PHYS 1443 – Section 001
Lecture #16

Wednesday, June 28, 2006

Dr. Jaehoon Yu

- Density and Specific Gravity
- Fluid and Pressure
- Absolute and Relative Pressure
- Pascal's Law
- Buoyant Force and Archimedes' Principle
- Flow Rate and Continuity Equation
- Bernoulli's Equation

Announcements

- Reading assignments
  - CH13 – 9 through 13 – 13
- Final exam
  - Date and time: 8 – 10am, Friday, June 30
  - Location: SH103
  - Covers: Ch 9 – 13
  - No class tomorrow

Density and Specific Gravity

Density, ρ (rho), of an object is defined as mass per unit volume

$$\rho \equiv \frac{M}{V}$$

Unit? kg/m^3

Dimension? $[ML^{-3}]$

Specific Gravity of a substance is defined as the ratio of the density of the substance to that of water at 4.0 °C (ρ_{H2O}=1.00g/cm^3).

$$SG \equiv \frac{\rho_{substance}}{\rho_{H_2O}}$$

Unit? None

Dimension? None

What do you think would happen of a substance in the water dependent on SG?

$SG > 1$ Sink in the water

$SG < 1$ Float on the surface

Fluid and Pressure

What are the three states of matter?

Solid, Liquid, and Gas

How do you distinguish them?

By the time it takes for a particular substance to change its shape in reaction to external forces.

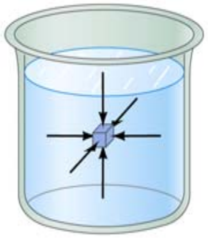
What is a fluid?

A collection of molecules that are randomly arranged and loosely bound by forces between them or by the external container.

We will first learn about mechanics of fluid at rest, *fluid statics*.

In what ways do you think fluid exerts stress on the object submerged in it?

Fluid cannot exert shearing or tensile stress. Thus, the only force the fluid exerts on an object immersed in it is the forces perpendicular to the surfaces of the object.



This force by the fluid on an object usually is expressed in the form of the force on a unit area at the given depth, the pressure, defined as

$$P \equiv \frac{F}{A}$$

Expression of pressure for an infinitesimal area dA by the force dF is

$$P = \frac{dF}{dA}$$

Note that pressure is a scalar quantity because it's the magnitude of the force on a surface area A.

What is the unit and dimension of pressure?

Unit: N/m^2

Dim.: $[M][L^{-1}][T^{-2}]$

Special SI unit for pressure is Pascal

$$1Pa \equiv 1N/m^2$$

Example for Pressure

The mattress of a water bed is 2.00m long by 2.00m wide and 30.0cm deep. a) Find the weight of the water in the mattress.

The volume density of water at the normal condition (0^oC and 1 atm) is $1000kg/m^3$. So the total mass of the water in the mattress is

$$m = \rho_W V_M = 1000 \times 2.00 \times 2.00 \times 0.300 = 1.20 \times 10^3 kg$$

Therefore the weight of the water in the mattress is

$$W = mg = 1.20 \times 10^3 \times 9.8 = 1.18 \times 10^4 N$$

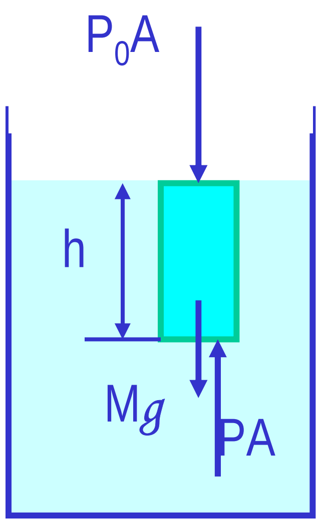
b) Find the pressure exerted by the water on the floor when the bed rests in its normal position, assuming the entire lower surface of the mattress makes contact with the floor.

