PHYS 1443 – Section 001
Lecture #2

Wednesday, May 30, 2007

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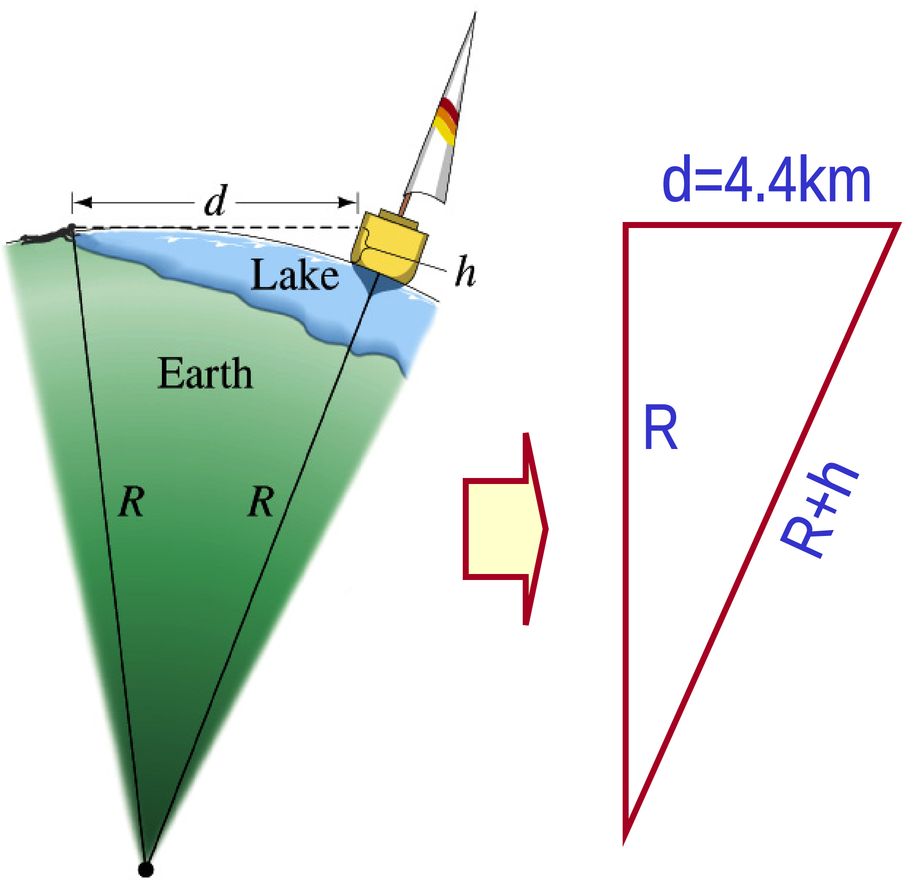
- Dimensional Analysis
- Fundamentals
- One Dimensional Motion: Average Velocity; Acceleration; Motion under constant acceleration; Free Fall
- Motion in Two Dimensions: Vector Properties and Operations; Motion under constant acceleration; Projectile Motion

Announcements

- Reading assignment #1: Read and follow through all sections in appendices A and B by Thursday, May 31
- There will be a quiz on tomorrow, Thursday, May 31, on this reading assignment.

Example 1.8

Estimate the radius of the Earth using triangulation as shown in the picture when d=4.4km and h=1.5m.



Pythagorian theorem

$$(R+h)^2 \approx d^2 + R^2$$

$$R^2 + 2hR + h^2 \approx d^2 + R^2$$

Solving for R

$$R \approx \frac{d^2 - h^2}{2h}$$

$$= \frac{(4400m)^2 - (1.5m)^2}{2 \times 1.5m}$$

$$= 6500km$$

Uncertainties

- Physical measurements have limited precision, however good they are, due to:
  - Stat.{ – Number of measurements
  - Syst.{ – Quality of instruments (meter stick vs micro-meter)
  - – Experience of the person doing measurements
  - – Etc
- In many cases, uncertainties are more important and difficult to estimate than the central (or mean) values

Significant Figures

- Significant figures denote the precision of the measured values
  - Significant figures: non-zero numbers or zeros that are not place-holders
    - 34 has two significant digits, 34.2 has 3, 0.001 has one because the 0's before 1 are place holders, 34.100 has 5, because the 0's after 1 indicates that the numbers in these digits are indeed 0's.
    - When there are many 0's, use scientific notation:
      - $31400000=3.14\times10^{7}$
      - $0.00012=1.2\times10^{-4}$

Significant Figures

- Operational rules:
  - Addition or subtraction: Keep the **smallest number of decimal place** in the result, independent of the number of significant digits: 12.001+ 3.1= 15.1
  - Multiplication or Division: Keep the **smallest significant figures** in the result: 12.001 x 3.1 = 37 , because the smallest significant figures is ?.

