# PHYS 1443 – Section 001
# Lecture #17

*Thursday, June 28, 2007*
*Dr. Jaehoon Yu*

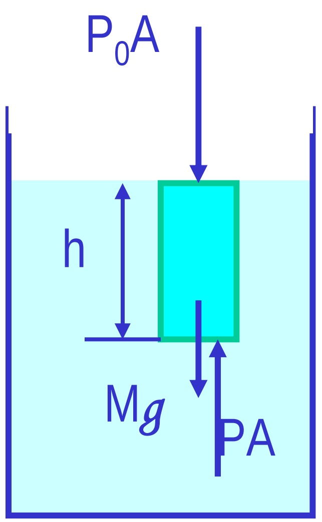
- Variation of Pressure vs Depth
- Pascal's Principle
- Absolute and Relative Pressure
- Buoyant Force and Archimedes' Principle
- Flow Rate and Continuity Equation
- Bernoulli's Equation

# Announcements

- Reading assignments
  - CH13 – 9 through 13 – 13
- Final exam
  - Date and time: 8 – 10am, Next Monday, July 2
  - Location: SH103
  - Covers: Ch 8.4 – 13

# Variation of Pressure and Depth

Water pressure increases as a function of depth, and the air pressure decreases as a function of altitude. Why?




It seems that the pressure has a lot to do with the total mass of the fluid above the object that puts weight on the object.

Let's imagine a liquid contained in a cylinder with height *h* and the cross sectional area *A* immersed in a fluid of density ρ at rest, as shown in the figure, and the system is in its equilibrium.

If the liquid in the cylinder is the same substance as the fluid, the mass of the liquid in the cylinder is $M = \rho V = \rho A h$

Since the system is in its equilibrium $PA - P_0A - Mg = PA - P_0A - \rho Ahg = 0$

Therefore, we obtain $P = P_0 + \rho g h$

The pressure at the depth *h* below the surface of a fluid open to the atmosphere is greater than atmospheric pressure by ρ*gh*.

Atmospheric pressure $P_0$ is

$$1.00\,atm = 1.013 \times 10^5\,Pa$$

# Pascal's Principle and Hydraulics

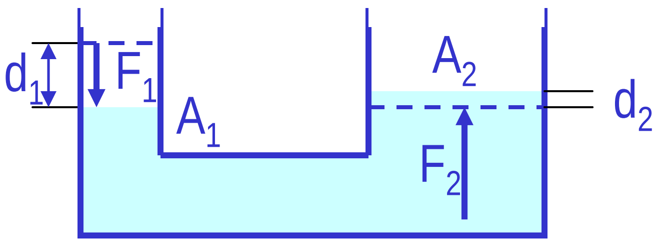
A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

$$P = P_0 + \rho g h$$

What happens if $P_0$ is changed?

The resultant pressure P at any given depth h increases as much as the change in $P_0$.

This is the principle behind hydraulic pressure. How?




Since the pressure change caused by the the force $F_1$ applied onto the area $A_1$ is transmitted to the $F_2$ on an area $A_2$.

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Therefore, the resultant force $F_2$ is

$$F_2 = \frac{A_2}{A_1} F_1$$

In other words, the force gets multiplied by the ratio of the areas $A_2/A_1$ and is transmitted to the force $F_2$ on the surface.

This seems to violate some kind of conservation law, doesn't it?

No, the actual displaced volume of the fluid is the same. And the work done by the forces are still the same.

$$F_2 = \frac{d_1}{d_2} F_1$$

# Example for Pascal's Principle

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of 5.00cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0cm. What force must the compressed air exert to lift a car weighing 13,300N? What air pressure produces this force?

Using the Pascal's principle, one can deduce the relationship between the forces, the force exerted by the compressed air is

$$F_1 = \frac{A_1}{A_2} F_2 \quad = \frac{\pi (0.05)^2}{\pi (0.15)^2} \times 1.33 \times 10^4 = 1.48 \times 10^3 \, N$$

Therefore the necessary pressure of the compressed air is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3}{\pi (0.05)^2} = 1.88 \times 10^5 \, Pa$$

# Example for Pascal's Principle

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of the pool with a depth 5.0 m.

We first need to find out the pressure difference that is being exerted on the eardrum. Then estimate the area of the eardrum to find out the force exerted on the eardrum.

