PHYS 1443 – Section 001
Lecture #9

Tuesday, June 12, 2007

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- Motion in Accelerated Frames
- Work done by a constant force
- Scalar Product of Vectors
- Work done by a varying force
- Work and Kinetic Energy Theorem

Today's homework is HW #5, due 7pm, Monday, June 18!!

Remember the mid-term exam 8 – 10am, Thursday, June 14!!

Motion in Accelerated Frames

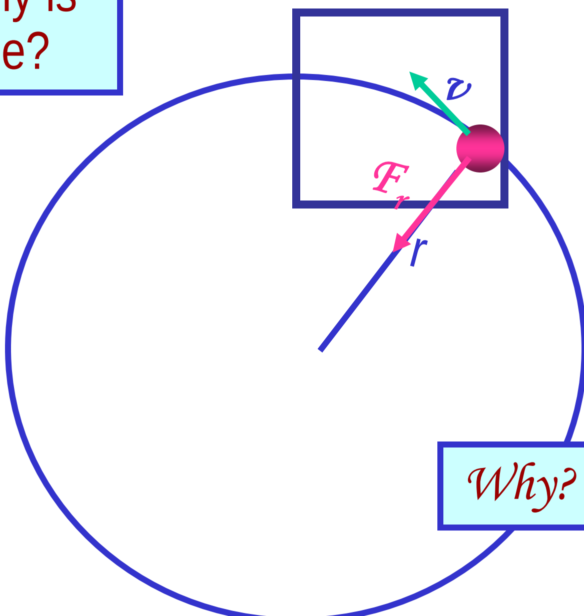
Newton's laws are valid only when observations are made in an inertial frame of reference. What happens in a non-inertial frame?

Fictitious forces are needed to apply Newton's second law in an accelerated frame.

This force does not exist when the observations are made in an inertial reference frame.

What does this mean and why is this true?

Let's consider a free ball inside a box under uniform circular motion.



How does this motion look like in an inertial frame (or frame outside a box)?

We see that the box has a radial force exerted on it but none on the ball directly

How does this motion look like in the box?

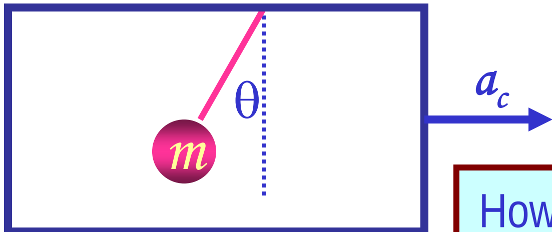
The ball is tumbled over to the wall of the box and feels that it is getting force that pushes it toward the wall.

Why?

According to Newton's first law, the ball wants to continue on its original movement but since the box is turning, the ball feels like it is being pushed toward the wall relative to everything else in the box.

Example of Motion in Accelerated Frames

A ball of mass m is hung by a cord to the ceiling of a boxcar that is moving with an acceleration *a*. What do the inertial observer at rest and the non-inertial observer traveling inside the car conclude? How do they differ?

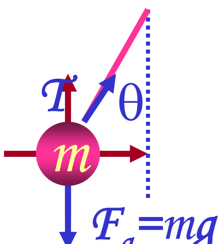


This is how the ball looks like no matter which frame you are in.

How do the free-body diagrams look for two frames?

How do the motions interpreted in these two frames? Any differences?

Inertial Frame



$$\sum \vec{F} = \vec{F}_g + \vec{T}$$

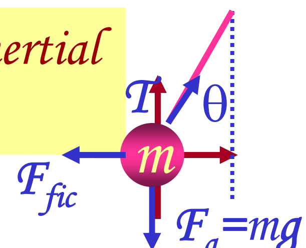
$$\sum F_x = ma_x = ma_c = T\sin\theta$$

$$\sum F_y = T\cos\theta - mg = 0$$

$$\boxed{T = \frac{mg}{\cos\theta}} \quad \boxed{a_c = g\tan\theta}$$

For an inertial frame observer, the forces being exerted on the ball are only T and F_g. The acceleration of the ball is the same as that of the box car and is provided by the x component of the tension force.