Dimension and Dimensional Analysis

- An extremely useful concept in solving physical problems
- Good to write physical laws in mathematical expressions
- No matter what units are used the base quantities are the same
  - *Length* (distance) is length whether meter or inch is used to express the size: Usually denoted as *[L]*
  - The same is true for *Mass ([M])* and *Time ([T])*
  - One can say "Dimension of Length, Mass or Time"
  - Dimensions are used as algebraic quantities: Can perform two algebraic operations; multiplication or division

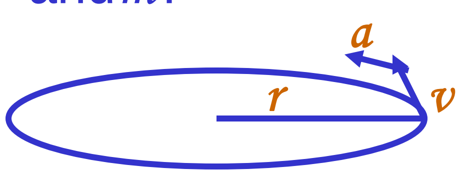
Dimension and Dimensional Analysis

- One can use dimensions only to check the validity of one's expression: Dimensional analysis
  - Eg: Speed $[v] = [L]/[T]=[L][T^{-1}]$
    - *Distance (L) traveled by a car running at the speed V in time T*
    - $L = V*T = [L/T]*[T]=[L]$
- More general expression of dimensional analysis is using exponents: eg. $[v]=[L^n T^m] =[L]\{T^{-1}]$ *where* $n = 1$ *and* $m = -1$

Examples

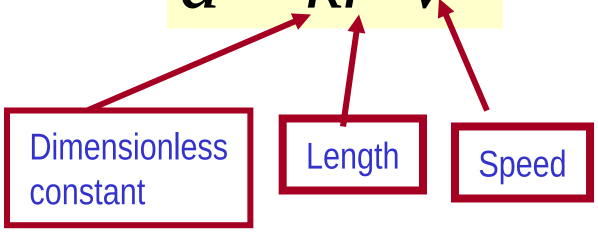
- Show that the expression $[v] = [at]$ is dimensionally correct
  - Speed: $[v]$ =L/T
  - Acceleration: $[a]$ =L/T^2
  - Thus, $[at]$ = (L/T^2)xT=LT$^{(-2+1)}$ =LT^{-1} =L/T= $[v]$

•Suppose the acceleration a of a circularly moving particle with speed v and radius r is proportional to r^n and v^m. What are n and m?



$$L^1T^{-2} = (L)^n\left(\frac{L}{T}\right)^m = L^{n+m}T^{-m}$$

$$a = kr^nv^m$$



$$-m = -2 \Rightarrow \quad m = 2$$

$$n + m = n + 2 \equiv 1 \Rightarrow \quad n = -1$$

$$a = kr^{-1}v^2 = \frac{v^2}{r}$$

Some Fundamentals

- Kinematics: Description of Motion without understanding the cause of the motion
- Dynamics: Description of motion accompanied with understanding the cause of the motion
- Vector and Scalar quantities:
  - Scalar: Physical quantities that require magnitude but no direction
    - Speed, length, mass, height, volume, area, magnitude of a vector quantity, etc
  - Vector: Physical quantities that require both magnitude and direction
    - Velocity, Acceleration, Force, Momentum
    - It does not make sense to say "I ran with velocity of 10miles/hour."
- Objects can be treated as point-like if their sizes are smaller than the scale in the problem
  - Earth can be treated as a point like object (or a particle)in celestial problems
    - Simplification of the problem (The first step in setting up to solve a problem…)
  - Any other examples?

