PHYS 1443 – Section 001
Lecture #6

Wednesday, June 6, 2007

Dr. Jaehoon Yu

- Reference Frame and Relative Velocity
- Newton's Laws of Motion
  - Force
  - Newton's Law of Inertia & Mass
  - Newton's second law of motion
  - Gravitational Force and Weight
  - Newton's third law of motion
- Application of Newton's Laws
  - Motion without friction
- Forces of Friction

Announcements

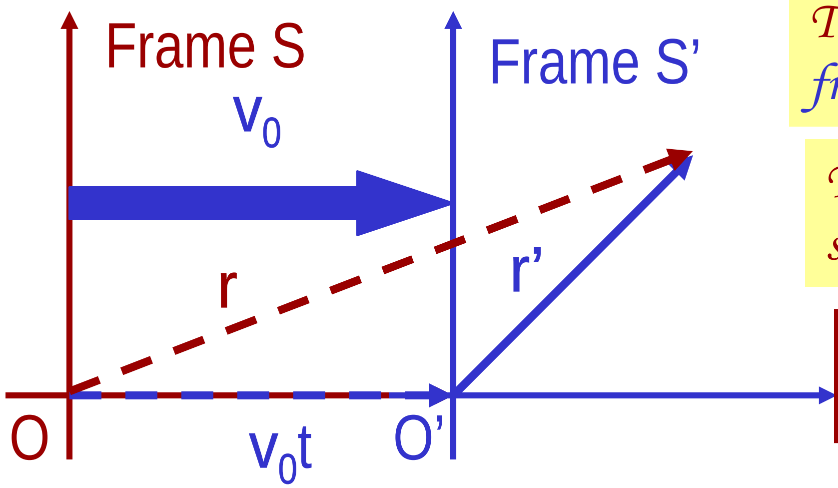
- Quiz tomorrow, Thursday, June 7
  – In the beginning of the class
  – CH 1 – what we cover today (CH 5?)
- Reading assignments:
  – CH 4 – 7, 5 – 4, 5 – 5
- Mid-term exam
  – 8:00 – 10am, Thursday, June 14, in class
  – CH 1 through what we cover Wednesday, June 13
- Tomorrow's lecture will be given by a mystery substitute instructor

Observations in Different Reference Frames

Results of physical measurements in different reference frames could be different

Observations of the same motion in a stationary frame would be different than the ones made in the frame moving together with the moving object.

Consider that you are driving a car. To you, the objects in the car do not move while to the person outside the car they are moving in the same speed and direction as your car is.



The position vector r' is still r' in the moving frame S' no matter how much time has passed!!

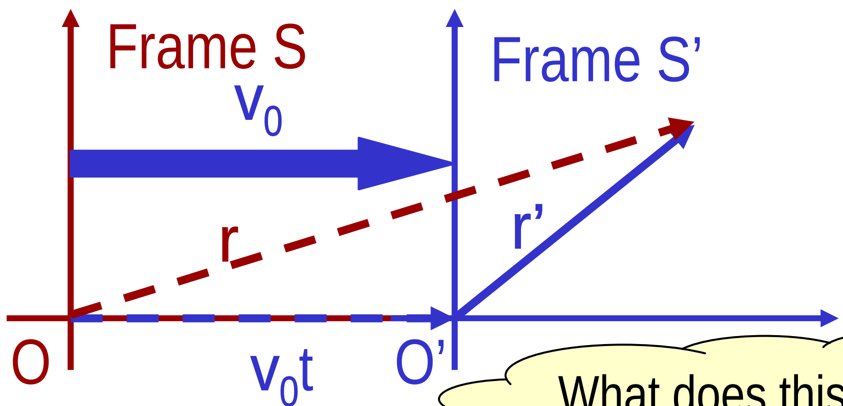
The position vector r is no longer r in the stationary frame S when time t has passed.

How are these position vectors related to each other?

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t$$

Relative Velocity and Acceleration

The velocity and acceleration in two different frames of references can be denoted, using the formula in the previous slide:



Galilean transformation equation

$$\vec{r}' = \vec{r} - \vec{v}_0 t$$

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{r}}{dt} - \vec{v}_0$$

$$\vec{v}' = \vec{v} - \vec{v}_0$$

What does this tell you?

$$\frac{d\vec{v}'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{v}_0}{dt}$$

$\vec{a}' = \vec{a}$, when $\vec{v}_0$ is constant

The accelerations measured in two frames are the same when the frames move at a constant velocity with respect to each other!!!

