does kinetic friction apply?

Opposite to the motion!

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Example w/ Friction

Suppose a block is placed on a rough surface inclined relative to the horizontal. The inclination angle is increased till the block starts to move. Show that by measuring this critical angle, θ_c, one can determine coefficient of static friction, μ_s.

n

a

Free-body Diagram

y

n

$f_s = \mu_k n$

x

F_g

θ

$F = -Mg$

θ
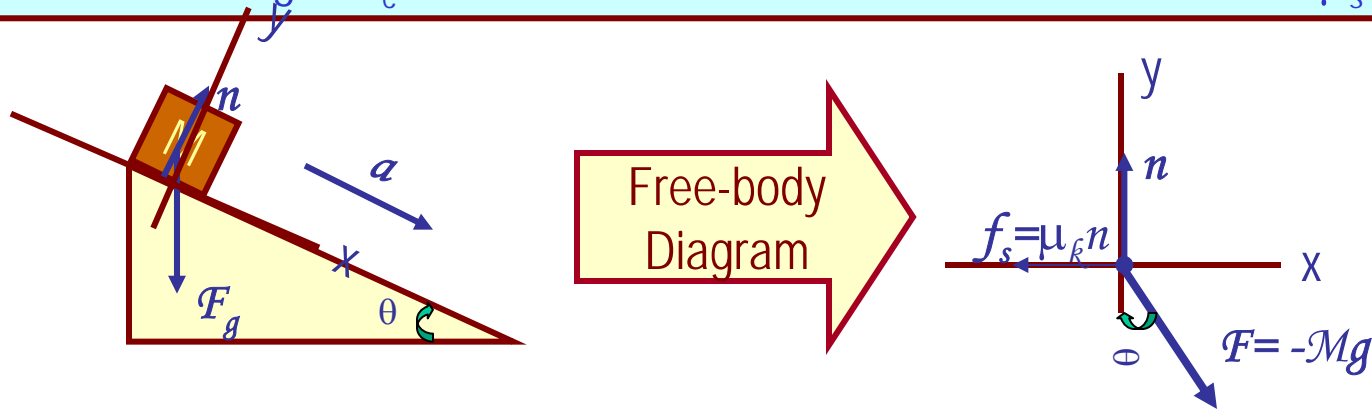
Net force $\qquad \vec{F} = M\vec{a} = \vec{F}_g + \vec{n} + \vec{f}_s$
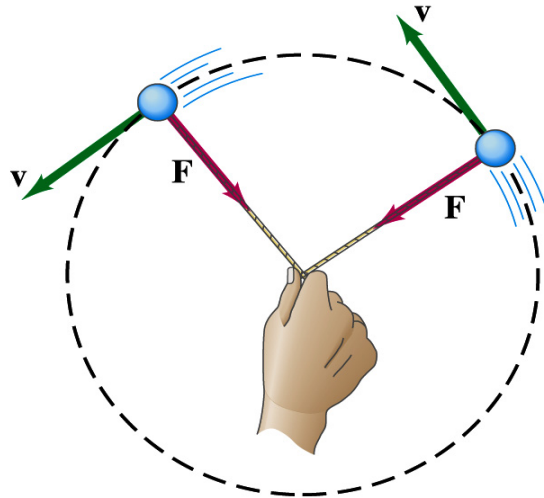
x comp. $\qquad F_x = F_{gx} - f_s = Mg \sin \theta - f_s = 0 \qquad f_s = \mu_s n = Mg \sin \theta_c$

y comp. $\qquad F_y = Ma_y = n - F_{gy} = n - Mg\cos\theta_c = 0 \qquad n = F_{gy} = Mg \cos \theta_c$

$$\mu_s = \frac{Mg \sin \theta_c}{n} = \frac{Mg \sin \theta_c}{Mg \cos \theta_c} = \tan \theta_c$$

Newton's Second Law & Uniform Circular Motion

*The centripetal acceleration is always perpendicular to velocity vector, **v**, for uniform circular motion.*

$$a_r = \frac{v^2}{r}$$

Are there forces in this motion? If so, what do they do?

The force that causes the centripetal acceleration acts toward the center of the circular path and causes the change in the direction of the velocity vector. This force is called the centripetal force.
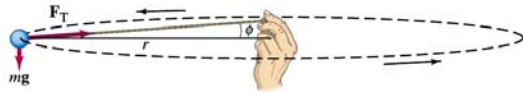
$$\sum F_r = ma_r = m\frac{v^2}{r}$$

What do you think will happen to the ball if the string that holds the ball breaks?

Based on Newton's 1st law, since the external force no longer exist. Therefore, the ball will continue its motion without change and will fly away following the tangential direction to the circle.