Some More Fundamentals

- Motions:Can be described as long as the position is known at any time (or position is expressed as a function of time)
  - Translation: Linear motion along a line
  - Rotation: Circular or elliptical motion
  - Vibration: Oscillation
- Dimensions
  - 0 dimension: A point
  - 1 dimension: Linear drag of a point, resulting in a line ➔ Motion in one-dimension is a motion on a line
  - 2 dimension: Linear drag of a line resulting in a surface
  - 3 dimension: Perpendicular Linear drag of a surface, resulting in a stereo object

Displacement, Velocity and Speed

One dimensional displacement is defined as:

$$\Delta x \equiv x_f - x_i$$

Displacement is the difference between initial and final potions of motion and is a vector quantity. *How is this different than distance?*

Average velocity is defined as:

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Displacement per unit time in the period throughout the motion

Average speed is defined as:

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

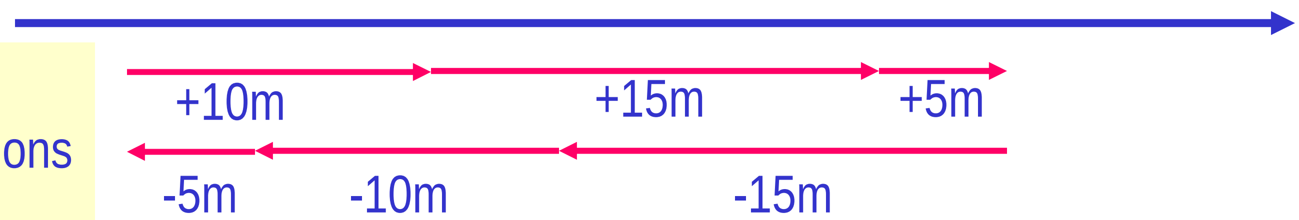
Can someone tell me what the difference between speed and velocity is?

Difference between Speed and Velocity

- Let's take a simple one dimensional translation that has many steps:

Let's call this line as X-axis

Let's have a couple of motions in a total time interval of 20 sec.



Total Displacement: $\Delta x \equiv x_f - x_i = x_i - x_i = 0(m)$

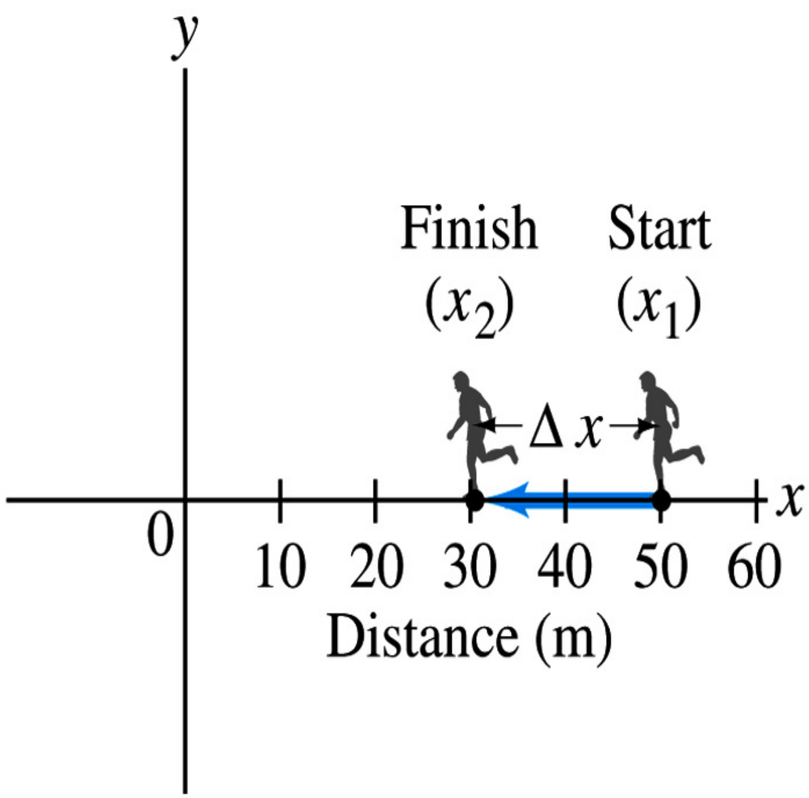
Average Velocity: $v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} = \frac{0}{20} = 0(m/s)$

Total Distance Traveled: $D = 10 + 15 + 5 + 15 + 10 + 5 = 60(m)$

Average Speed: $v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}} = \frac{60}{20} = 3(m/s)$

Example 2.1

The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from x_1=50.0m to x_2=30.5 m, as shown in the figure. What was the runner's average velocity? What was the average speed?