The earth's gravitational acceleration is the same in a frame moving at a constant velocity wrt the earth.

Force

We've been learning kinematics; describing motion without understanding what the cause of the motion is. Now we are going to learn dynamics!!

Can someone tell me what FORCE is?

FORCE is what causes an object to move.

The above statement is not entirely correct. Why?

Because when an object is moving with a constant velocity no force is exerted on the object!!!

FORCEs are what cause any change in the velocity of an object!!

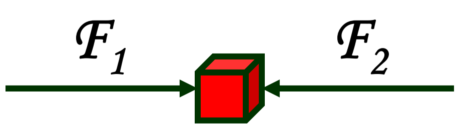
What does this statement mean?

When there is force, there is change of velocity!!

What does force cause? It causes an acceleration.!!

What happens if there are several forces being exerted on an object?

Forces are vector quantities, so vector sum of all forces, the NET FORCE, determines the direction of the acceleration of the object.



NET FORCE, $F = F_1 + F_2$

When the net force on an object is **0**, *it has constant velocity and is at its equilibrium!!*

More Force

There are various classes of forces

Contact Forces: Forces exerted by physical contact of objects

Examples of Contact Forces: Baseball hit by a bat, Car collisions

Field Forces: Forces exerted without physical contact of objects

Examples of Field Forces: Gravitational Force, Electro-magnetic force

What are possible ways to measure strength of Force?

A calibrated spring whose length changes linearly with the force exerted .

Forces are vector quantities, so addition of multiple forces must be done following the rules of vector additions.

Newton's First Law and Inertial Frames

Aristotle (384-322BC): *A natural state of a body is rest. Thus force is required to move an object. To move faster, ones needs larger forces.*

Galileo's statement on natural states of matter: *Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed!!*

Galileo's statement is formulated by Newton into the 1st law of motion (Law of Inertia): *In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.*

What does this statement tell us?

- When no force is exerted on an object, the acceleration of the object is 0.
- Any isolated object, the object that do not interact with its surrounding, is either at rest or moving at a constant velocity.
- Objects would like to keep its current state of motion, as long as there are no forces that interfere with the motion. This tendency is called the Inertia.

A frame of reference that is moving at a constant velocity is called an ***Inertial Frame***

Is a frame of reference with an acceleration an *Inertial Frame?*

NO!

Mass

Mass: *A measure of the inertia of a body* or *quantity of matter*

- Independent of the object's surroundings: The same no matter where you go.
- Independent of method of measurement: The same no matter how you measure it.

The heavier the object gets the bigger the inertia !!

It is harder to make changes of motion of a heavier object than the lighter ones.

The same forces applied to two different masses result in different acceleration depending on the mass.

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1}$$

Note that the mass and the weight of an object are two different quantities!!

Weight of an object is the magnitude of the gravitational force exerted on the object.

Not an inherent property of an object!!!

Weight will change if you measure on the Earth or on the moon but the mass won't!!

Newton's Second Law of Motion

The acceleration of an object is directly proportional to the net force exerted on it and is inversely proportional to the object's mass.

How do we write the above statement in a mathematical expression?

$$\sum_i \vec{F}_i = m\vec{a}$$

Newton's 2nd Law of Motion

Since it's a vector expression, each component should also satisfy:

$$\sum_i F_{ix} = ma_x \qquad \sum_i F_{iy} = ma_y \qquad \sum_i F_{iz} = ma_z$$

From the above vector expression, what do you conclude the dimension and unit of force are?

The dimension of force is $[m][a] = [M][LT^{-2}]$

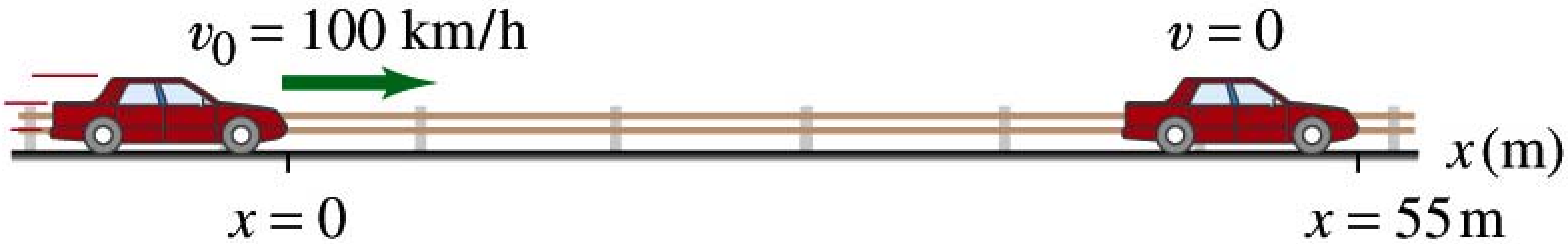
The unit of force in SI is $[Force] = [m][a] = [M][LT^{-2}] = kg \cdot m/s^2$