Non-Inertial Frame



$$\sum \vec{F} = \vec{F}_g + \vec{T} + \vec{F}_{fic}$$

$$\sum F_x = T\sin\theta - F_{fic} = 0 \quad \boxed{F_{fic} = ma_{fic} = T\sin\theta}$$

$$\sum F_y = T\cos\theta - mg = 0$$

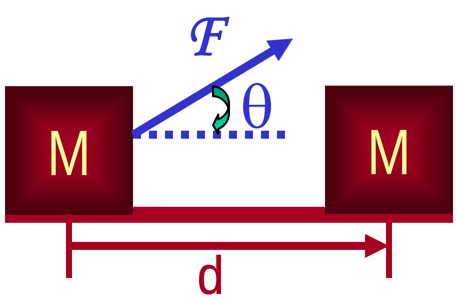
$$\boxed{T = \frac{mg}{\cos\theta}} \quad \boxed{a_{fic} = g\tan\theta}$$

In the non-inertial frame observer, the forces being exerted on the ball are T, F_g, and F_{fic}. For some reason the ball is under a force, F_{fic}, that provides acceleration to the ball.

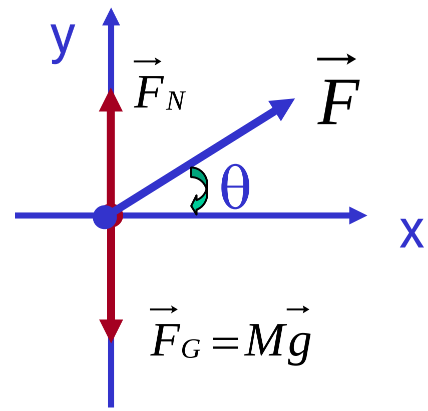
While the mathematical expression of the acceleration of the ball is identical to that of inertial frame observer's, the cause of the force is dramatically different.

Work Done by a Constant Force

A meaningful work in physics is done only when a sum of forces exerted on an object made a motion to the object.



Free Body Diagram



Which force did the work? Force $\vec{F}$ Why?

How much work did it do? $W = \left(\sum \vec{F}\right) \cdot \vec{d} = Fd\cos\theta$ Unit? $N{\bullet}m$ $= J$ (for Joule)

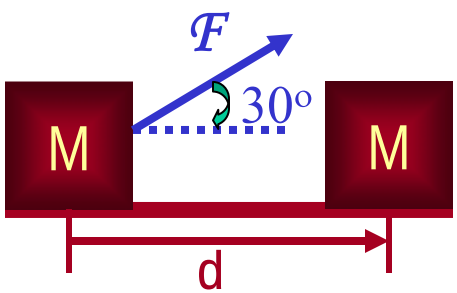
What does this mean?

Physical work is done only by the component of the force along the movement of the object.

Work is an <u>energy transfer</u>!!

Example of Work w/ Constant Force

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude F=50.0N at an angle of 30.0° with East. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced by 3.00m to East.



$$W = \left(\sum \vec{F}\right)\cdot\vec{d} = \left|\left(\sum \vec{F}\right)\right|\left|\vec{d}\right|\cos\theta$$

$$W = 50.0 \times 3.00 \times \cos 30^{\circ} = 130J$$

Does work depend on mass of the object being worked on?

Yes

Why don't I see the mass term in the work at all then?

It is reflected in the force. If an object has smaller mass, it would take less force to move it at the same acceleration than a heavier object. So it would take less work. Which makes perfect sense, doesn't it?

Scalar Product of Two Vectors

- Product of magnitude of the two vectors and the cosine of the angle between them $\vec{A} \bullet \vec{B} \equiv \left|\vec{A}\right|\left|\vec{B}\right| \cos\theta$
- Operation is commutative $\vec{A} \bullet \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \cos\theta = \left|\vec{B}\right|\left|\vec{A}\right| \cos\theta = \vec{B} \bullet \vec{A}$
- Operation follows the distribution law of multiplication $\vec{A} \bullet \left(\vec{B} + \vec{C}\right) = \vec{A} \bullet \vec{B} + \vec{A} \bullet \vec{C}$
- Scalar products of Unit Vectors $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1 \quad \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$
- How does scalar product look in terms of components?