- Displacement:

$$\Delta x \equiv x_f - x_i = x_2 - x_1 = 30.5 - 50.0 = -19.5(m)$$

- Average Velocity:

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{-19.5}{3.00} = -6.50(m/s)$$

- Average Speed:

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}}$$

$$= \frac{50.0 - 30.5}{3.00} = \frac{+19.5}{3.00} = +6.50(m/s)$$

Instantaneous Velocity and Speed

- Can average quantities tell you the detailed story of the whole motion?
- Instantaneous velocity is defined as:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

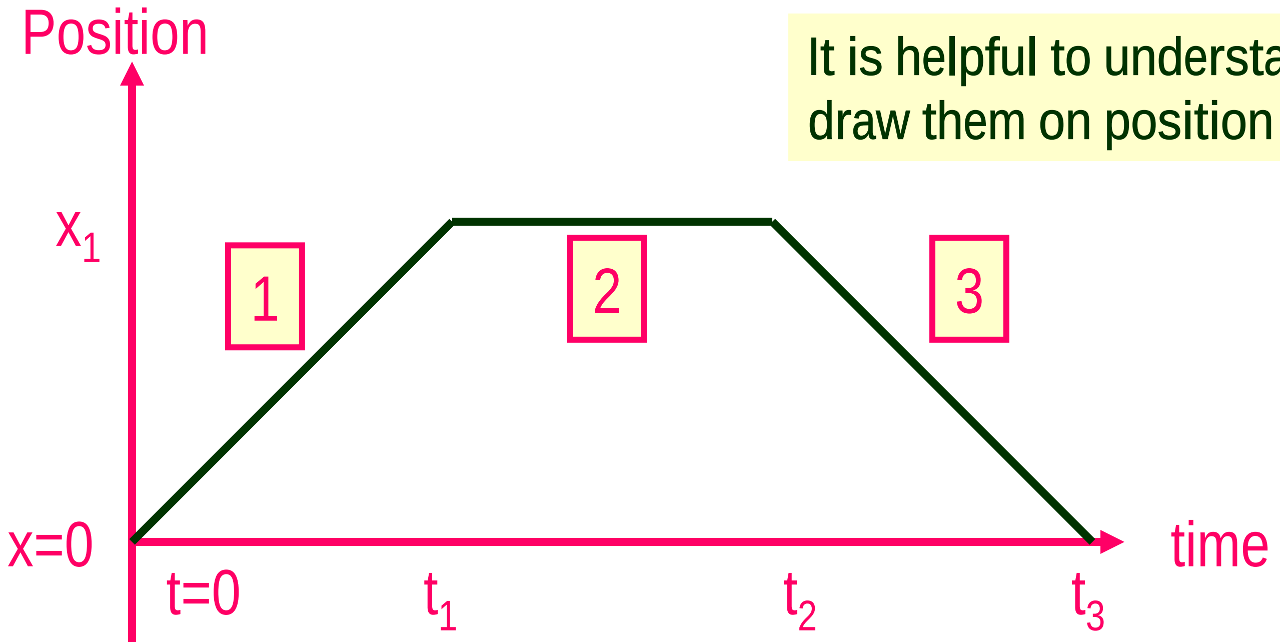
  - What does this mean?
    - Displacement in an infinitesimal time interval
    - Mathematically: Slope of the position variation as a function of time

- Instantaneous speed is the size (magnitude) of the velocity vector:

$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right| = \left| \frac{dx}{dt} \right|$$

