

PHYS 1443 – Section 001

Lecture #3

Thursday, May 31, 2007

Dr. Jaehoon Yu

- One Dimensional Motion:
 - Motion under constant acceleration
 - Free Fall
- Motion in Two Dimensions
 - Vector Properties and Operations
 - Motion under constant acceleration
 - Projectile Motion

Today's homework is HW #2, due 7pm, Monday, June 4!!



Acceleration

Change of velocity in time (what kind of quantity is this?)

- Average acceleration:

$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} \quad \text{analogous to} \quad v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

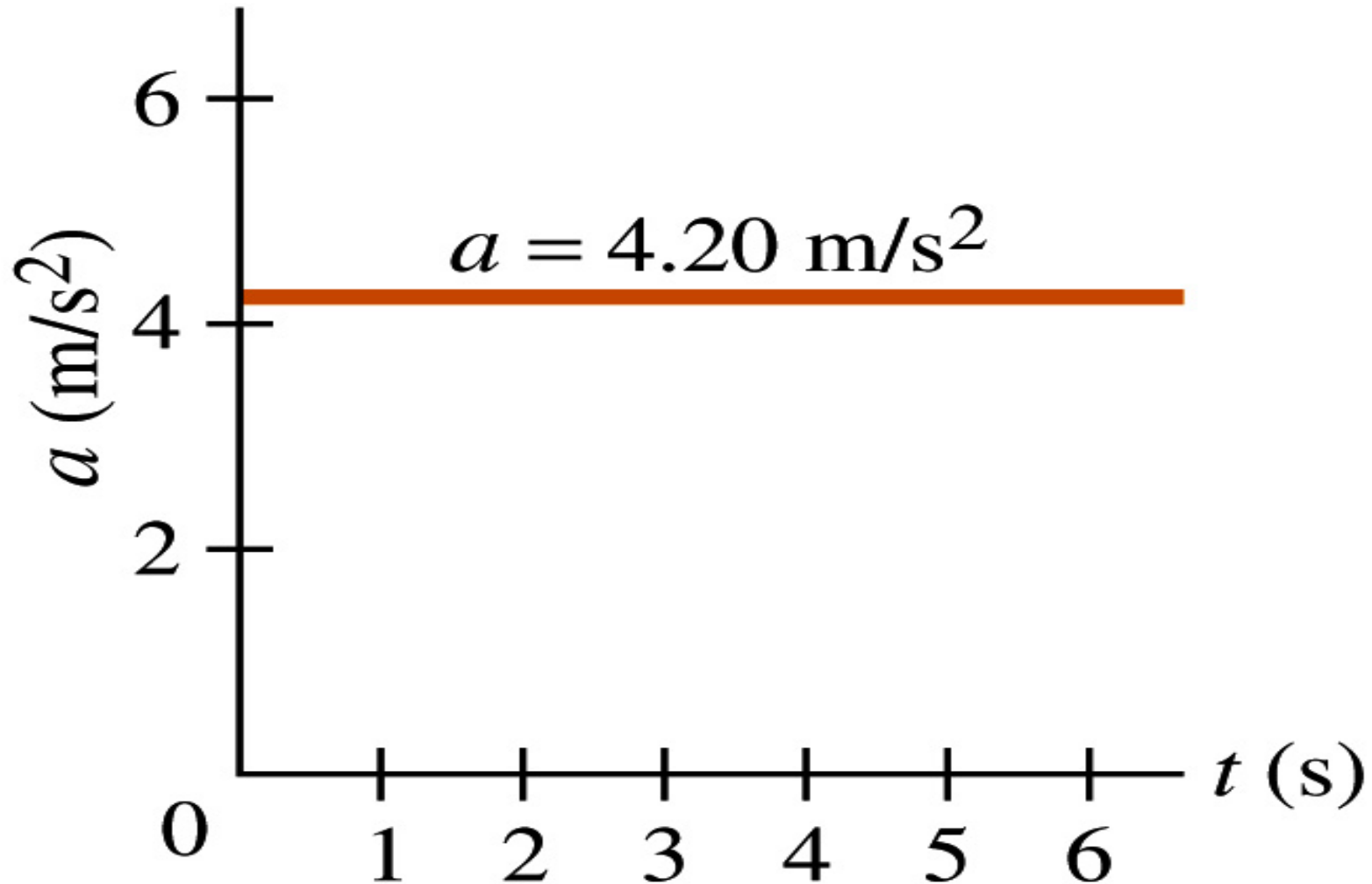
- Instantaneous acceleration:

$$a_x \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d^2 x}{dt^2} \quad \text{analogous to} \quad v_x \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- In calculus terms: The slope (derivative) of the velocity vector with respect to time or the change of slopes of position as a function of time



