PHYS 1443 – Section 001
Lecture #8

Monday, June 12, 2006

Dr. Jaehoon Yu

- Quiz Solutions
- Work done by a varying force
- Work and Kinetic Energy Theorem

Announcements

- Quiz result
  - Average: 3.8/8
    - 46/100
    - Quiz 1: 76
  - Top score: 7/8
- Mid-term exam
  - 8:00 – 10am, this Thursday, June 15, in class
  - CH 1 – 8

Work Done by Varying Force

- If the force depends on the position of the object in motion
  - one must consider work in small segments of the position where the force can be considered constant

$$\Delta W = F_x \cdot \Delta x$$

  - Then add all work-segments throughout the entire motion ($x_i \rightarrow x_f$)

$$W \approx \sum_{x_i}^{x_f} F_x \cdot \Delta x$$

In the limit where $\Delta x \rightarrow 0$

$$\lim_{\Delta x \rightarrow 0} \sum_{x_i}^{x_f} F_x \cdot \Delta x = \int_{x_i}^{x_f} F_x dx = W$$

  - If more than one force is acting, the net work is done by the net force

$$W(net) = \int_{x_i}^{x_f} \left(\sum F_{ix} \right) dx$$

One of the forces depends on position is force by a spring

$$\boxed{F_s = -kx}$$

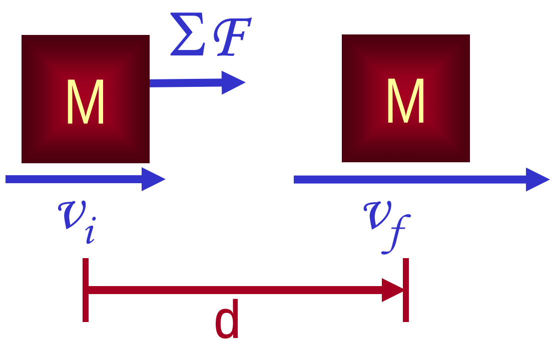
Hooke's Law

The work done by the spring force is

$$W = \int_{-x_{\max}}^{0} F_s dx = \int_{-x_{\max}}^{0} (-kx) dx = \frac{1}{2} kx^2$$

Kinetic Energy and Work-Kinetic Energy Theorem

- Some problems are hard to solve using Newton's second law
  - If forces exerting on the object during the motion are so complicated
  - Relate the work done on the object by the net force to the change of the speed of the object



Suppose net force ΣF was exerted on an object for displacement d to increase its speed from v_i to v_f.

The work on the object by the net force ΣF is

$$W = \left(\sum \vec{F}\right)\cdot\vec{d} = (ma)d\cos 0 = (ma)d$$

Displacement $$d = \frac{1}{2}\left(v_f + v_i\right)t$$

Acceleration $$a = \frac{v_f - v_i}{t}$$

Work $$W = (ma)d = \left[m\left(\frac{v_f - v_i}{t}\right)\right]\frac{1}{2}\left(v_f + v_i\right)t = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Kinetic Energy $$\boxed{KE \equiv \frac{1}{2}mv^2}$$

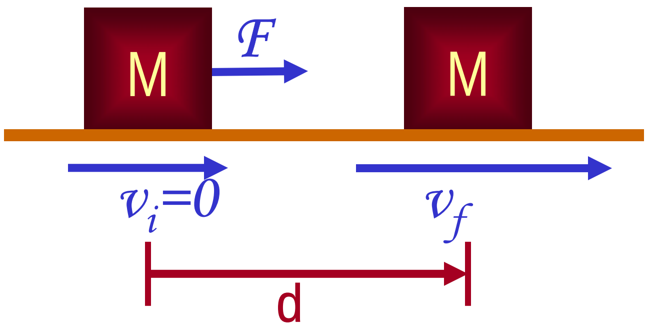
Work $$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$$

Work done by the net force causes change of object's kinetic energy.