Since the surface area of the mattress is $4.00\ m^2$, the pressure exerted on the floor is

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{1.18 \times 10^4}{4.00} = 2.95 \times 10^3$$

Variation of Pressure and Depth

Water pressure increases as a function of depth, and the air pressure decreases as a function of altitude. Why?



It seems that the pressure has a lot to do with the total mass of the fluid above the object that puts weight on the object.

Let's imagine a liquid contained in a cylinder with height h and the cross sectional area A immersed in a fluid of density ρ at rest, as shown in the figure, and the system is in its equilibrium.

If the liquid in the cylinder is the same substance as the fluid, the mass of the liquid in the cylinder is $M = \rho V = \rho A h$

Since the system is in its equilibrium $PA - P_0 A - Mg = PA - P_0 A - \rho A h g = 0$

Therefore, we obtain $P = P_0 + \rho g h$

Atmospheric pressure P_0 is

$1.00\,atm = 1.013 \times 10^5\,Pa$

The pressure at the depth h below the surface of a fluid open to the atmosphere is greater than atmospheric pressure by $\rho g h$.

Pascal's Principle and Hydraulics

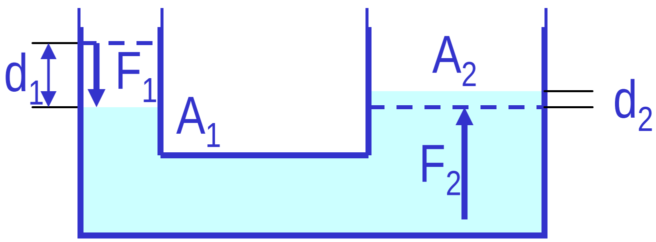
A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

$$P = P_0 + \rho g h$$

What happens if P_0 is changed?

The resultant pressure P at any given depth h increases as much as the change in P_0.

This is the principle behind hydraulic pressure. How?



Since the pressure change caused by the the force F_1 applied on to the area A_1 is transmitted to the F_2 on an area A_2.

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Therefore, the resultant force F_2 is

$$F_2 = \frac{A_2}{A_1} F_1$$

In other words, the force gets multiplied by the ratio of the areas A_2/A_1 and is transmitted to the force F_2 on the surface.

This seems to violate some kind of conservation law, doesn't it?

No, the actual displaced volume of the fluid is the same. And the work done by the forces are still the same.

$$F_2 = \frac{d_1}{d_2} F_1$$

Example for Pascal's Principle

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of 5.00cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0cm. What force must the compressed air exert to lift a car weighing 13,300N? What air pressure produces this force?

Using the Pascal's principle, one can deduce the relationship between the forces, the force exerted by the compressed air is

$$F_1 = \frac{A_1}{A_2} F_2 \quad = \frac{\pi (0.05)^2}{\pi (0.15)^2} \times 1.33 \times 10^4 = 1.48 \times 10^3 \, N$$

Therefore the necessary pressure of the compressed air is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3}{\pi (0.05)^2} = 1.88 \times 10^5 \, Pa$$

Example for Pascal's Principle

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of the pool with a depth 5.0 m.

We first need to find out the pressure difference that is being exerted on the eardrum. Then estimate the area of the eardrum to find out the force exerted on the eardrum.

Since the outward pressure in the middle of the eardrum is the same as normal air pressure

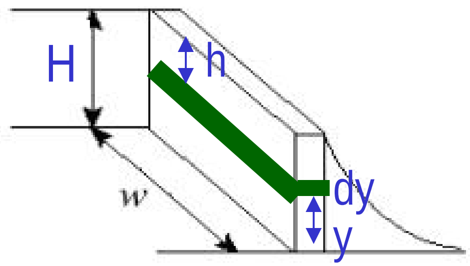
$$P - P_0 = \rho_W gh = 1000 \times 9.8 \times 5.0 = 4.9 \times 10^4 Pa$$

Estimating the surface area of the eardrum at $1.0cm^2 = 1.0x10^{-4}$ m^2, we obtain

$$F = (P - P_0)A \approx 4.9 \times 10^4 \times 1.0 \times 10^{-4} \approx 4.9N$$

Example for Pascal's Principle

Water is filled to a height H behind a dam of width w. Determine the resultant force exerted by the water on the dam.