Acceleration vs Time Plot



Meanings of Acceleration

- When an object is moving in a constant velocity ($v=v_0$), there is no acceleration ($a=0$)
 - Is there any acceleration when an object is not moving?
- When an object speeds up as time goes on, ($v=v(t)$), acceleration has the same sign as v .
- When an object slows down as time goes on, ($v=v(t)$), acceleration has the opposite sign as v .
- Is there acceleration if an object moves in a constant speed but changes direction? **YES!!**



One Dimensional Motion

- Let's start with the simplest case: acceleration is constant ($a=a_0$)
- Using definitions of average acceleration and velocity, we can draw equation of motion (description of motion, position *wrt* time)

$$a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} \xrightarrow{\text{If } t_f=t \text{ and } t_i=0} a_x = \frac{v_{xf} - v_{xi}}{t} \xrightarrow{\text{Solve for } v_{xf}} \boxed{v_{xf} = v_{xi} + a_x t}$$

For constant acceleration,
simple numeric average

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2} a_x t$$

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i} \xrightarrow{\text{If } t_f=t \text{ and } t_i=0} \bar{v}_x = \frac{x_f - x_i}{t} \xrightarrow{\text{Solve for } x_f} x_f = x_i + \bar{v}_x t$$

Resulting Equation of
Motion becomes

$$\boxed{x_f = x_i + \bar{v}_x t = x_i + v_{xi} t + \frac{1}{2} a_x t^2}$$



Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v_{xf}(t) = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f - x_i = \frac{1}{2} \bar{v}_x t = \frac{1}{2} (v_{xf} + v_{xi}) t$$

Displacement as a function of velocities and time

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!



How do we solve a problem using a kinematic formula for constant acceleration?

- Identify what information is given in the problem.
 - Initial and final velocity?
 - Acceleration?
 - Distance?
 - Time?
- Convert the units of all quantities to SI units to be consistent.
- Identify what the problem wants.
- Identify which kinematic formula is **appropriate and easiest** to solve for what the problem wants.
 - Frequently multiple formulae can give you the answer for the quantity you are looking for. → Do not just use any formula but use the one that can be easiest to solve.
- Solve the equation for the quantity wanted.



Example 2.11

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop?  As long as it takes for it to crumple.

The initial speed of the car is $v_{xi} = 100\text{km} / \text{h} = \frac{100000\text{m}}{3600\text{s}} = 28\text{m} / \text{s}$

We also know that $v_{xf} = 0\text{m} / \text{s}$ and $x_f - x_i = 1\text{m}$

Using the kinematic formula $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$

The acceleration is $a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{0 - (28\text{m} / \text{s})^2}{2 \times 1\text{m}} = -390\text{m} / \text{s}^2$

Thus the time for air-bag to deploy is $t = \frac{v_{xf} - v_{xi}}{a} = \frac{0 - 28\text{m} / \text{s}}{-390\text{m} / \text{s}^2} = 0.07\text{s}$



Free Fall

- Free fall is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
 - A motion under constant acceleration
 - All kinematic formulae we learned can be used to solve for falling motions.
- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The magnitude of the gravitational acceleration is $|g|=9.80\text{m/s}^2$ on the surface of the Earth, most of the time.
- The direction of gravitational acceleration is ALWAYS toward the center of the earth, which we normally call (-y); where up and down directions are indicated with the variable "y"
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g = -9.80\text{m/s}^2$

Down to the center of the Earth!

Note the negative sign!!



Example for Using 1D Kinematic Equations on a Falling object

Stone was thrown straight upward at $t=0$ with $+20.0\text{m/s}$ initial velocity on the roof of a 50.0m high building,

What is the acceleration in this motion?


$$g = -9.80\text{m/s}^2$$

(a) Find the time the stone reaches at the maximum height.

