PHYS 1443 – Section 001
Lecture #12

Wednesday, June 21, 2006
Dr. Jaehoon Yu

- Center of Mass
- CM and the Center of Gravity

Announcements

- Reading assignments: CH. 9 – 10.
- Quiz tomorrow
  - Early in the class
  - Covers Ch. 8.5 – Ch. 9
- Mid-term grade discussions today
- Exam results
  - Average: 70/102
  - Top score: 100/102
- Grade proportions
  - Exam constitutes 22.5% each, totaling 45%
  - Homework: 25%
  - Lab: 20%
  - Quizzes: 10%
  - Extra credit: 10%

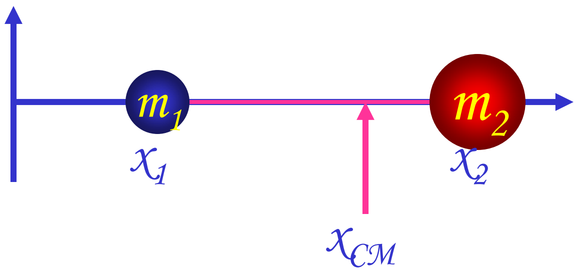
Center of Mass

We've been solving physical problems treating objects as sizeless points with masses, but in realistic situation objects have shapes with masses distributed throughout the body.

Center of mass of a system is the average position of the system's mass and represents the motion of the system as if all the mass is on the point.

What does above statement tell you concerning forces being exerted on the system?

The total external force exerted on the system of total mass M causes the center of mass to move at an acceleration given by $\vec{a} = \sum \vec{F} / M$ *as if all the mass of the system is concentrated on the center of mass.*



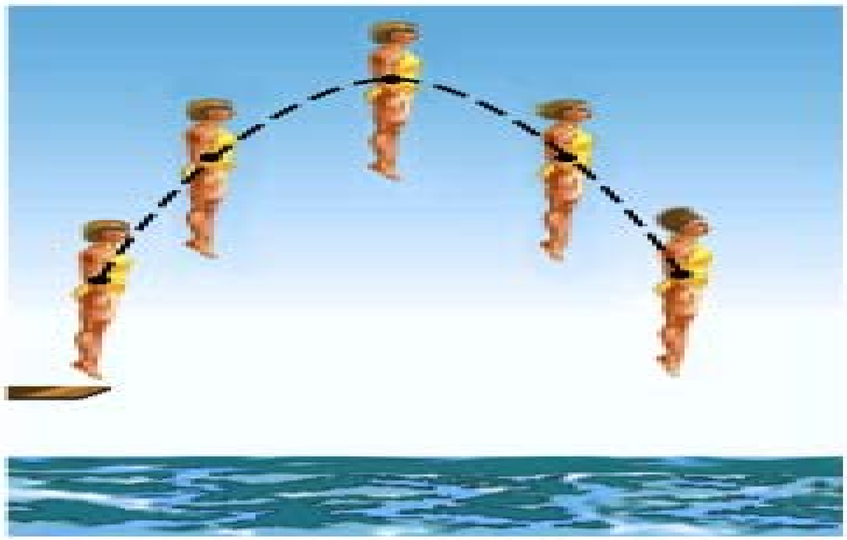
Consider a massless rod with two balls attached at either end.

The position of the center of mass of this system is the mass averaged position of the system

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

CM is closer to the heavier object

Motion of a Diver and the Center of Mass



(a)

Diver performs a simple dive.
The motion of the center of mass follows a parabola since it is a projectile motion.

