PHYS 1443 – Section 001
Lecture #5

Tuesday, June 5, 2007

Dr. Jaehoon Yu

- Motion in Two Dimensions
  - Motion under constant acceleration
  - Projectile Motion
  - Maximum ranges and heights

Today's homework is HW #3, due 7pm, Friday, June 8!!

Announcements

- Quiz results
  - Class average: 9.9/14
    - Equivalent to: 70.7/100
  - Top score: 13/14
- Quiz this Thursday, June 7
  - At the beginning of the class
  - CH 1 – what we cover tomorrow
- Mail distribution list
  - 20 of you are on the list as of last night
  - Extra credit
    - 3 points if done by tomorrow Wednesday, June 7

Kinematic Quantities in 1D and 2D

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{r}}{dt^2}$

What is the difference between 1D and 2D quantities?

2-dim Motion Under Constant Acceleration

- Position vectors in x-y plane:

$$\vec{r}_i = x_i\vec{i} + y_i\vec{j} \qquad \vec{r}_f = x_f\vec{i} + y_f\vec{j}$$

- Velocity vectors in x-y plane:

$$\vec{v}_i = v_{xi}\vec{i} + v_{yi}\vec{j} \qquad \vec{v}_f = v_{xf}\vec{i} + v_{yf}\vec{j}$$

Velocity vectors in terms of acceleration vector

X-comp $v_{xf} = v_{xi} + a_x t$ Y-comp $v_{yf} = v_{yi} + a_y t$

$$\vec{v}_f = (v_{xi} + a_x t)\vec{i} + (v_{yi} + a_y t)\vec{j} = (v_{xi}\vec{i} + v_{yi}\vec{j}) + (a_x\vec{i} + a_y\vec{j})t =$$

$$= \vec{v}_i + \vec{a}t$$

2-dim Motion Under Constant Acceleration

- How are the position vectors written in acceleration vectors?

Position vector components

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Putting them together in a vector form

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} =$$

$$= \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right)\vec{i} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2\right)\vec{j}$$

Regrouping the above

$$= \left(x_i\vec{i} + y_i\vec{j}\right) + \left(v_{xi}\vec{i} + v_{yi}\vec{j}\right)t + \frac{1}{2}\left(a_x\vec{i} + a_y\vec{j}\right)t^2$$

$$= \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

Example for 2-D Kinematic Equations

A particle starts at origin when t=0 with an initial velocity **v**=(20**i**-15**j**)m/s. The particle moves in the xy plane with $a_x=4.0m/s^2$. Determine the components of the velocity vector at any time t.

$$v_{xf}=v_{xi}+a_x t=20+4.0t\left(m/s\right) \quad v_{yf}=v_{yi}+a_y t=-15+0t=-15\left(m/s\right)$$

Velocity vector

$$\vec{v}\left(t\right)=v_x\left(t\right)\vec{i}+v_y\left(t\right)\vec{j}=\left(20+4.0t\right)\vec{i}-15\vec{j}\left(m/s\right)$$

Compute the velocity and speed of the particle at t=5.0 s.

$$\vec{v}_{t=5}=v_{x,t=5}\vec{i}+v_{y,t=5}\vec{j}=\left(20+4.0\times5.0\right)\vec{i}-15\vec{j}=\left(40\vec{i}-15\vec{j}\right)\ m/s$$

$$speed=\left|\vec{v}\right|=\sqrt{\left(v_x\right)^2+\left(v_y\right)^2}=\sqrt{\left(40\right)^2+\left(-15\right)^2}=43m/s$$

Example for 2-D Kinematic Eq. Cnt'd

Angle of the Velocity vector

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = \tan^{-1}\left(\frac{-3}{8}\right) = -21^\circ$$

Determine the x and y components of the particle at t=5.0 s.

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = 20\times 5 + \frac{1}{2}\times 4\times 5^2 = 150(m)$$