What is so special about the maximum height?

$$V=0$$

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00\text{m/s}$$



$$t = \frac{20.0}{9.80} = 2.04\text{s}$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$
$$= 50.0 + 20.4 = 70.4(\text{m})$$



Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

(e) Find the velocity and position of the stone at $t=5.00s$.

Velocity

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0(m/s)$$

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

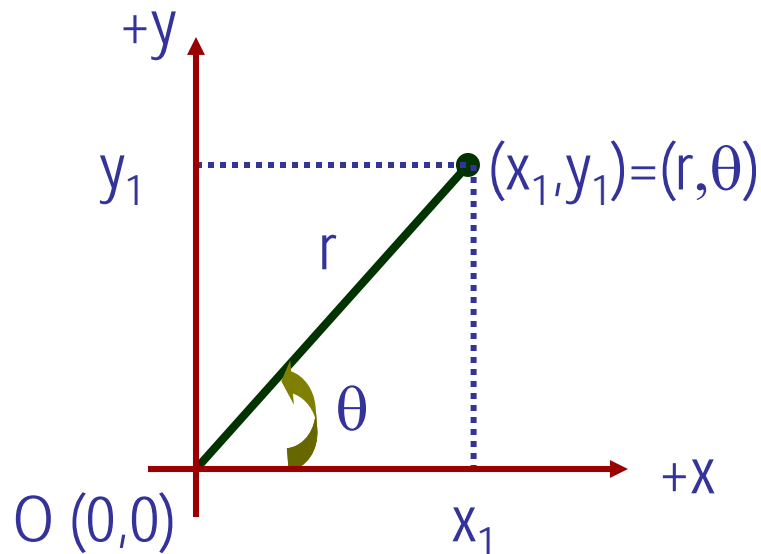
Position

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$



Coordinate Systems

- They make it easy to express locations or positions
- Two commonly used systems, depending on convenience, are
 - Cartesian (Rectangular) Coordinate System
 - Coordinates are expressed in (x,y)
 - Polar Coordinate System
 - Coordinates are expressed in (r,θ)
- Vectors become a lot easier to express and compute



How are Cartesian and Polar coordinates related?

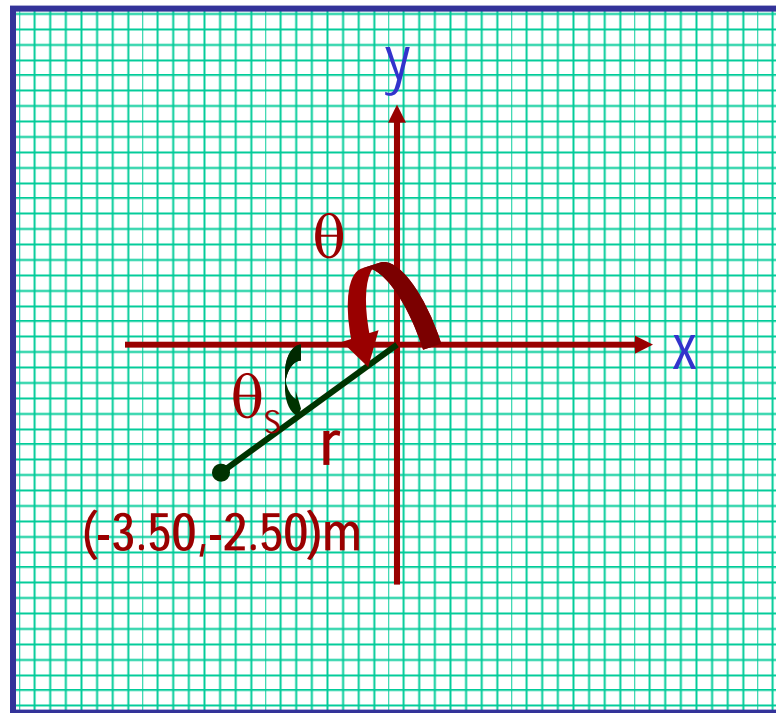
$$x_1 = r \cos \theta \quad r = \sqrt{(x_1^2 + y_1^2)}$$

$$y_1 = r \sin \theta \quad \tan \theta = \frac{y_1}{x_1}$$



Example

Cartesian Coordinate of a point in the xy plane are $(x,y) = (-3.50, -2.50)\text{m}$. Find the equivalent polar coordinates of this point.