Example of Uniform Circular Motion

A ball of mass 0.500kg is attached to the end of a 1.50m long cord. The ball is moving in a horizontal circle. If the string can withstand maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks?

Centripetal acceleration:

$$a_r = \frac{v^2}{r}$$

When does the string break?

$$\sum F_r = ma_r = m\frac{v^2}{r} > T$$

*when the centripetal force is greater than the sustainable tension.*

$$m\frac{v^2}{r} = T \qquad v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{50.0 \times 1.5}{0.500}} = 12.2\,(m/s)$$

Calculate the tension of the cord when speed of the ball is 5.00m/s.

$$T = m\frac{v^2}{r} = 0.500 \times \frac{(5.00)^2}{1.5} = 8.33\,(N)$$

Example of Banked Highway

(a) For a car traveling with speed v around a curve of radius r, determine a formula for the angle at which a road should be banked so that no friction is required to keep the car from skidding.
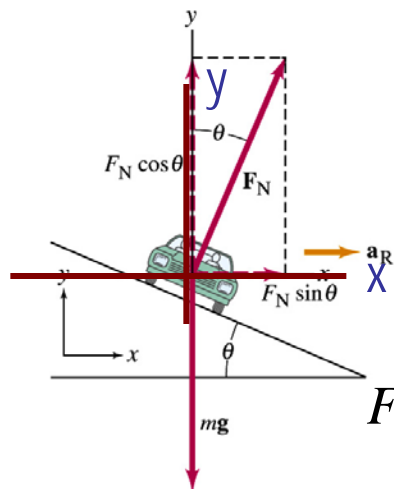
x comp. $\sum F_x = F_N \sin\theta - ma_r = F_N \sin\theta - \dfrac{mv^2}{r} = 0$

$$F_N \sin\theta = \frac{mv^2}{r}$$

y comp. $\sum F_y = F_N \cos\theta - mg = 0$ $\qquad F_N \cos\theta = mg$

$F_N = \dfrac{mg}{\sin\theta}$ $\quad F_N \sin\theta = \dfrac{mg\sin\theta}{\cos\theta} = mg\tan\theta = \dfrac{mv^2}{r}$ $\boxed{\tan\theta = \dfrac{v^2}{gr}}$

(b) What is this angle for an expressway off-ramp curve of radius 50m at a design speed of 50km/h?
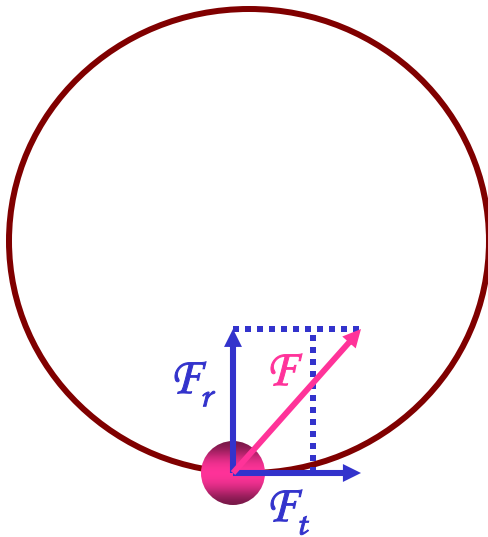
$$v = 50 km/hr = 14 m/s \qquad \tan\theta = \frac{(14)^2}{50 \times 9.8} = 0.4 \qquad \theta = \tan^{-1}(0.4) = 22°$$

Forces in Non-uniform Circular Motion

The object has both tangential and radial accelerations.

What does this statement mean?

The object is moving under both tangential and radial forces.

$$\vec{F} = \vec{F}_r + \vec{F}_t$$

These forces cause not only the velocity but also the speed of the ball to change. The object undergoes a curved motion under the absence of constraints, such as a string.

What is the magnitude of the net acceleration? $a = \sqrt{a_r^2 + a_t^2}$

Example for Non-Uniform Circular Motion

A ball of mass m is attached to the end of a cord of length R. The ball is moving in a vertical circle.   Determine the tension of the cord at any instant when the speed of the ball is v and the cord makes an angle θ with vertical.
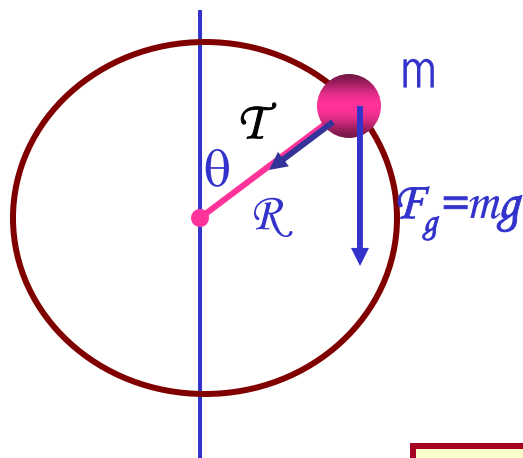
What are the forces involved in this motion?

