PHYS 1443 – Section 001
Lecture #11

Tuesday, June 19, 2007

Dr. Jaehoon Yu

- Conservation of Momentum
- Impulse and Momentum Change

Today's homework is HW #6, due 7pm, Friday, June 22!!

Announcements

- Quiz this Thursday
  - Early in the class
- Mid-term grade discussions today
  - Bottom half of the class
- Exam results
  - Average: 62/106
  - Top score: 94/106
- Grade proportions
  - Two exams constitute 45%, 22.5% each
  - Homework: 25%
  - Lab: 20%
  - Quizzes: 10%
  - Extra credit: 10%
- Tomorrow's lecture will be given by Mr. Atman

Conservation of Linear Momentum in a Two Particle System

Consider an isolated system with two particles that do not have any external forces exerting on it. What is the impact of Newton's 3rd Law?

*If particle #1 exerts force on particle #2, there must be another force that the particle #2 exerts on #1 as the reaction force. Both the forces are internal forces, and the net force in the entire SYSTEM is **still 0**.*

Now how would the momenta of these particles look like?

Let say that the particle #1 has momentum $\mathbf{p_1}$ *and #2 has* $\mathbf{p_2}$ *at some point of time.*

Using momentum-force relationship

$$\vec{F}_{21} = \frac{d\vec{p}_1}{dt} \quad \text{and} \quad \vec{F}_{12} = \frac{d\vec{p}_2}{dt}$$