$$r = \sqrt{(x^2 + y^2)}$$

$$= \sqrt{((-3.50)^2 + (-2.50)^2)}$$
$$= \sqrt{18.5} = 4.30(m)$$

$$\theta = 180 + \theta_s$$

$$\tan \theta_s = \frac{-2.50}{-3.50} = \frac{5}{7}$$

$$\theta_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$$

$$\therefore \theta = 180 + \theta_s = 180^\circ + 35.5^\circ = 216^\circ$$

Vector and Scalar

Vector quantities have both magnitudes (sizes) and directions

Force, gravitational acceleration, momentum

Normally denoted in **BOLD** letters, \mathbf{F} , or a letter with arrow on top \vec{F}

Their sizes or magnitudes are denoted with normal letters, F , or absolute values: $|\vec{F}|$ or $|\mathbf{F}|$

Scalar quantities have magnitudes only

Can be completely specified with a value and its unit

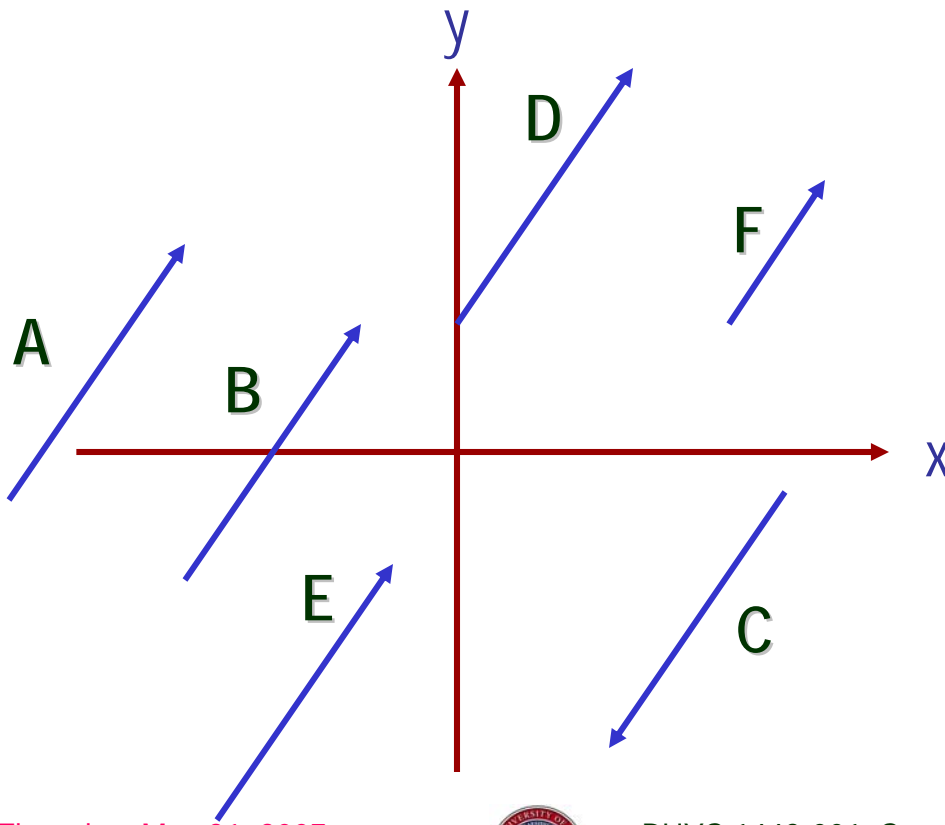
Normally denoted in normal letters, E

Energy, heat, mass, weight

Both have units!!!

Properties of Vectors

- Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system!!



Which ones are the same vectors?

$A=B=E=D$

Why aren't the others?

C: The same magnitude but opposite direction:
 $C=-A$: A negative vector

F: The same direction but different magnitude

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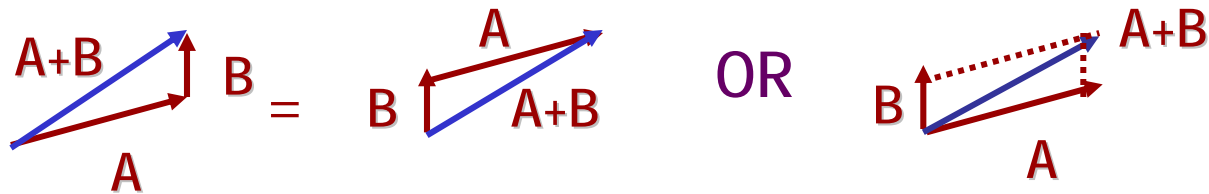