Since the water pressure varies as a function of depth, we will have to do some calculus to figure out the total force.

The pressure at the depth h is

$$P = \rho g h = \rho g \left(H - y\right)$$

The infinitesimal force dF exerting on a small strip of dam dy is

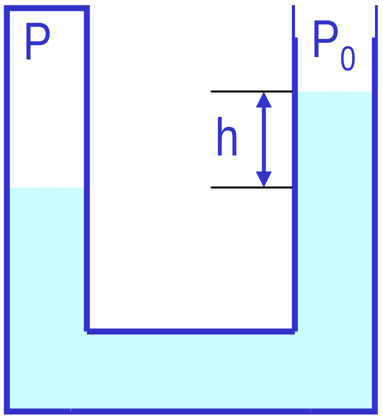
$$dF = P dA = \rho g \left(H - y\right) w dy$$

Therefore the total force exerted by the water on the dam is

$$F = \int_{y=0}^{y=H} \rho g \left(H - y\right) w dy = \rho g w \left[H y - \frac{1}{2} y^2 \right]_{y=0}^{y=H} = \frac{1}{2} \rho g w H^2$$

Absolute and Relative Pressure

How can one measure pressure?



One can measure the pressure using an open-tube manometer, where one end is connected to the system with unknown pressure P and the other open to air with pressure P_0.

The measured pressure of the system is $P = P_0 + \rho g h$

This is called the absolute pressure, because it is the actual value of the system's pressure.

In many cases we measure pressure difference with respect to atmospheric pressure due to isolate the changes in P_0 that depends on the environment. This is called gauge or relative pressure.

$$P_G = P - P_0 = \rho g h$$

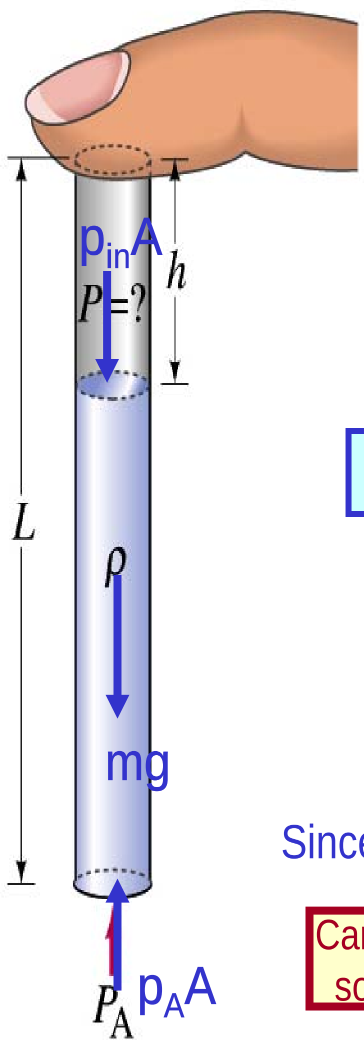
The common barometer which consists of a mercury column with one end closed at vacuum and the other open to the atmosphere was invented by Evangelista Torricelli.

Since the closed end is at vacuum, it does not exert any force. 1 atm is

$$P_0 = \rho g h = (13.595 \times 10^3 kg/m^3)(9.80665\, m/s^2)(0.7600\, m)$$
$$= 1.013 \times 10^5 Pa = 1 atm$$

If one measures the tire pressure with a gauge at 220kPa the actual pressure is 101kPa+220kPa=303kPa.

Finger Holds Water in Straw



You insert a straw of length *L* into a tall glass of your favorite beverage. You place your finger over the top of the straw so that no air can get in or out, and then lift the straw from the liquid. You find that the straw strains the liquid such that the distance from the bottom of your finger to the top of the liquid is *h*. Does the air in the space between your finger and the top of the liquid have a pressure P that is (a) greater than, (b) equal to, or (c) less than, the atmospheric pressure P_A outside the straw?