*Magnitude of Vectors are expressed in absolute values

Position vs Time Plot

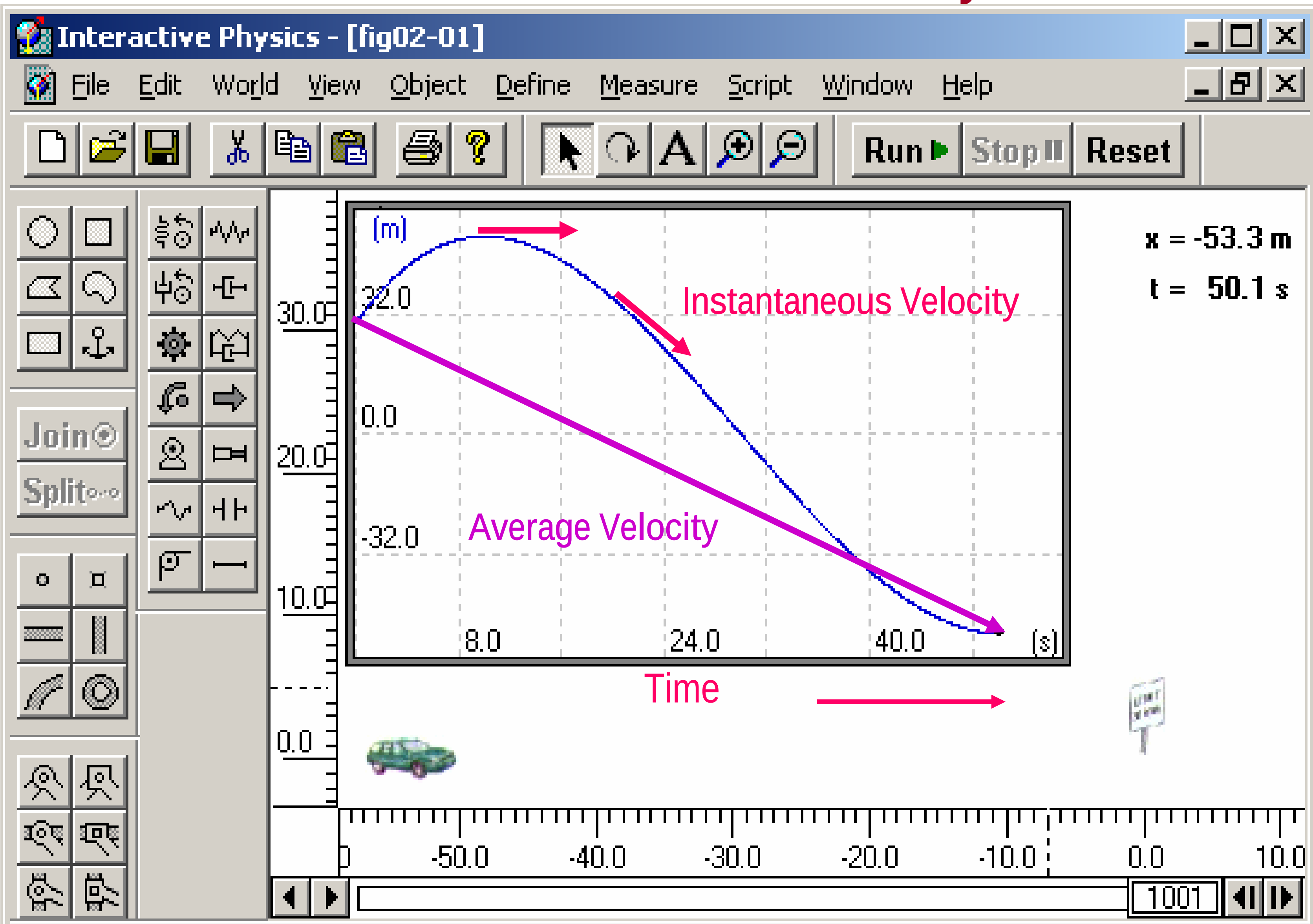


It is helpful to understand motions to draw them on position vs time plots.

1. Running at a constant velocity (go from x=0 to $x=x_1$ in t_1, Displacement is + x_1 in t_1 time interval)
2. Velocity is 0 (go from x_1 to x_1 no matter how much time changes)
3. Running at a constant velocity but in the reverse direction as 1. (go from x_1 to x=0 in t_3-t_2 time interval, Displacement is - x_1 in t_3-t_2 time interval)

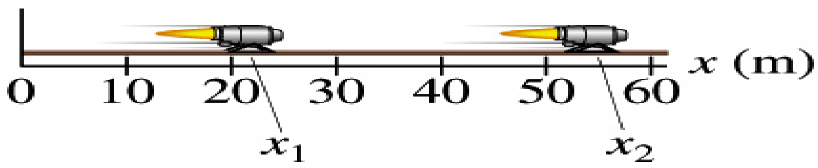
Does this motion physically make sense?