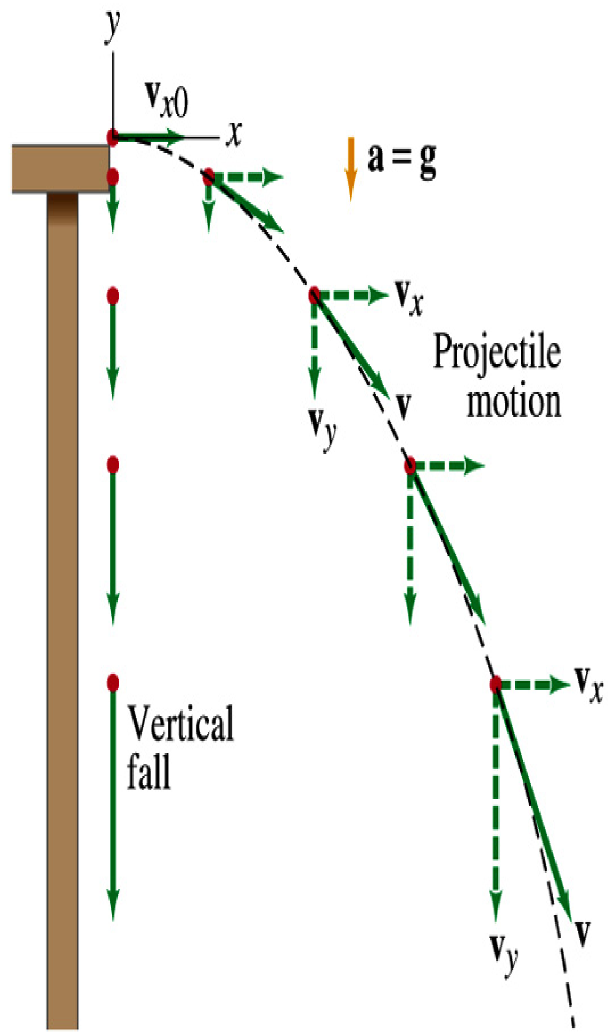
$$y_f = v_{yi}t = -15\times 5 = -75\,(m)$$

Can you write down the position vector at t=5.0s?

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} = 150\vec{i} - 75\vec{j}\,(m)$$

Projectile Motion

- A 2-dim motion of an object under the gravitational acceleration with the following assumptions
  - Free fall acceleration, g, is constant over the range of the motion
    - $\vec{g} = -9.8\vec{j}\left(m/s^2\right)$
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity (no acceleration)
  - Vertical motion under constant acceleration (g)



Show that a projectile motion is a parabola!!!

x-component $v_{xi} = v_i \cos\theta_i$

y-component $v_{yi} = v_i \sin\theta_i$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = -g\vec{j}$$

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

a_x=0

$$x_f = v_{xi}t = v_i \cos\theta_i t$$

$$t = \frac{x_f}{v_i \cos\theta_i}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)\,t^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