Less

What are the forces in this problem?

Gravitational force on the mass of the liquid $F_g = mg = \rho A(L-h)g$

Force exerted on the top surface of the liquid by inside air pressure $F_{in} = p_{in}A$

Force exerted on the bottom surface of the liquid by outside air $F_{out} = -p_A A$

Since it is at equilibrium $F_{out} + F_g + F_{in} = 0$ ⇒ $-p_A \cancel{A} + \rho g(L-h)\cancel{A} + p_{in}\cancel{A} = 0$

Cancel A and solve for p_{in} ⇒ $p_{in} = p_A - \rho g(L-h)$

So p_{in} is less than P_A by $\rho g(L-h)$.

Buoyant Forces and Archimedes' Principle

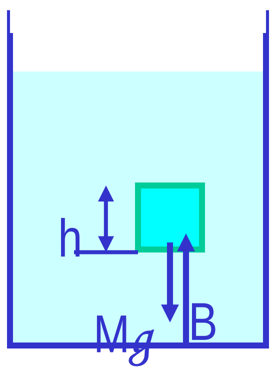
Why is it so hard to put an inflated beach ball under water while a small piece of steel sinks in the water easily?

The water exerts force on an object immersed in the water.
This force is called the buoyant force.

How does the buoyant force work?

The magnitude of the buoyant force always equals the weight of the fluid in the volume displaced by the submerged object.

This is called, Archimedes' principle. What does this mean?



Let's consider a cube whose height is h and is filled with fluid and at in its equilibrium so that its weight Mg is balanced by the buoyant force B.

$$B = F_g = Mg$$

The pressure at the bottom of the cube is larger than the top by ρgh.

Therefore, $\Delta P = B / A = \rho gh$

$$B = \Delta PA = \rho ghA = \rho Vg$$

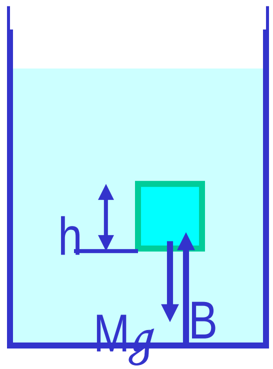
$$B = \rho Vg = Mg = F_g$$

Where Mg is the weight of the fluid.

More Archimedes' Principle

Let's consider buoyant forces in two special cases.

Case 1: Totally submerged object



Let's consider an object of mass M, with density ρ_0, is immersed in the fluid with density ρ_f.

The magnitude of the buoyant force is $B = \rho_f V g$

The weight of the object is $F_g = Mg = \rho_0 V g$

Therefore total force of the system is $F = B - F_g = (\rho_f - \rho_0) V g$

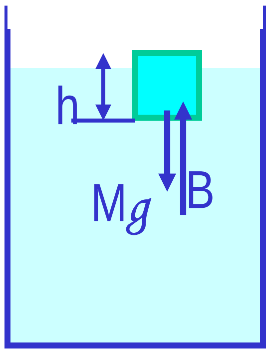
What does this tell you?

The total force applies to different directions depending on the difference of the density between the object and the fluid.

1. If the density of the object is smaller than the density of the fluid, the buoyant force will push the object up to the surface.
2. If the density of the object is larger that the fluid's, the object will sink to the bottom of the fluid.

More Archimedes' Principle

Case 2: Floating object



Let's consider an object of mass M, with density ρ_0, is in static equilibrium floating on the surface of the fluid with density ρ_f, and the volume submerged in the fluid is V_f.

The magnitude of the buoyant force is $B = \rho_f V_f g$

The weight of the object is $F_g = Mg = \rho_0 V_0 g$

Therefore total force of the system is $F = B - F_g = \rho_f V_f g - \rho_0 V_0 g = 0$

Since the system is in static equilibrium $\rho_f V_f g = \rho_0 V_0 g$

$$\frac{\rho_0}{\rho_f} = \frac{V_f}{V_0}$$

What does this tell you?