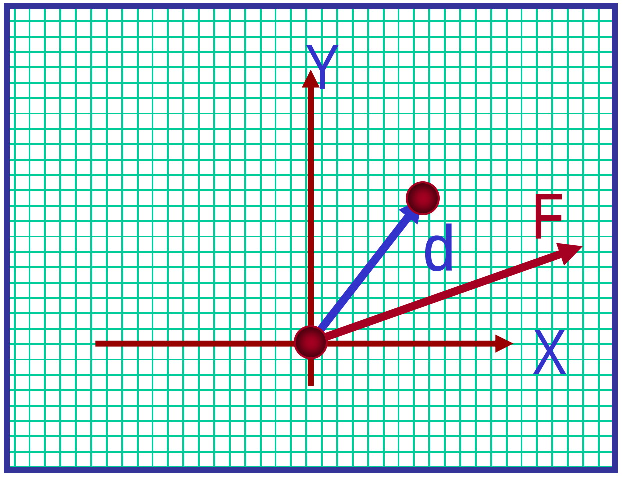
$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \qquad \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \bullet \vec{B} = \left(A_x \hat{i} + A_y \hat{j} + A_z \hat{k}\right) \bullet \left(B_x \hat{i} + B_y \hat{j} + B_z \hat{k}\right) = \left(A_x B_x \hat{i} \bullet \hat{i} + A_y B_y \hat{j} \bullet \hat{j} + A_z B_z \hat{k} \bullet \hat{k}\right) + \underbrace{cross\ terms}_{=0}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Example of Work by Scalar Product

A particle moving on the xy plane undergoes a displacement **d**=(2.0**i**+3.0**j**)m as a constant force F=(5.0**i**+2.0**j**) N acts on the particle.



a) Calculate the magnitude of the displacement and that of the force.

$$\left|\vec{d}\right| = \sqrt{d_x^2 + d_y^2} = \sqrt{(2.0)^2 + (3.0)^2} = 3.6m$$

$$\left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = 5.4N$$

b) Calculate the work done by the force F.

$$W = \vec{F} \cdot \vec{d} = \left(2.0\hat{i} + 3.0\hat{j}\right) \cdot \left(5.0\hat{i} + 2.0\hat{j}\right) = 2.0 \times 5.0\,\hat{i} \cdot \hat{i} + 3.0 \times 2.0\,\hat{j} \cdot \hat{j} = 10 + 6 = 16(J)$$

Can you do this using the magnitudes and the angle between **d** and F?

$$W = \vec{F} \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right| \cos\theta$$

Work Done by Varying Force

- If the force depends on the position of the object in motion
  - one must consider work in small segments of the position where the force can be considered constant

$$\Delta W = F_x \cdot \Delta x$$

  - Then add all the work-segments throughout the entire motion ($x_i \rightarrow x_f$)

$$W \approx \sum_{x_i}^{x_f} F_x \cdot \Delta x$$

In the limit where $\Delta x \rightarrow 0$

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \cdot \Delta x = \int_{x_i}^{x_f} F_x dx = W$$

  - If more than one force is acting, the net work done by the net force is

$$W(net) = \int_{x_i}^{x_f} \left(\sum F_{ix} \right) dx$$

One of the position dependent forces is the force by a spring $\boxed{F_s = -kx}$

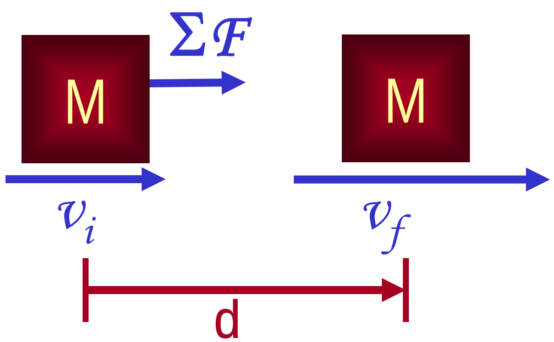
Hooke's Law

The work done by the spring force is

$$W = \int_{-x_{\max}}^{0} F_s dx = \int_{-x_{\max}}^{0} (-kx)\, dx = \frac{1}{2} kx^2$$

Kinetic Energy and Work-Kinetic Energy Theorem

- Some problems are hard to solve using Newton's second law
  - If forces exerting on an object during the motion are complicated
  - Relate the work done on the object by the net force to the change of the speed of the object



Suppose net force $\Sigma\mathcal{F}$ was exerted on an object for displacement d to increase its speed from v_i to v_f.