Since the outward pressure in the middle of the eardrum is the same as normal air pressure

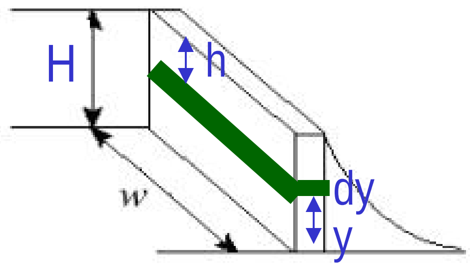
$$P - P_0 = \rho_W gh = 1000 \times 9.8 \times 5.0 = 4.9 \times 10^4 Pa$$

Estimating the surface area of the eardrum at $1.0cm^2 = 1.0x10^{-4}$ $m^2$, we obtain

$$F = (P - P_0)A \approx 4.9 \times 10^4 \times 1.0 \times 10^{-4} \approx 4.9 N$$

# Example for Pascal's Principle

Water is filled to a height H behind a dam of width w. Determine the resultant force exerted by the water on the dam.




Since the water pressure varies as a function of depth, we will have to do some calculus to figure out the total force.

The pressure at the depth h is

$$P = \rho g h = \rho g \left(H - y\right)$$

The infinitesimal force dF exerting on a small strip of dam dy is

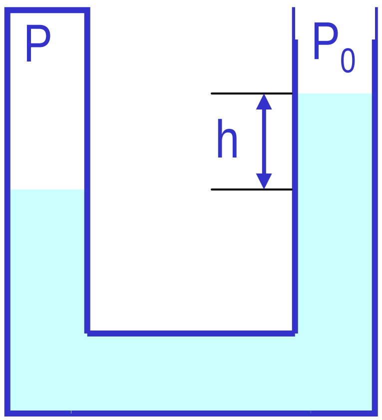
$$dF = PdA = \rho g \left(H - y\right) w dy$$

Therefore the total force exerted by the water on the dam is

$$F = \int_{y=0}^{y=H} \rho g \left(H - y\right) w dy = \rho g w \left[ H y - \frac{1}{2} y^2 \right]_{y=0}^{y=H} = \frac{1}{2} \rho g w H^2$$

# Absolute and Relative Pressure

How can one measure pressure?




One can measure the pressure using an open-tube manometer, where one end is connected to the system with unknown pressure P and the other open to air with pressure $P_0$.

The measured pressure of the system is $$P = P_0 + \rho g h$$

This is called the absolute pressure, because it is the actual value of the system's pressure.

In many cases we measure the pressure difference with respect to the atmospheric pressure due to isolate the changes in $P_0$ that depends on the environment. This is called gauge or relative pressure.

$$P_G = P - P_0 = \rho g h$$

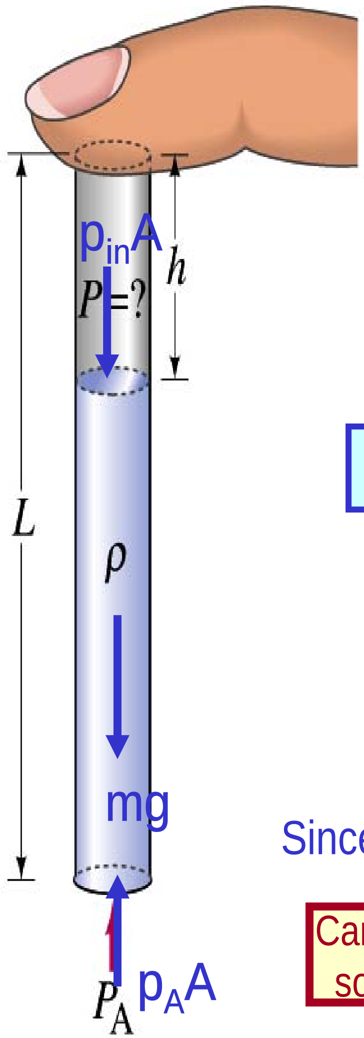
The common barometer which consists of a mercury column with one end closed at vacuum and the other open to the atmosphere was invented by Evangelista Torricelli.

Since the closed end is at vacuum, it does not exert any force. 1 atm of air pressure pushes mercury up 76cm. So 1 atm is

$$P_0 = \rho g h = (13.595 \times 10^3 \, kg/m^3)(9.80665 \, m/s^2)(0.7600 \, m)$$
$$= 1.013 \times 10^5 \, Pa = 1 \, atm$$

If one measures the tire pressure with a gauge at 220kPa the actual pressure is 101kPa+220kPa=303kPa.

# Finger Holds Water in Straw




You insert a straw of length $L$ into a tall glass of your favorite beverage. You place your finger over the top of the straw so that no air can get in or out, and then lift the straw from the liquid. You find that the straw strains the liquid such that the distance from the bottom of your finger to the top of the liquid is $h$. Does the air in the space between your finger and the top of the liquid have a pressure P that is (a) greater than, (b) equal to, or (c) less than, the atmospheric pressure $P_A$ outside the straw?