Since the object is floating its density is always smaller than that of the fluid.

The ratio of the densities between the fluid and the object determines the submerged volume under the surface.

Example for Archimedes' Principle

Archimedes was asked to determine the purity of the gold used in the crown. The legend says that he solved this problem by weighing the crown in air and in water. Suppose the scale read 7.84N in air and 6.86N in water. What should he have to tell the king about the purity of the gold in the crown?

In the air the tension exerted by the scale on the object is the weight of the crown

$$T_{air} = mg = 7.84N$$

In the water the tension exerted by the scale on the object is

$$T_{water} = mg - B = 6.86N$$

Therefore the buoyant force B is

$$B = T_{air} - T_{water} = 0.98N$$

Since the buoyant force B is

$$B = \rho_w V_w g = \rho_w V_c g = 0.98N$$

The volume of the displaced water by the crown is

$$V_c = V_w = \frac{0.98N}{\rho_w g} = \frac{0.98}{1000 \times 9.8} = 1.0 \times 10^{-4} m^3$$

Therefore the density of the crown is

$$\rho_c = \frac{m_c}{V_c} = \frac{m_c g}{V_c g} = \frac{7.84}{V_c g} = \frac{7.84}{1.0 \times 10^{-4} \times 9.8} = 8.3 \times 10^3 kg/m^3$$

Since the density of pure gold is $19.3 \times 10^3 kg/m^3$, this crown is not made of pure gold.

Example for Buoyant Force

What fraction of an iceberg is submerged in the sea water?

Let's assume that the total volume of the iceberg is V_i. Then the weight of the iceberg F_{gi} is

$$F_{gi} = \rho_i V_i g$$

Let's then assume that the volume of the iceberg submerged in the sea water is V_w. The buoyant force B caused by the displaced water becomes

$$B = \rho_w V_w g$$

Since the whole system is at its static equilibrium, we obtain

$$\rho_i V_i g = \rho_w V_w g$$

Therefore the fraction of the volume of the iceberg submerged under the surface of the sea water is

$$\frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{917\,kg/m^3}{1030\,kg/m^3} = 0.890$$

About 90% of the entire iceberg is submerged in the water!!!

Flow Rate and the Equation of Continuity

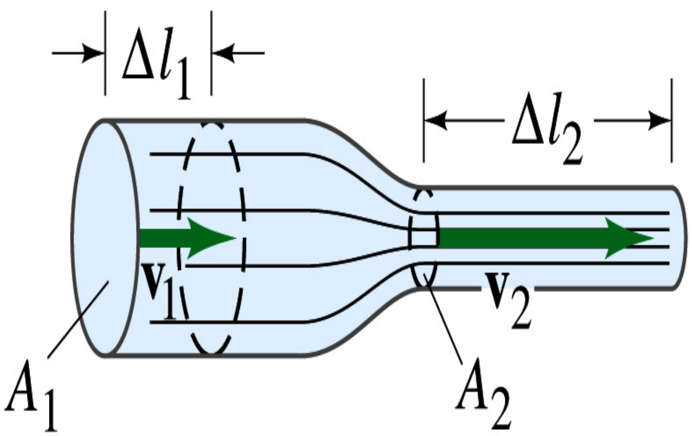
Study of fluid in motion: Fluid Dynamics

If the fluid is water: Water dynamics?? Hydro-dynamics

Two main types of flow

- **Streamline or Laminar flow**: Each particle of the fluid follows a smooth path, a streamline
- **Turbulent flow**: Erratic, small, whirlpool-like circles called eddy current or eddies which absorbs a lot of energy

Flow rate: the mass of fluid that passes a given point per unit time $\Delta m / \Delta t$



$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta l_1}{\Delta t} = \rho_1 A_1 v_1$$