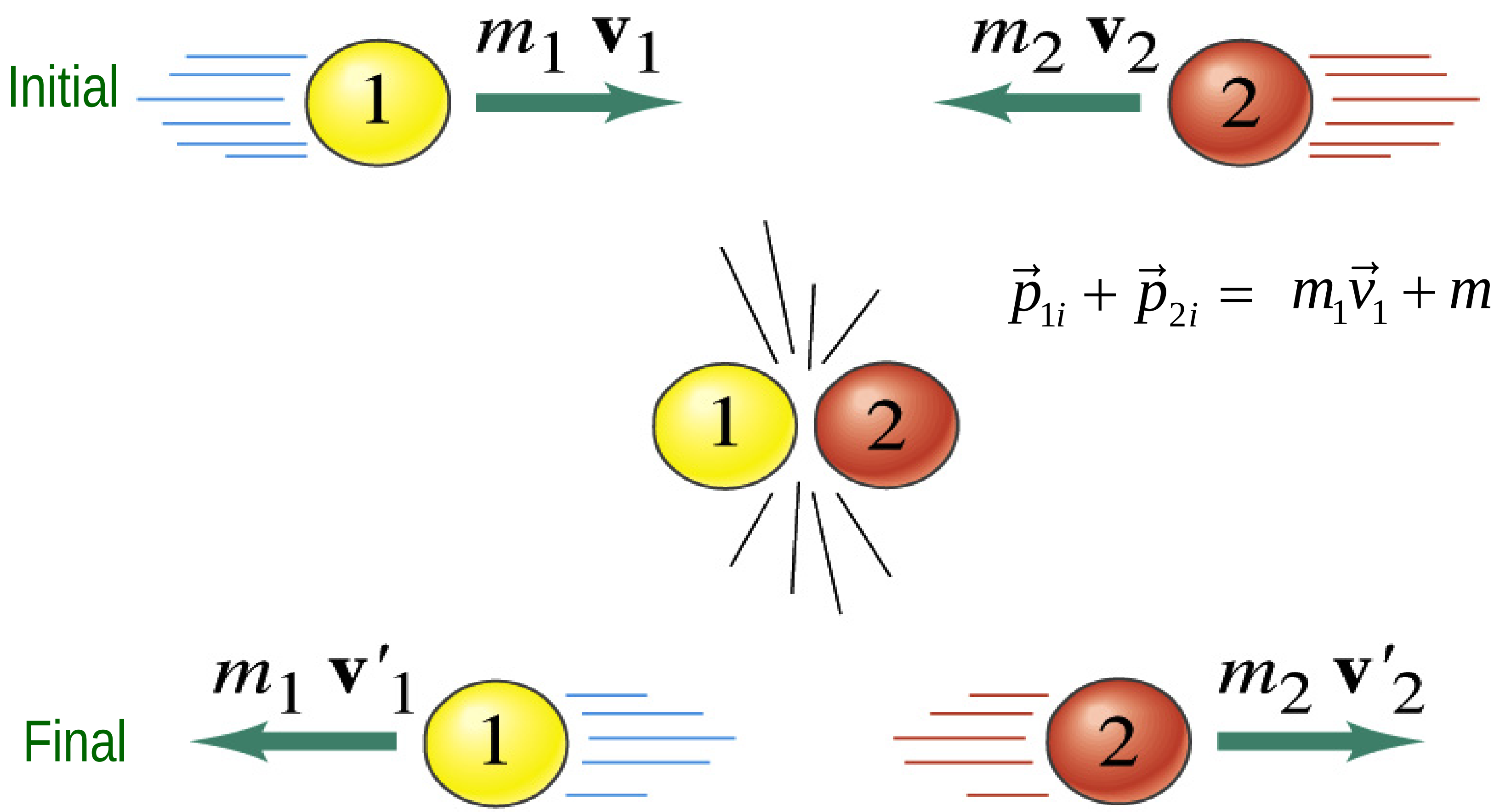
And since net force of this system is 0

$$\sum \vec{F} = \vec{F}_{12} + \vec{F}_{21} = \frac{d\vec{p}_2}{dt} + \frac{d\vec{p}_1}{dt} = \frac{d}{dt}\left(\vec{p}_2 + \vec{p}_1\right) = 0$$

Therefore $\vec{p}_2 + \vec{p}_1 = const$

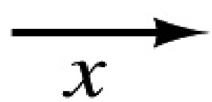
The total linear momentum of the system is conserved!!!

Linear Momentum Conservation



$$\vec{p}_{1i} + \vec{p}_{2i} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\vec{p}_{1f} + \vec{p}_{2f} = m_1\vec{v}'_1 + m_2\vec{v}'_2$$



$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

More on Conservation of Linear Momentum in a Two Body System

From the previous slide we've learned that the total momentum of the system is conserved if no external forces are exerted on the system.

$$\sum \vec{p} = \vec{p}_2 + \vec{p}_1 = const$$

What does this mean?

As in the case of energy conservation, this means that the total vector sum of all momenta in the system is the same before and after any interactions

Mathematically this statement can be written as

$$\vec{p}_{2i} + \vec{p}_{1i} = \vec{p}_{2f} + \vec{p}_{1f}$$

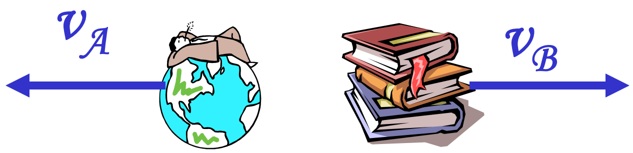
$$\sum_{system} P_{xi} = \sum_{system} P_{xf} \qquad \sum_{system} P_{yi} = \sum_{system} P_{yf} \qquad \sum_{system} P_{zi} = \sum_{system} P_{zf}$$

This can be generalized into conservation of linear momentum in many particle systems.

Whenever two or more particles in an ***isolated system*** *interact, the total momentum of the system remains constant.*

Example for Linear Momentum Conservation

Estimate an astronaut's resulting velocity after he throws his book to a direction in the space to move to a direction.



From momentum conservation, we can write

$$\vec{p}_i = 0 = \vec{p}_f = m_A \vec{v}_A + m_B \vec{v}_B$$

Assuming the astronaut's mass is 70kg, and the book's mass is 1kg and using linear momentum conservation

$$\vec{v}_A = -\frac{m_B \vec{v}_B}{m_A} = -\frac{1}{70} \vec{v}_B$$

Now if the book gained a velocity of 20 m/s in +x-direction, the Astronaut's velocity is

$$\vec{v}_A = -\frac{1}{70}\left(20\vec{i}\right) = -0.3\vec{i} \quad (m/s)$$

Impulse and Linear Momentum

Net force causes change of momentum → Newton's second law

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = \vec{F}dt$$

By integrating the above equation in a time interval t_i to t_f, one can obtain impulse I.

$$\int_{t_i}^{t_f} d\vec{p} = \vec{p}_f - \vec{p}_i = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}dt = \vec{I}$$

So what do you think an impulse is?

Effect of the force F acting on an object over the time interval $\Delta t = t_f - t_i$ is equal to the change of the momentum of the object caused by that force. Impulse is the degree of which an external force changes an object's momentum.

The above statement is called the impulse-momentum theorem and is equivalent to Newton's second law.

What are the dimension and unit of Impulse? What is the direction of an impulse vector?