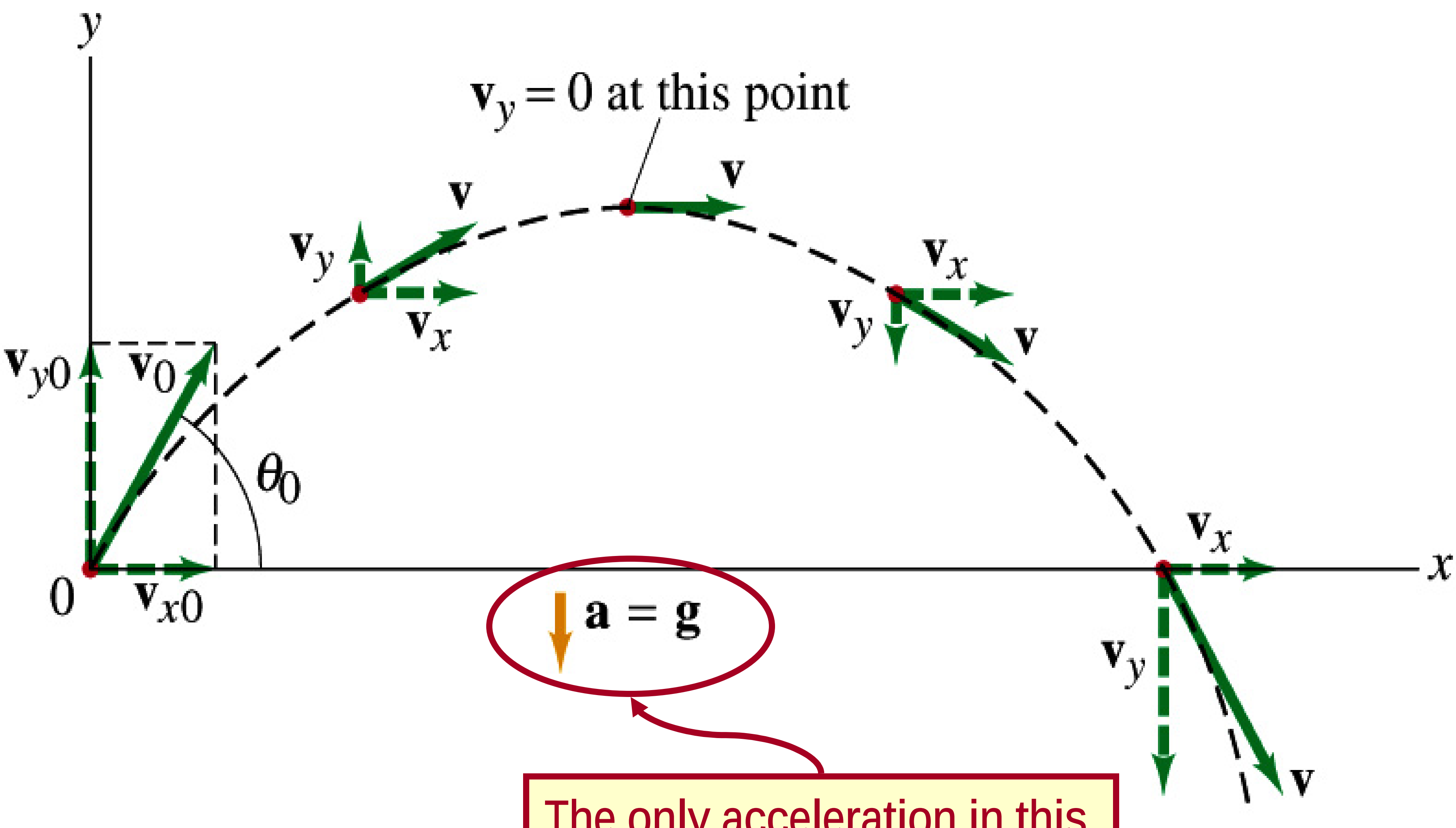
Plug t into the above

$$y_f = v_i \sin\theta_i \left(\frac{x_f}{v_i \cos\theta_i}\right) - \frac{1}{2}g\left(\frac{x_f}{v_i \cos\theta_i}\right)^2$$

$$y_f = x_f \tan\theta_i - \left(\frac{g}{2v_i^2 \cos^2\theta_i}\right)x_f^2$$

What kind of parabola is this?

Projectile Motion



The only acceleration in this motion. It is a constant!!

Example for Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance the ball is from the original position when landed.

Which component determines the flight time and the distance?

Flight time is determined by *y* component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

So the possible solutions are…

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\sec$$

$$\therefore\ t \approx 8\sec$$

Why isn't 0 the solution?

Distance is determined by *x* components in 2-dim, because the ball is at *y=0* position when it completed it's flight.

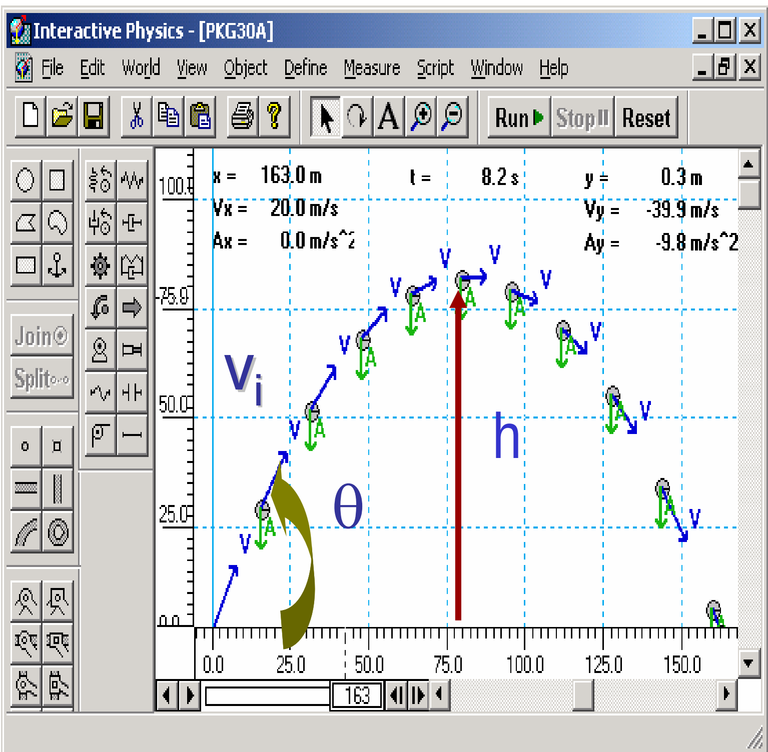
$$x_f = v_{xi}t = 20 \times 8 = 160(m)$$

Horizontal Range and Max Height

- Based on what we have learned in the previous pages, one can analyze a projectile motion in more detail
  - Maximum height an object can reach
  - Maximum range

What happens at the maximum height?