The work on the object by the net force $\Sigma\mathcal{F}$ is

$$W = \left(\sum \vec{F}\right)\cdot \vec{d} = (ma)d\cos 0 = (ma)d$$

Displacement $$d = \frac{1}{2}\left(v_f + v_i\right)t$$

Acceleration $$a = \frac{v_f - v_i}{t}$$

Work $$W = (ma)d = \left[m\left(\frac{v_f - v_i}{t}\right)\right]\frac{1}{2}\left(v_f + v_i\right)t = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Kinetic Energy $$\boxed{KE \equiv \frac{1}{2}mv^2}$$

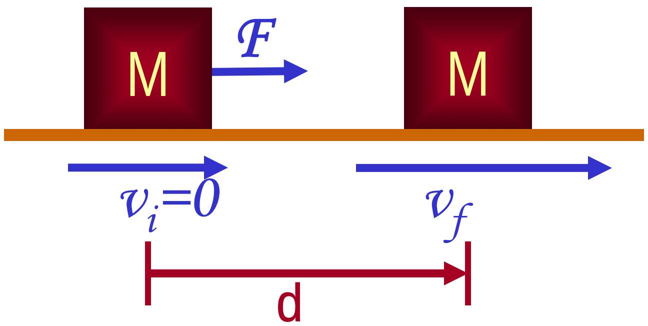
Work $$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$$

Work done by the net force causes change of object's kinetic energy.

Work-Kinetic Energy Theorem

Example of Work-KE Theorem

A 6.0kg block initially at rest is pulled to East along a horizontal, frictionless surface by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W = \vec{F} \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right| \cos\theta = 12 \times 3.0 \cos 0 = 36(J)$$

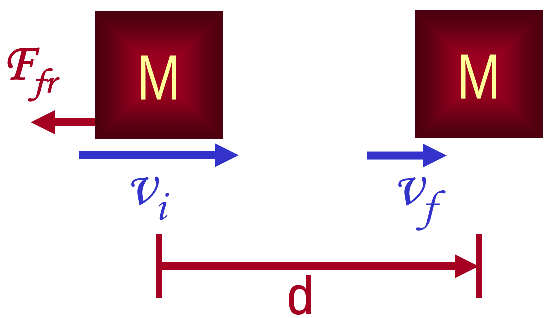
From the work-kinetic energy theorem, we know $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

Since initial speed is 0, the above equation becomes $W = \frac{1}{2}mv_f^2$

Solving the equation for v_f, we obtain $v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 36}{6.0}} = 3.5m/s$

Work and Energy Involving Kinetic Friction

- What do you think the work looks like if there is friction?
  - Static friction does not matter! Why?
    It isn't there when the object is moving.
  - Then which friction matters?
    Kinetic Friction



Friction force $\mathcal{F}_{fr}$ works on the object to slow down

The work on the object by the friction $\mathcal{F}_{fr}$ is

$$W_{fr} = F_{fr} d \cos(180) = -F_{fr} d \quad \Delta KE = -F_{fr} d$$

The final kinetic energy of an object, taking into account its initial kinetic energy, friction force and other source of work, is

$$KE_f = KE_i + \sum W - F_{fr} d$$

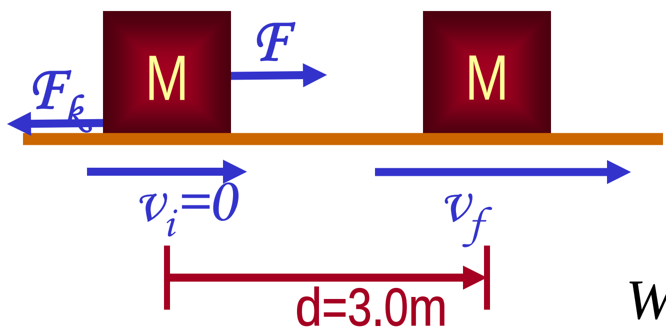
t=0, KE_i

Friction, Engine work

t=T, KE_f

Example of Work Under Friction

A 6.0kg block initially at rest is pulled to East along a horizontal surface with coefficient of kinetic friction μ_k=0.15 by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W_F = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36\left(J\right)$$

Work done by friction $\mathcal{F}_k$ is

$$W_k = \vec{F}_k \bullet \vec{d} = \left|\vec{F}_k\right|\left|\vec{d}\right|\cos\theta = \left|\mu_k m g\right|\left|\vec{d}\right|\cos\theta$$

$$= 0.15\times 6.0\times 9.8\times 3.0\cos 180 = -26\left(J\right)$$

Thus the total work is

$$W = W_F + W_k = 36 - 26 = 10(J)$$

Using work-kinetic energy theorem and the fact that initial speed is 0, we obtain

$$W = W_F + W_k = \frac{1}{2}mv_f^2$$

Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 10}{6.0}} = 1.8m/s$$