Defining a time-averaged force

$$\bar{\vec{F}} \equiv \frac{1}{\Delta t}\sum_i \vec{F}_i \Delta t$$

Impulse can be rewritten

$$\vec{I} \equiv \bar{\vec{F}}\Delta t$$

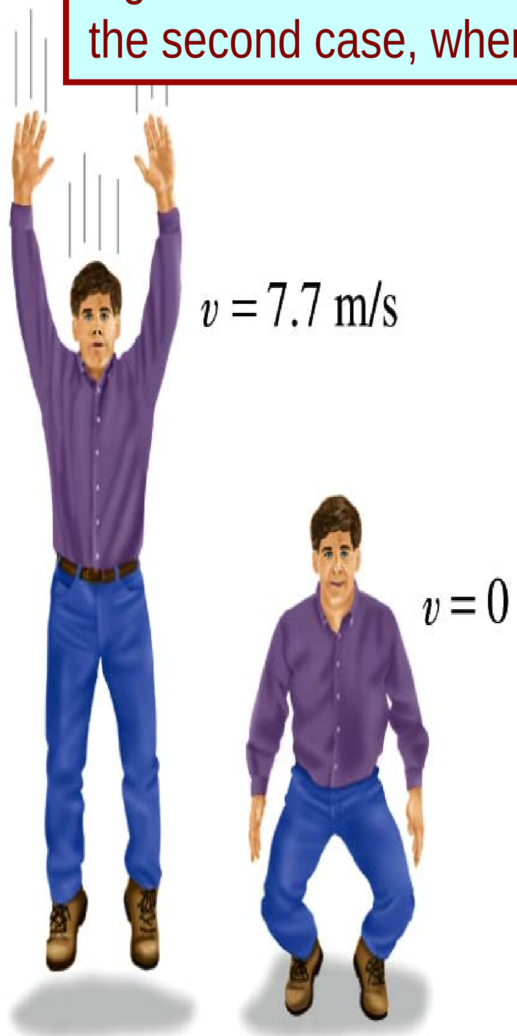
If force is constant

$$\vec{I} \equiv \vec{F}\Delta t$$

It is generally assumed that the impulse force acts on a short time but much greater than any other forces present.

Example 9-6

(a) Calculate the impulse experienced when a 70 kg person lands on firm ground after jumping from a height of 3.0 m. Then estimate the average force exerted on the person's feet by the ground, if the landing is (b) stiff-legged and (c) with bent legs. In the former case, assume the body moves 1.0cm during the impact, and in the second case, when the legs are bent, about 50 cm.



We don't know the force. How do we do this?

Obtain velocity of the person before striking the ground.

$$KE = -\Delta PE \qquad \frac{1}{2}mv^2 = -mg\left(y - y_i\right) = mgy_i$$

Solving the above for velocity v, we obtain

$$v = \sqrt{2gy_i} = \sqrt{2\cdot 9.8\cdot 3} = 7.7m/s$$

Then as the person strikes the ground, the momentum becomes 0 quickly giving the impulse

$$I = \bar{F}\Delta t = \Delta p = p_f - p_i = 0 - mv =$$

$$= -70kg\cdot 7.7m/s = -540N\cdot s$$

Example 9 – 6 cont'd

In coming to rest, the body decelerates from 7.7m/s to 0m/s in a distance d=1.0cm=0.01m.

The average speed during this period is $$\bar{v} = \frac{0 + v_i}{2} = \frac{7.7}{2} = 3.8m/s$$

The time period the collision lasts is $$\Delta t = \frac{d}{\bar{v}} = \frac{0.01m}{3.8m/s} = 2.6 \times 10^{-3}s$$

Since the magnitude of impulse is $$I = \bar{F}\Delta t = 540N \cdot s$$

The average force on the feet during this landing is $$\bar{F} = \frac{I}{\Delta t} = \frac{540}{2.6 \times 10^{-3}} = 2.1 \times 10^5 N$$

How large is this average force? $$Weight = 70kg \cdot 9.8m/s^2 = 6.9 \times 10^2 N$$

$$\bar{F} = 2.1 \times 10^5 N = 304 \times 6.9 \times 10^2 N = 304 \times Weight$$

If landed in stiff legged, the feet must sustain 300 times the body weight. The person will likely break his leg.

For bent legged landing: $$\Delta t = \frac{d}{\bar{v}} = \frac{0.50m}{3.8m/s} = 0.13s$$

$$\bar{F} = \frac{540}{0.13} = 4.1 \times 10^3 N = 5.9Weight$$

Another Example for Impulse

In a crash test, an automobile of mass 1500kg collides with a wall. The initial and final velocities of the automobile are v_i= -15.0i m/s and v_f=2.60i m/s. If the collision lasts for 0.150 seconds, what would be the impulse caused by the collision and the average force exerted on the automobile?

Let's assume that the force involved in the collision is a lot larger than any other forces in the system during the collision. From the problem, the initial and final momentum of the automobile before and after the collision is

$$\vec{p}_i = m\vec{v}_i = 1500 \times (-15.0)\vec{i} = -22500\vec{i}\ kg \cdot m/s$$

$$\vec{p}_f = m\vec{v}_f = 1500 \times (2.60)\vec{i} = 3900\vec{i}\ kg \cdot m/s$$

Therefore the impulse on the automobile due to the collision is

$$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (3900 + 22500)\vec{i}\ kg \cdot m/s$$

$$= 26400\vec{i}\ kg \cdot m/s = 2.64 \times 10^4\vec{i}\ kg \cdot m/s$$

The average force exerted on the automobile during the collision is

$$\overline{\vec{F}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{2.64 \times 10^4}{0.150}\vec{i}$$

$$= 1.76 \times 10^5\vec{i}\ kg \cdot m/s^2 = 1.76 \times 10^5\vec{i}\ \mathrm{N}$$