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Vector Operations

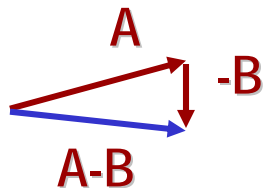
- Addition:

- Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
- Parallelogram method: Connect the tails of the two vectors and extend
- Addition is commutative: Changing order of operation does not affect the results
 $A+B=B+A$, $A+B+C+D+E=E+C+A+B+D$



- Subtraction:

- The same as adding a negative vector: $A - B = A + (-B)$



Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude A , $B=2A$

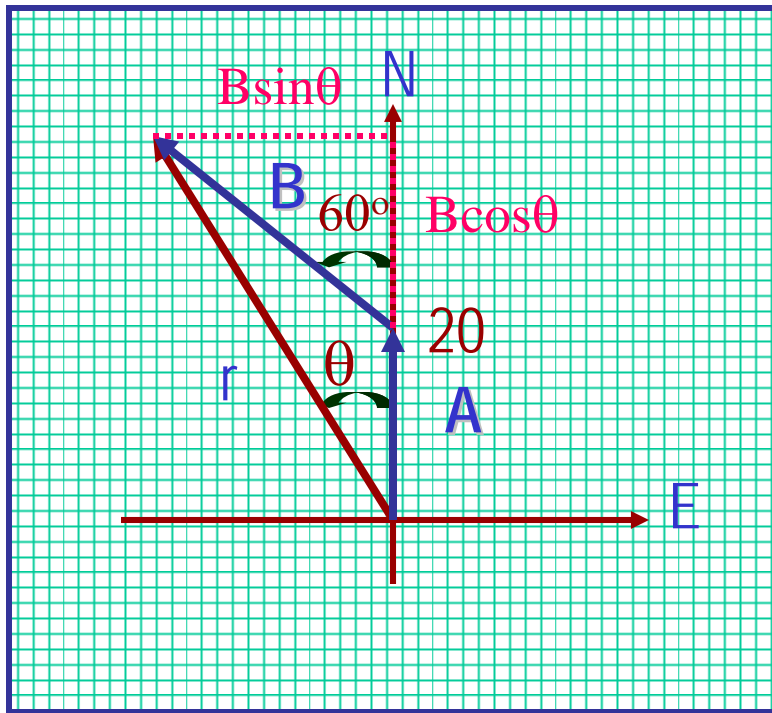


Thursd $|B| = 2|A|$



Example for Vector Addition

A car travels 20.0km due north followed by 35.0km in a direction 60.0° west of north. Find the magnitude and direction of resultant displacement.



$$r = \sqrt{(A + B \cos \theta)^2 + (B \sin \theta)^2}$$

$$= \sqrt{A^2 + B^2 (\cos^2 \theta + \sin^2 \theta) + 2AB \cos \theta}$$

$$= \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$= \sqrt{(20.0)^2 + (35.0)^2 + 2 \times 20.0 \times 35.0 \cos 60}$$

$$= \sqrt{2325} = 48.2(\text{km})$$

$$\theta = \tan^{-1} \frac{|\vec{B}| \sin 60}{|\vec{A}| + |\vec{B}| \cos 60}$$

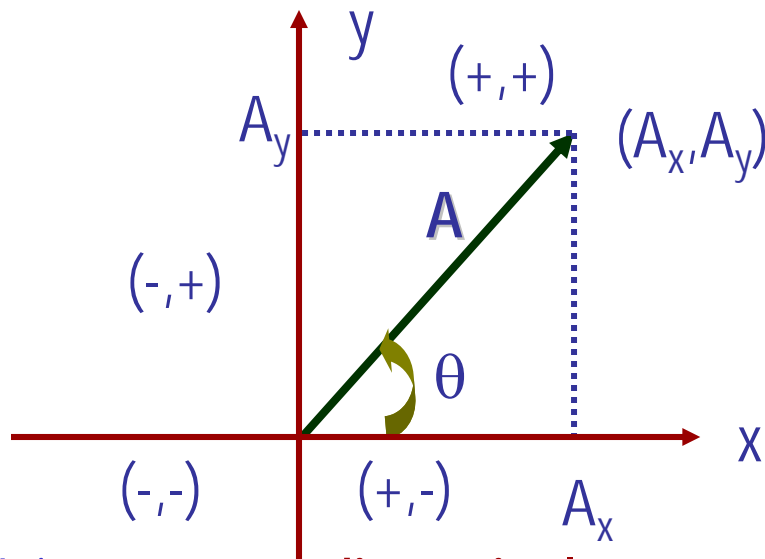
$$= \tan^{-1} \frac{35.0 \sin 60}{20.0 + 35.0 \cos 60}$$