- The gravitational force F_g
- The radial force, **T**, providing tension.
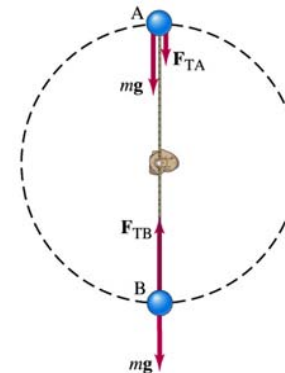
tangential comp.

$$\sum F_t = mg \sin \theta = ma_t$$

$$a_t = g \sin \theta$$

Radial comp.

$$\sum F_r = T + mg\cos\theta = ma_r = m\frac{v^2}{R}$$

$$T = m\left(\frac{v^2}{R} - g \cos \theta \right)$$

At what angles the tension becomes maximum and minimum. What are the tensions?

Motion in Resistive Forces

Medium can exert resistive forces on an object moving through it due to viscosity or other types frictional properties of the medium.

Some examples? Air resistance, viscous force of liquid, etc

These forces are exerted on moving objects in opposite direction of the movement.

These forces are proportional to such factors as speed. They almost always increase with increasing speed.
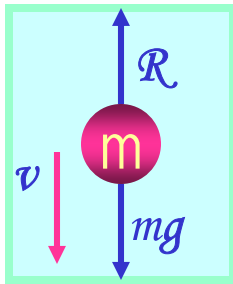
$$\mathbf{F}_D = -b\mathbf{v}$$

Two different cases of proportionality:
1. Forces linearly proportional to speed:
   Slowly moving or very small objects
2. Forces proportional to square of speed:
   Large objects w/ reasonable speed

mg
(a)

v_T
v
t
(b)
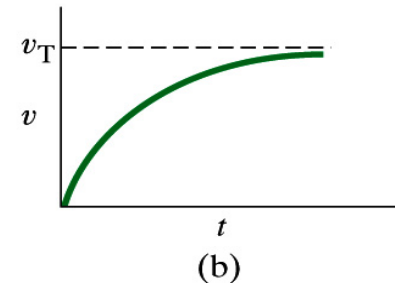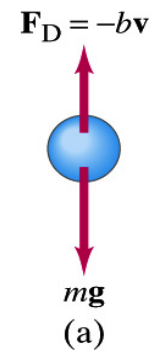
Resistive Force Proportional to Speed

Since the resistive force is proportional to speed, we can write R=bv.

Let's consider that a ball of mass m is falling through a liquid.

$$\sum \vec{F} = \vec{F}_g + \vec{R} = m\vec{a} \qquad \sum F_x = 0$$

In other words

$$\sum F_y = mg - bv = ma = m\frac{dv}{dt} \qquad \frac{dv}{dt} = g - \frac{b}{m}v \qquad \frac{dv}{dt} = g - \frac{b}{m}v = g, \text{ when } v = 0$$

The above equation also tells us that as time goes on the speed increases and the acceleration decreases, eventually reaching 0.

What does this mean?

An object moving in a viscous medium will obtain speed to a certain speed (terminal speed) and then maintain the same speed without any more acceleration.

What is the terminal speed in above case?

How do the speed and acceleration depend on time?

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right); \quad v = 0 \text{ when } t = 0;$$

$$a = \frac{dv}{dt} = \frac{mg}{b}\frac{b}{m}e^{-bt/m} = ge^{-t/\tau}; \quad a = g \text{ when } t = 0;$$

$$\frac{dv}{dt} = \frac{mg}{b}\frac{b}{m}e^{-t/\tau} = \frac{mg}{b}\frac{b}{m}\left(1 - 1 + e^{-t/\tau}\right) = g - \frac{b}{m}v$$

The time needed to reach 63.2% of the terminal speed is defined as the time constant, $\tau = m/b$.

$$\frac{dv}{dt} = g - \frac{b}{m}v = 0; \quad v_t = \frac{mg}{b}$$

Newton's Law of Universal Gravitation

People have been very curious about stars in the sky, making observations for a long time. But the data people collected have not been explained until Newton has discovered the law of gravitation.

Every particle in the Universe attracts every other particle with a force that is directly **proportional to the product of their masses** and **inversely proportional to the square of the distance** between them.

How would you write this law mathematically?

$$F_g \propto \frac{m_1 m_2}{r_{12}^2}$$

With G

$$F_g = G \frac{m_1 m_2}{r_{12}^2}$$

G is the universal gravitational constant, and its value is

$$G = 6.673 \times 10^{-11}$$

Unit? $N \cdot m^2 / kg^2$

This constant is not given by the theory but must be measured by experiments.