For ease of use, we define a new derived unit called, Newton (N)

$$1N \equiv 1kg \cdot m/s^2 \approx \frac{1}{4} lbs$$

Example

What constant net force is required to bring a 1500kg car to rest from a speed of 100km/h within a distance of 55m?



What do we need to know to figure out the force?

Acceleration!!

What are given? Initial speed: $v_{xi} = 100\ km/h = 28\ m/s$ Final speed: $v_{xf} = 0\ m/s$

Displacement: $\Delta x = x_f - x_i = 55\ m$

This is a one dimensional motion. Which kinetic formula do we use to find acceleration?

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Acceleration →

$$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{-(28m/s)^2}{2(55m)} = -7.1m/s^2$$

Thus, the force needed to stop the car is

$$F_x = ma_x = 1500kg \times (-7.1m/s^2) = -1.1 \times 10^4 N$$

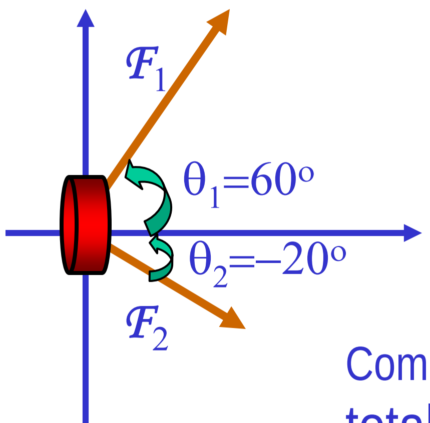
Given the force how far does the car move till it stops?

$$\Delta x = x_f - x_i = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{m(v_{xf}^2 - v_{xi}^2)}{2ma_x} = \frac{m(v_{xf}^2 - v_{xi}^2)}{2F_x}$$

- Linearly proportional to the mass of the car
- Squarely proportional to the speed of the car
- Inversely proportional to the force by the brake

Example for Newton's 2nd Law of Motion

Determine the magnitude and direction of acceleration of the puck whose mass is 0.30kg and is being pulled by two forces, F1 and F2, as shown in the picture, whose magnitudes of the forces are 8.0 N and 5.0 N, respectively.



Components of $\mathcal{F}_1$

$$F_{1x} = \left|\vec{F}_1\right| \cos\theta_1 = 8.0 \times \cos\left(60^\circ\right) = 4.0N$$

$$F_{1y} = \left|\vec{F}_1\right| \sin\theta_1 = 8.0 \times \sin\left(60^\circ\right) = 6.9N$$

Components of $\mathcal{F}_2$

$$F_{2x} = \left|\vec{F}_2\right| \cos\theta_2 = 5.0 \times \cos\left(-20^\circ\right) = 4.7N$$

$$F_{2y} = \left|\vec{F}_2\right| \sin\theta_2 = 5.0 \times \sin\left(-20^\circ\right) = -1.7N$$

Components of total force $\mathcal{F}$

$$F_x = F_{1x} + F_{2x} = 4.0 + 4.7 = 8.7N = ma_x$$

$$F_y = F_{1y} + F_{2y} = 6.9 - 1.7 = 5.2N = ma_y$$

Magnitude and direction of acceleration a

$$a_x = \frac{F_x}{m} = \frac{8.7}{0.3} = 29m/s^2 \quad a_y = \frac{F_y}{m} = \frac{5.2}{0.3} = 17m/s^2 \quad \left|\vec{a}\right| = \sqrt{\left(a_x\right)^2 + \left(a_y\right)^2} = \sqrt{\left(29\right)^2 + \left(17\right)^2} = 34m/s^2$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^\circ$$

Acceleration Vector a

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \left(29\hat{i} + 17\hat{j}\right)m/s^2$$