$$= \tan^{-1} \frac{30.3}{37.5} = 38.9^\circ \text{ to W wrt N}$$

Find other ways to solve this problem...

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$\left. \begin{aligned} A_x &= |\vec{A}| \cos \theta \\ A_y &= |\vec{A}| \sin \theta \end{aligned} \right\} \text{Components}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2} \quad \left. \right\} \text{Magnitude}$$

$$\begin{aligned} |\vec{A}| &= \sqrt{\left(|\vec{A}| \cos \theta\right)^2 + \left(|\vec{A}| \sin \theta\right)^2} \\ &= \sqrt{|\vec{A}|^2 (\cos^2 \theta + \sin^2 \theta)} = |\vec{A}| \end{aligned}$$

- Unit vectors are dimensionless vectors whose magnitude are exactly 1
 - Unit vectors are usually expressed in $\mathbf{i}, \mathbf{j}, \mathbf{k}$ or $\vec{i}, \vec{j}, \vec{k}$
 - Vectors can be expressed using components and unit vectors

So the above vector **A** can be written as

$$\vec{A} = A_x \vec{i} + A_y \vec{j} = |\vec{A}| \cos \theta \vec{i} + |\vec{A}| \sin \theta \vec{j}$$



Examples of Vector Operations

Find the resultant vector which is the sum of $\mathbf{A}=(2.0\mathbf{i}+2.0\mathbf{j})$ and $\mathbf{B}=(2.0\mathbf{i}-4.0\mathbf{j})$

$$\begin{aligned}\vec{C} &= \vec{A} + \vec{B} = (2.0\vec{i} + 2.0\vec{j}) + (2.0\vec{i} - 4.0\vec{j}) \\ &= (2.0 + 2.0)\vec{i} + (2.0 - 4.0)\vec{j} = 4.0\vec{i} - 2.0\vec{j} (m)\end{aligned}$$

$$\begin{aligned}|\vec{C}| &= \sqrt{(4.0)^2 + (-2.0)^2} \\ &= \sqrt{16 + 4.0} = \sqrt{20} = 4.5(m)\end{aligned}\quad \theta = \tan^{-1} \frac{C_y}{C_x} = \tan^{-1} \frac{-2.0}{4.0} = -27^\circ$$

Find the resultant displacement of three consecutive displacements:
 $\mathbf{d}_1=(15\mathbf{i}+30\mathbf{j}+12\mathbf{k})\text{cm}$, $\mathbf{d}_2=(23\mathbf{i}+14\mathbf{j}-5.0\mathbf{k})\text{cm}$, and $\mathbf{d}_3=(-13\mathbf{i}+15\mathbf{j})\text{cm}$

$$\begin{aligned}\vec{D} &= \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = (15\vec{i} + 30\vec{j} + 12\vec{k}) + (23\vec{i} + 14\vec{j} - 5.0\vec{k}) + (-13\vec{i} + 15\vec{j}) \\ &= (15 + 23 - 13)\vec{i} + (30 + 14 + 15)\vec{j} + (12 - 5.0)\vec{k} = 25\vec{i} + 59\vec{j} + 7.0\vec{k} (cm)\end{aligned}$$

Magnitude

$$|\vec{D}| = \sqrt{(25)^2 + (59)^2 + (7.0)^2} = 65(cm)$$



Displacement, Velocity, and Acceleration in 2-dim

- Displacement:

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

- Average Velocity:

$$\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

- Instantaneous Velocity:

$$\vec{v} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

- Average Acceleration

$$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

- Instantaneous Acceleration:

$$\vec{a} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt} \left(\frac{d\vec{r}}{dt} \right) = \frac{d^2 \vec{r}}{dt^2}$$

How is each of these quantities defined in 1-D?