Work-Kinetic Energy Theorem

Example of Work-KE Theorem

A 6.0kg block initially at rest is pulled to East along a horizontal, frictionless surface by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W = \vec{F} \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right| \cos\theta = 12 \times 3.0 \cos 0 = 36(J)$$

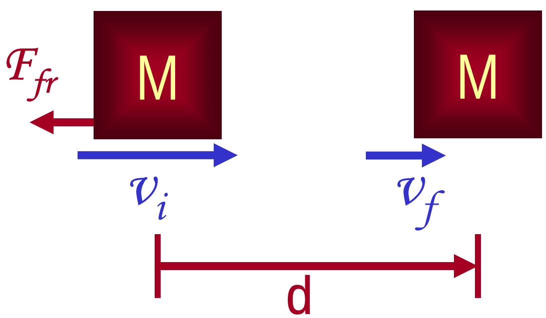
From the work-kinetic energy theorem, we know $W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

Since initial speed is 0, the above equation becomes $W = \frac{1}{2} m v_f^2$

Solving the equation for v_f, we obtain $v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 36}{6.0}} = 3.5 m/s$

Work and Energy Involving Kinetic Friction

- What do you think the work looks like if there is friction?
  - Static friction does not matter! Why? It isn't there when the object is moving.
  - Then which friction matters? Kinetic Friction



Friction force $\mathcal{F}_{fr}$ works on the object to slow down

The work on the object by the friction $\mathcal{F}_{fr}$ is

$$W_{fr} = F_{fr} d \cos(180) = -F_{fr} d \quad \Delta KE = -F_{fr} d$$

The final kinetic energy of an object, taking into account its initial kinetic energy, friction force and other source of work, is

$$KE_f = KE_i + \sum W - F_{fr} d$$

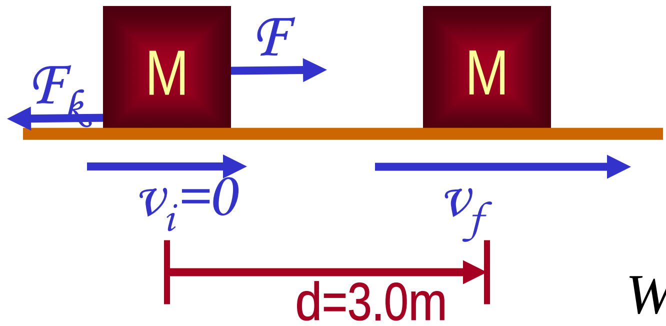
t=0, KE_i

Friction, Engine work

t=T, KE_f

Example of Work Under Friction

A 6.0kg block initially at rest is pulled to East along a horizontal surface with coefficient of kinetic friction μ_k=0.15 by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W_F = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36\,(J)$$

Work done by friction $\mathcal{F}_k$ is

$$W_k = \vec{F}_k \bullet \vec{d} = \left|\vec{F}_k\right|\left|\vec{d}\right|\cos\theta = \left|\mu_k m g\right|\left|\vec{d}\right|\cos\theta$$

$$= 0.15\times 6.0\times 9.8\times 3.0\cos 180 = -26\,(J)$$

Thus the total work is

$$W = W_F + W_k = 36 - 26 = 10(J)$$

Using work-kinetic energy theorem and the fact that initial speed is 0, we obtain

$$W = W_F + W_k = \frac{1}{2}mv_f^2$$

Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 10}{6.0}} = 1.8m/s$$

Work and Kinetic Energy

Work in physics is done only when the sum of forces exerted on an object made a motion to the object.

What does this mean?

However much tired your arms feel, if you were just holding an object without moving it you have not done any physical work to the object.

Mathematically, work is written in a product of magnitudes of the net force vector, the magnitude of the displacement vector and the angle between them.

$$W = \sum\left(\vec{F}_i\right)\bullet\vec{d} = \left|\sum\left(\vec{F}_i\right)\right|\left|\vec{d}\right|\cos\theta$$

Kinetic Energy is the energy associated with motion and capacity to perform work. Work causes change of energy after the completion ← Work-Kinetic energy theorem

$$K = \frac{1}{2}mv^2$$

$$\sum W = K_f - K_i = \Delta K$$

Nm=Joule