Gravitational Force and Weight

Gravitational Force, $\mathcal{F}_g$

The attractive force exerted on an object by the Earth

$$\vec{F}_G = m\vec{a} = m\vec{g}$$

Weight of an object with mass M is

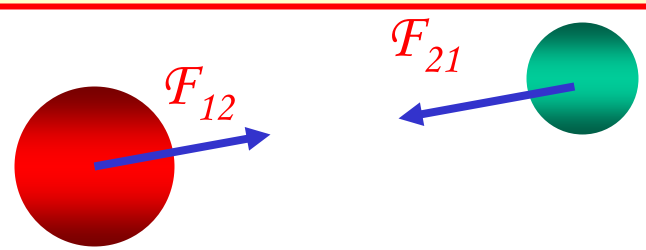
$$W = \left|\vec{F}_G\right| = M\left|\vec{g}\right| = Mg$$

Since weight depends on the magnitude of gravitational acceleration, ***g****, it varies depending on geographical location.*

By measuring the forces one can determine masses. This is why you can measure mass using the spring scale.

Newton's Third Law (Law of Action and Reaction)

If two objects interact, the force F_{21} exerted on object 1 by object 2 is equal in magnitude and opposite in direction to the force F_{12} exerted on object 2 by object 1.



$$\vec{F}_{12} = -\vec{F}_{21}$$

The action force is equal in magnitude to the reaction force but in opposite direction. These two forces always act ***on different objects***.

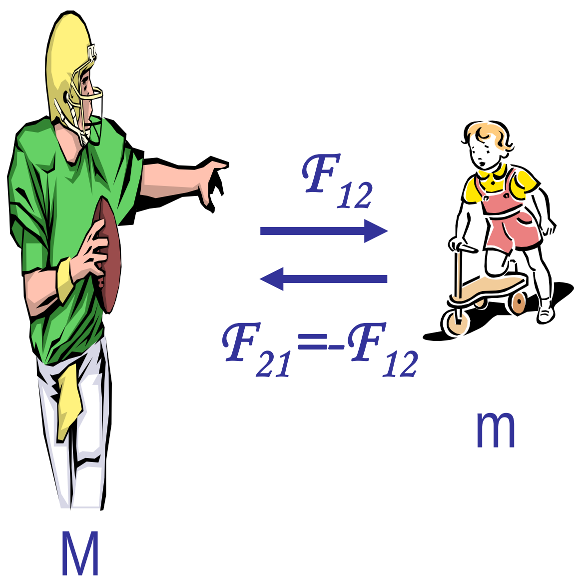
What is the reaction force to the force of a free fall object?

The gravitational force exerted by the object to the Earth!

Stationary objects on top of a table has a reaction force (normal force) from table to balance the action force, the gravitational force.

Example of Newton's 3rd Law

A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push against each other so that they move apart. a) Who moves away with the higher speed and by how much?



$$\vec{F}_{12} = -\vec{F}_{21} \quad \Rightarrow \quad \left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = F$$

$$\vec{F}_{12} = m\vec{a}_b \quad \Rightarrow \quad F_{12x} = ma_{bx}, \quad F_{12y} = ma_{by} = 0$$

$$\vec{F}_{21} = M\vec{a}_M \quad \Rightarrow \quad F_{21x} = Ma_{Mx}, \quad F_{21y} = Ma_{My} = 0$$

Since $\vec{F}_{12} = -\vec{F}_{21}$ and $\left|\vec{F}_{12}\right| = \left|-\vec{F}_{21}\right| = F$

Establish the equation $\Rightarrow$ $ma_{bx} = F = Ma_{Mx}$ Divide by m $\Rightarrow$ $a_{bx} = \frac{F}{m} = \frac{M}{m}a_{Mx}$

Example of Newton's 3rd Law

Man's velocity
$$v_{Mxf} = v_{Mxi} + a_{Mx}t = a_{Mx}t$$

Boy's velocity
$$v_{bxf} = v_{bxi} + a_{bx}t = a_{bx}t = \frac{M}{m}a_{Mx}t = \frac{M}{m}v_{Mxf}$$

So boy's velocity if higher than man's, if M>m, by the ratio of the masses.

b) Who moves farther while their hands are in contact?

Boy's displacement
$$x_b = v_{bxi}t + \frac{1}{2}a_{bx}t^2 = \frac{M}{2m}a_{Mx}t^2$$

$$x_b = \frac{M}{m}\left(\frac{1}{2}a_{Mx}t^2\right) = \frac{M}{m}x_M$$

Man's displacement

Given in the same time interval, since the boy has higher acceleration and thereby higher speed, he moves farther than the man.