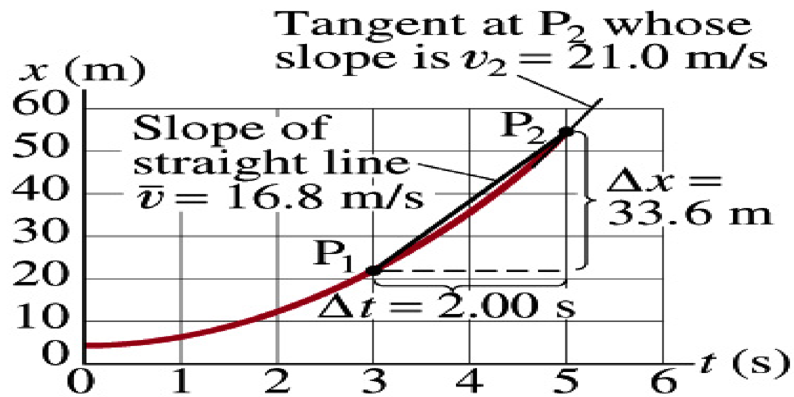
Instantaneous Velocity



Example 2.3

A jet engine moves along a track. Its position as a function of time is given by the equation $x=At^2+B$ where A=2.10m/s^2 and B=2.80m.





(a) Determine the displacement of the engine during the interval from t_1=3.00s to t_2=5.00s.

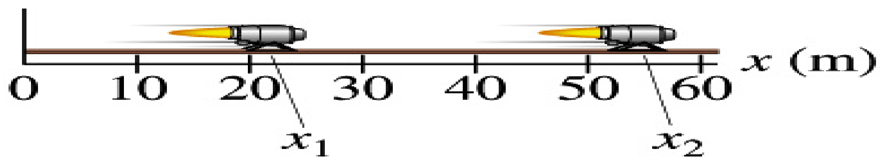
$$x_1 = x_{t_1=3.00} = 2.10\times(3.00)^2 + 2.80 = 21.7m \qquad x_2 = x_{t_2=5.00} = 2.10\times(5.00)^2 + 2.80 = 55.3m$$

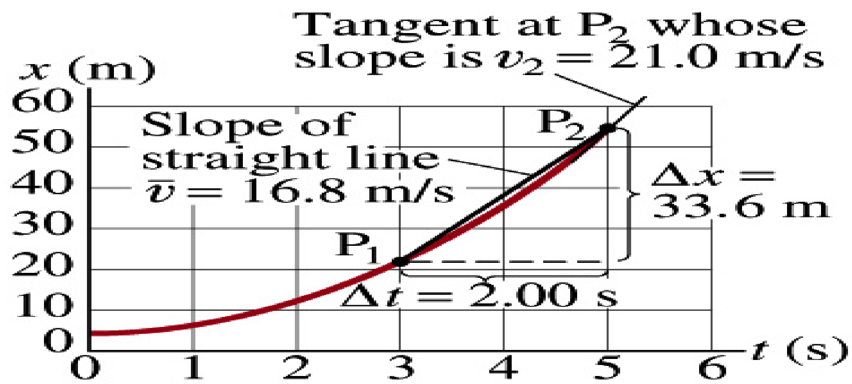
Displacement is, therefore: $$\Delta x = x_2 - x_1 = 55.3 - 21.7 = +33.6(m)$$

(b) Determine the average velocity during this time interval.

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{33.6}{5.00-3.00} = \frac{33.6}{2.00} = 16.8(m/s)$$

Example 2.3 cont'd





(c) Determine the instantaneous velocity at t=t[2]=5.00s.

Calculus formula for derivative $\frac{d}{dt}\left(Ct^n\right) = nCt^{n-1}$ and $\frac{d}{dt}\left(C\right) = 0$

The derivative of the engine's equation of motion is

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt}\left(At^2 + B\right) = 2At$$

The instantaneous velocity at t=5.00s is

$$v_x\left(t = 5.00s\right) = 2A \times 5.00 = 2.10 \times 10.0 = 21.0\left(m/s\right)$$

Displacement, Velocity and Speed

Displacement

$$\Delta x \equiv x_f - x_i$$

Average velocity

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Average speed

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

Instantaneous velocity

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous speed

$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right| = \left| \frac{dx}{dt} \right|$$