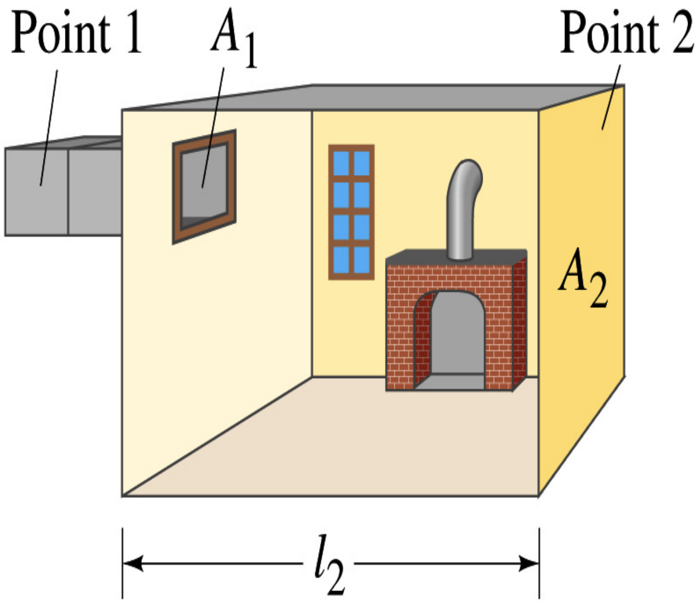
since the total flow must be conserved

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \Rightarrow \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Equation of Continuity

Example for Equation of Continuity

How large must a heating duct be if air moving at 3.0m/s along it can replenish the air every 15 minutes, in a room of 300m^3 volume? Assume the air's density remains constant.



Using equation of continuity

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Since the air density is constant

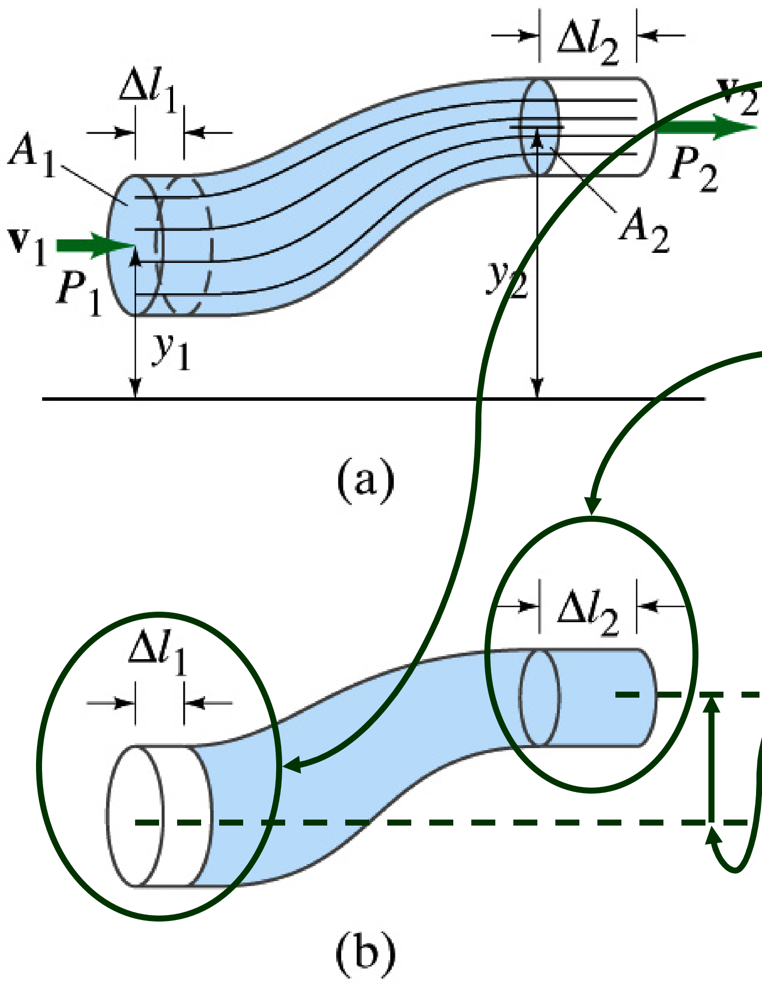
$$A_1 v_1 = A_2 v_2$$

Now let's imagine the room as the large section of the duct

$$A_1 = \frac{A_2 v_2}{v_1} = \frac{A_2 l_2 / t}{v_1} = \frac{V_2}{v_1 \cdot t} = \frac{300}{3.0 \times 900} = 0.11 m^2$$