At the maximum height the object's vertical motion stops to turn around!!



$$v_{yf} = v_{yi} + a_y t$$

$$= v_i \sin\theta_i - g t_A = 0$$

Solve for t_A

$$\therefore t_A = \frac{v_i \sin\theta_i}{g}$$

Horizontal Range and Max Height

Since no acceleration is in x direction, it still flies even if v_y=0.

$$R = v_{xi}t = v_{xi}\left(2t_A\right) = 2v_i\cos\theta_i\left(\frac{v_i\sin\theta_i}{g}\right)$$

Range

$$R = \left(\frac{v_i^2\sin 2\theta_i}{g}\right)$$

$$y_f = h = v_{yi}t + \frac{1}{2}(-g)t^2 = v_i\sin\theta_i\left(\frac{v_i\sin\theta_i}{g}\right) - \frac{1}{2}g\left(\frac{v_i\sin\theta_i}{g}\right)^2$$

Height

$$y_f = h = \left(\frac{v_i^2\sin^2\theta_i}{2g}\right)$$

Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left(\frac{v_i^2 \sin^2 \theta_i}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_i = 90^o$!!!

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i = 90^o$, i.e., $\theta_i = 45^o$!!!

Example for a Projectile Motion

- A stone was thrown upward from the top of a cliff at an angle of 37º to horizontal with initial speed of 65.0m/s. If the height of the cliff is 125.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yi} = v_i \sin\theta_i = 65.0 \times \sin 37^\circ = 39.1 m/s$$

$$y_f = -125.0 = v_{yi}t - \frac{1}{2}gt^2$$

Becomes

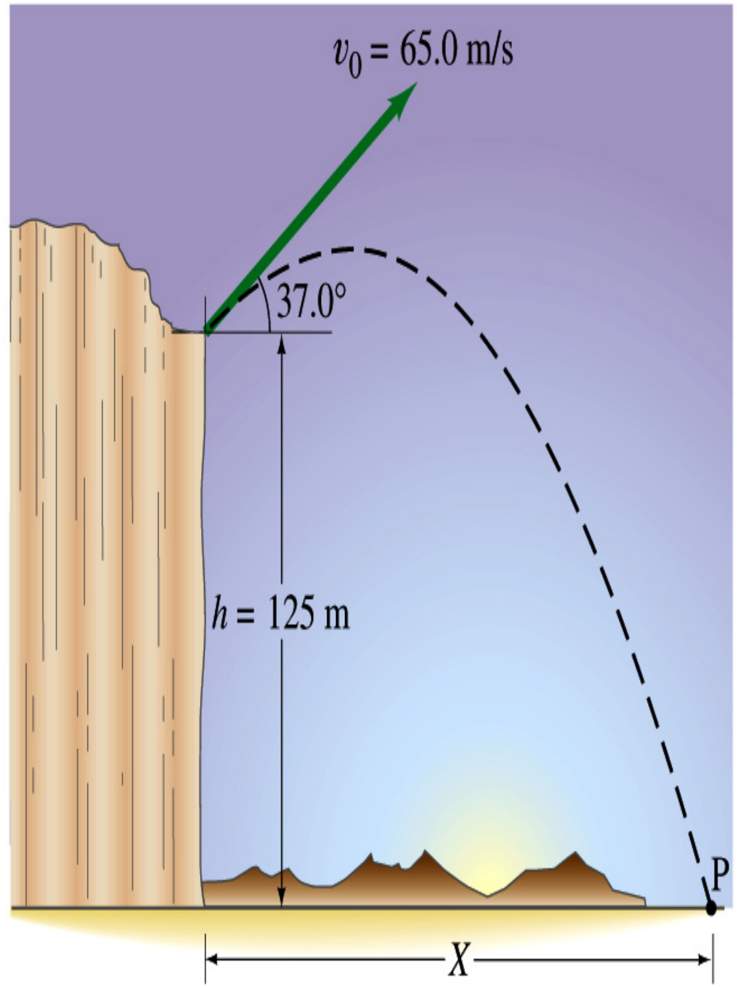
$$gt^2 - 78.2t - 250 = 9.80t^2 - 78.2t - 250 = 0$$

$$t = \frac{78.2 \pm \sqrt{(-78.2)^2 - 4 \times 9.80 \times (-250)}}{2 \times 9.80}$$

$$t = -2.43 s \quad \text{or} \quad t = 10.4 s$$

$$t = 10.4 s$$

Since negative time does not exist.



Example cont'd

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yf} = v_{yi} - gt = v_i \sin\theta_i - gt = 39.1 - 9.80 \times 10.4 = -62.8 m/s$$

$$|v| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{51.9^2 + (-62.8)^2} = 81.5 m/s$$

- What are the maximum height and the maximum range of the stone?

Do these yourselves at home for fun!!!