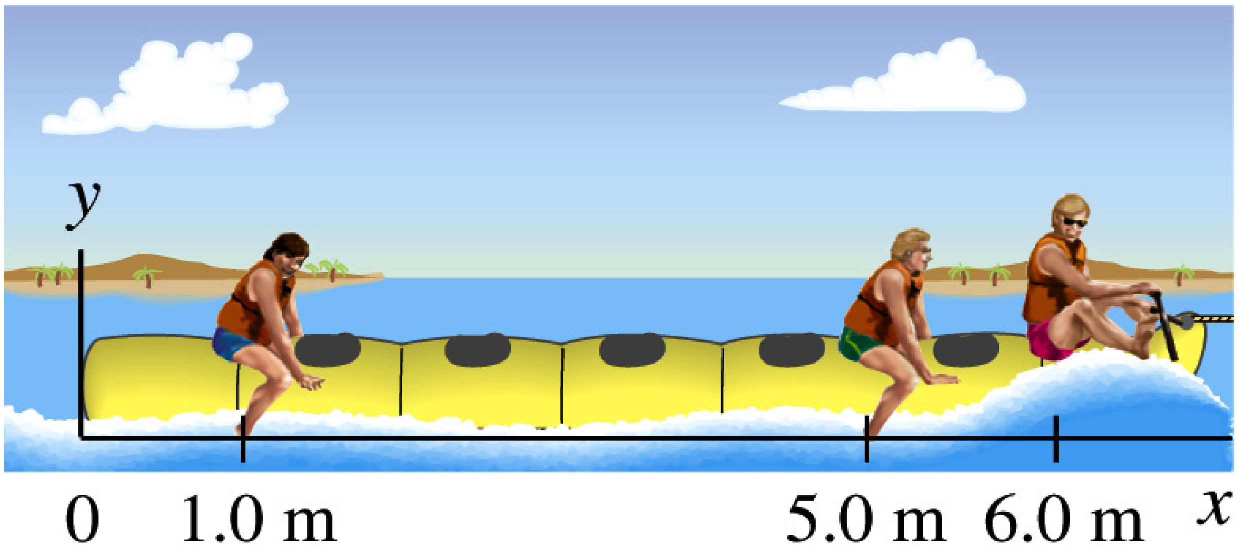
(b)

Diver performs a complicated dive.
The motion of the center of mass still follows the same parabola since it still is a projectile motion.

The motion of the center of mass of the diver is always the same.

Example 9-12

Thee people of roughly equivalent mass M on a lightweight (air-filled) banana boat sit along the x axis at positions x_1=1.0m, x_2=5.0m, and x_3=6.0m. Find the position of CM.



Using the formula for CM

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$= \frac{M \cdot 1.0 + M \cdot 5.0 + M \cdot 6.0}{M + M + M} = \frac{12.0M}{3M} = 4.0(m)$$

Center of Mass of a Rigid Object

The formula for CM can be expanded to Rigid Object or a system of many particles

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + \cdots + m_nx_n}{m_1 + m_2 + \cdots + m_n} = \frac{\sum_i m_ix_i}{\sum_i m_i}$$

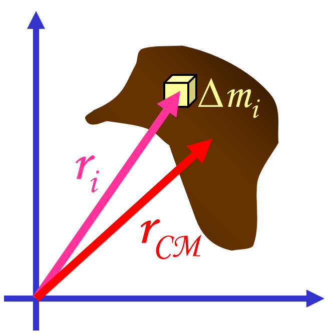
$$y_{CM} = \frac{\sum_i m_iy_i}{\sum_i m_i}$$

$$z_{CM} = \frac{\sum_i m_iz_i}{\sum_i m_i}$$

The position vector of the center of mass of a many particle system is

$$\vec{r}_{CM} = x_{CM}\vec{i} + y_{CM}\vec{j} + z_{CM}\vec{k} = \frac{\sum_i m_ix_i\vec{i} + \sum_i m_iy_i\vec{j} + \sum_i m_iz_i\vec{k}}{\sum_i m_i}$$

$$\vec{r}_{CM} = \frac{\sum_i m_i\vec{r}_i}{M}$$



A rigid body – an object with shape and size with mass spread throughout the body, ordinary objects – can be considered as a group of particles with mass m_i densely spread throughout the given shape of the object

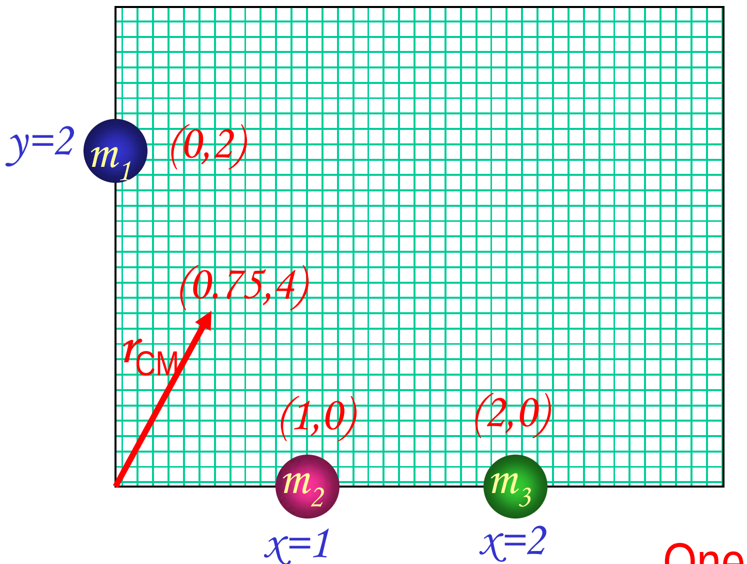
$$x_{CM} \approx \frac{\sum_i \Delta m_ix_i}{M}$$