This form of forces is known as **the inverse-square law**, because the magnitude of the force is inversely proportional to the square of the distances between the objects.

More on Law of Universal Gravitation

Consider two particles exerting gravitational forces to each other.

$\hat{r}_{12}$

m_2

r

m_1

$\mathcal{F}_{11}$

$\mathcal{F}_{21}$
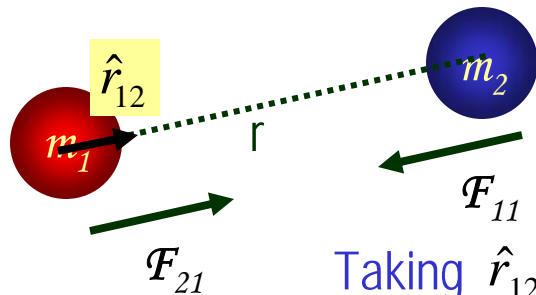
Two objects exert gravitational force on each other following Newton's 3rd law.

Taking $\hat{r}_{12}$ as the unit vector, we can write the force m_2 experiences as

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}_{12}$$

What do you think the negative sign means?

It means that the force exerted on the particle 2 by particle 1 is an attractive force, pulling #2 toward #1.

Gravitational force is a field force: Forces act on object without physical contact between the objects at all times, independent of medium between them.

The gravitational force exerted by a finite size, spherically symmetric mass distribution on an object outside of it is the same as when the entire mass of the distributions is concentrated at the center of the object.

What do you think the gravitational force on the surface of the earth looks?

$$F_g = G\frac{M_E m}{R_E^2}$$

Example for Gravitation

Since the gravitational acceleration is

$$g = G \frac{M_E}{R_E{}^2} = 6.67 \times 10^{-11} \frac{M_E}{R_E{}^2}$$

Solving for ME

$$M_E = \frac{R_E{}^2 g}{G}$$

Therefore the density of the Earth is

$$\rho = \frac{M_E}{V_E} = \frac{\dfrac{R_E{}^2 g}{G}}{\dfrac{4\pi}{3} R_E{}^3} = \frac{3g}{4\pi G R_E}$$

$$= \frac{3 \times 9.80}{4\pi \times 6.67 \times 10^{-11} \times 6.37 \times 10^6} = 5.50 \times 10^3 kg/m^3$$

Free Fall Acceleration & Gravitational Force

Weight of an object with mass m is mg. Using the force exerting on a particle of mass m on the surface of the Earth, one can obtain

$$mg = G \frac{M_E m}{R_E^2}$$

$$g = G \frac{M_E}{R_E^2}$$

What would the gravitational acceleration be if the object is at an altitude h above the surface of the Earth?

$$F_g = mg' = G \frac{M_E m}{r^2} = G \frac{M_E m}{\left(R_E + h\right)^2}$$

$$g' = G \frac{M_E}{\left(R_E + h\right)^2}$$

Distance from the center of the Earth to the object at the altitude h.

What do these tell us about the gravitational acceleration?

•The gravitational acceleration is independent of the mass of the object
•The gravitational acceleration decreases as the altitude increases
•If the distance from the surface of the Earth gets infinitely large, the weight of the object approaches 0.

Example for Gravitational Force

The international space station is designed to operate at an altitude of 350km. When completed, it will have a weight (measured on the surface of the Earth) of 4.22×10^6 N. What is its weight when in its orbit?
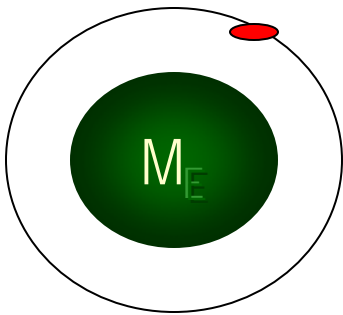
The total weight of the station on the surface of the Earth is

$$F_{GE} = mg = G \frac{M_E m}{R_E^{2}} = 4.22 \times 10^6 \, N$$

Since the orbit is at 350km above the surface of the Earth, the gravitational force at that height is

$$F_O = mg' = G \frac{M_E m}{(R_E + h)^2} = \frac{R_E^2}{(R_E + h)^2} F_{GE}$$

Therefore the weight in the orbit is

$$F_O = \frac{R_E^2}{(R_E + h)^2} F_{GE} = \frac{(6.37 \times 10^6)^2}{(6.37 \times 10^6 + 3.50 \times 10^5)^2} \times 4.22 \times 10^6 = 3.80 \times 10^6 \, N$$

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