Bernoulli's Principle

Bernoulli's Principle: Where the velocity of fluid is high, the pressure is low, and where the velocity is low, the pressure is high.



Amount of work done by the force, F_1, that exerts pressure, P_1, at point 1

$$W_1 = F_1 \Delta l_1 = P_1 A_1 \Delta l_1$$

Amount of work done on the other section of the fluid is

$$W_2 = -P_2 A_2 \Delta l_2$$

Work done by the gravitational force to move the fluid mass, m, from y_1 to y_2 is

$$W_3 = -mg\left(y_2 - y_1\right)$$

Bernoulli's Equation cont'd

The net work done on the fluid is

$$W = W_1 + W_2 + W_3 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mgy_2 + mgy_1$$

From the work-energy principle

$$\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mgy_2 + mgy_1$$

Since mass, m, is contained in the volume that flowed in the motion

$$A_1 \Delta l_1 = A_2 \Delta l_2 \quad \text{and} \quad m = \rho A_1 \Delta l_1 = \rho A_2 \Delta l_2$$

Thus,

$$\frac{1}{2} \rho A_2 \Delta l_2 v_2^2 - \frac{1}{2} \rho A_1 \Delta l_1 v_1^2$$

$$= P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - \rho A_2 \Delta l_2 g y_2 + \rho A_1 \Delta l_1 g y_1$$

Bernoulli's Equation cont'd

Since

$$\frac{1}{2}\rho \cancel{A_2}\Delta l_2 v_2^2 - \frac{1}{2}\rho \cancel{A_1}\Delta l_1 v_1^2 = P_1 \cancel{A_1}\Delta l_1 - P_2 \cancel{A_2}\Delta l_2 - \rho \cancel{A_2}\Delta l_2 g y_2 + \rho \cancel{A_1}\Delta l_1 g y_1$$

We obtain

$$\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_1 - P_2 - \rho g y_2 + \rho g y_1$$

Re-organize

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Bernoulli's Equation

Thus, for any two points in the flow

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = \ const.$$

Result of Energy conservation!

For static fluid

$$P_2 = P_1 + \rho g \left(y_1 - y_2 \right) = P_1 + \rho g h$$

Pascal's Law

For the same heights

$$P_2 = P_1 + \frac{1}{2}\rho \left(v_1^2 - v_2^2 \right)$$

The pressure at the faster section of the fluid is smaller than slower section.

Example for Bernoulli's Equation

Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.5m/s through a 4.0cm diameter pipe in the basement under a pressure of 3.0atm, what will be the flow speed and pressure in a 2.6cm diameter pipe on the second 5.0m above? Assume the pipes do not divide into branches.

Using the equation of continuity, flow speed on the second floor is

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi r_1^2 v_1}{\pi r_2^2} = 0.5 \times \left(\frac{0.020}{0.013}\right)^2 = 1.2 m/s$$

Using Bernoulli's equation, the pressure in the pipe on the second floor is

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g\left(y_1 - y_2\right)$$

$$= 3.0\times10^5 + \frac{1}{2}1\times10^3\left(0.5^2 - 1.2^2\right) + 1\times10^3\times9.8\times(-5)$$

$$= 2.5\times10^5 N/m^2$$

Congratulations!!!!

You all have done very well!!!

I certainly had a lot of fun with ya'll!

Good luck with your exam!!!

Have a safe summer!!