$$x_{CM} = \lim_{\Delta m_i \to 0} \frac{\sum_i \Delta m_ix_i}{M} = \frac{1}{M}\int x\,dm$$

$$\vec{r}_{CM} = \frac{1}{M}\int \vec{r}\,dm$$

Example for Center of Mass in 2-D

A system consists of three particles as shown in the figure. Find the position of the center of mass of this system.



Using the formula for CM for each position vector component

$$x_{CM}=\frac{\sum_i m_i x_i}{\sum_i m_i} \qquad y_{CM}=\frac{\sum_i m_i y_i}{\sum_i m_i}$$

One obtains $\vec{r}_{CM}=x_{CM}\vec{i}+y_{CM}\vec{j}=\frac{(m_2+2m_3)\vec{i}+2m_1\vec{j}}{m_1+m_2+m_3}$

$$x_{CM}=\frac{\sum_i m_i x_i}{\sum_i m_i}=\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}=\frac{m_2+2m_3}{m_1+m_2+m_3}$$

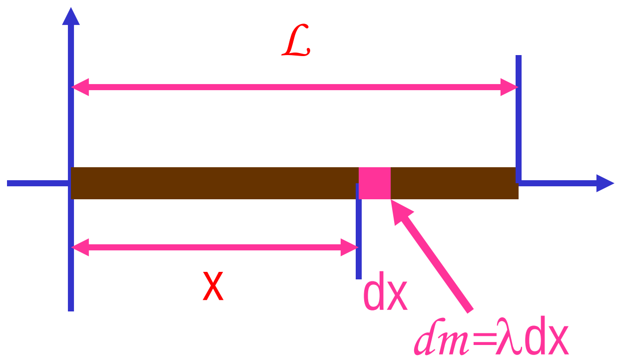
$$y_{CM}=\frac{\sum_i m_i y_i}{\sum_i m_i}=\frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}=\frac{2m_1}{m_1+m_2+m_3}$$

If $m_1=2kg; m_2=m_3=1kg$

$$\vec{r}_{CM}=\frac{3\vec{i}+4\vec{j}}{4}=0.75\vec{i}+\vec{j}$$

Example of Center of Mass; Rigid Body

Show that the center of mass of a rod of mass $\mathcal{M}$ and length $\mathcal{L}$ lies in midway between its ends, assuming the rod has a uniform mass per unit length.



The formula for CM of a continuous object is

$$\boxed{x_{CM} = \frac{1}{M}\int_{x=0}^{x=L} x\,dm}$$

Since the density of the rod (λ) is constant; $\lambda = M/L$

The mass of a small segment $dm = \lambda dx$

Therefore $x_{CM} = \frac{1}{M}\int_{x=0}^{x=L} \lambda x dx = \frac{1}{M}\left[\frac{1}{2}\lambda x^2\right]_{x=0}^{x=L} = \frac{1}{M}\left(\frac{1}{2}\lambda L^2\right) = \frac{1}{M}\left(\frac{1}{2}ML\right) = \frac{L}{2}$

Find the CM when the density of the rod non-uniform but varies linearly as a function of x, λ=α x

$$M = \int_{x=0}^{x=L} \lambda dx = \int_{x=0}^{x=L} \alpha x dx$$

$$= \left[\frac{1}{2}\alpha x^2\right]_{x=0}^{x=L} = \frac{1}{2}\alpha L^2$$

$$x_{CM} = \frac{1}{M}\int_{x=0}^{x=L} \lambda x dx = \frac{1}{M}\int_{x=0}^{x=L} \alpha x^2 dx = \frac{1}{M}\left[\frac{1}{3}\alpha x^3\right]_{x=0}^{x=L}$$

$$x_{CM} = \frac{1}{M}\left(\frac{1}{3}\alpha L^3\right) = \frac{1}{M}\left(\frac{2}{3}ML\right) = \frac{2L}{3}$$