**Less**

What are the forces in this problem?

Gravitational force on the mass of the liquid $F_g = mg = \rho A(L-h)g$

Force exerted on the top surface of the liquid by inside air pressure $F_{in} = p_{in}A$

Force exerted on the bottom surface of the liquid by the outside air $F_{out} = -p_A A$

Since it is at equilibrium $F_{out} + F_g + F_{in} = 0$ ⇒ $-p_A \cancel{A} + \rho g(L-h)\cancel{A} + p_{in}\cancel{A} = 0$

Cancel A and solve for $p_{in}$ ⇒ $p_{in} = p_A - \rho g(L-h)$

So $p_{in}$ is less than $P_A$ by $\rho g(L-h)$.

# Buoyant Forces and Archimedes' Principle

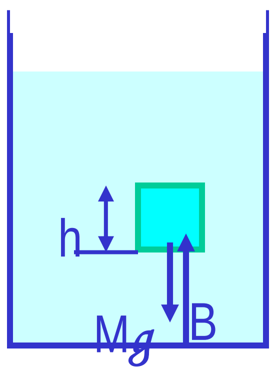
Why is it so hard to put an inflated beach ball under water while a small piece of steel sinks in the water easily?

The water exerts force on an object immersed in the water.
This force is called the buoyant force.

How does the buoyant force work?

The magnitude of the buoyant force always equals the weight of the fluid in the volume displaced by the submerged object.

This is called Archimedes' principle. What does this mean?




Let's consider a cube whose height is h and is filled with fluid and at in its equilibrium so that its weight Mg is balanced by the buoyant force B.

$$B = F_g = Mg$$

The pressure at the bottom of the cube is larger than the top by $\rho gh$.

Therefore, $\Delta P = B/A = \rho gh$

$$B = \Delta PA = \rho ghA = \rho Vg$$

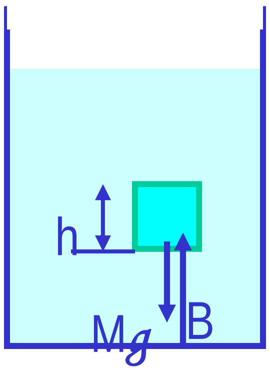
$$B = \rho Vg = Mg = F_g$$

Where Mg is the weight of the fluid.

# More Archimedes' Principle

Let's consider buoyant forces in two special cases.

Case 1: Totally submerged object




Let's consider an object of mass M, with density $\rho_0$, is immersed in the fluid with density $\rho_f$.

The magnitude of the buoyant force is $B = \rho_f V g$

The weight of the object is $F_g = Mg = \rho_0 V g$

Therefore total force in the system is $F = B - F_g = (\rho_f - \rho_0) V g$

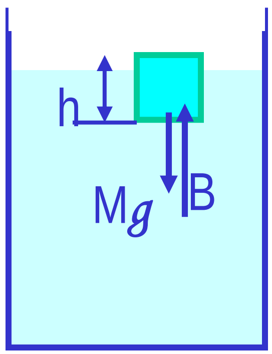
What does this tell you?

The total force applies to different directions depending on the difference of the density between the object and the fluid.

1. If the density of the object is smaller than the density of the fluid, the buoyant force will push the object up to the surface.
2. If the density of the object is larger than the fluid's, the object will sink to the bottom of the fluid.

# More Archimedes' Principle

Case 2: Floating object




Let's consider an object of mass M, with density $\rho_0$, is in static equilibrium floating on the surface of the fluid with density $\rho_f$, and the volume submerged in the fluid is $V_f$.

The magnitude of the buoyant force is $B = \rho_f V_f g$

The weight of the object is $F_g = Mg = \rho_0 V_0 g$

Therefore total force of the system is $F = B - F_g = \rho_f V_f g - \rho_0 V_0 g = 0$

Since the system is in static equilibrium $\rho_f V_f g = \rho_0 V_0 g$

$$\frac{\rho_0}{\rho_f} = \frac{V_f}{V_0}$$

What does this tell you?

Since the object is floating, its density is smaller than that of the fluid.

The ratio of the densities between the fluid and the object determines the submerged volume under the surface.

# Example for Archimedes' Principle

Archimedes was asked to determine the purity of the gold used in the crown. The legend says that he solved this problem by weighing the crown in air and in water. Suppose the scale read 7.84N in air and 6.86N in water. What should he have to tell the king about the purity of the gold in the crown?

In the air the tension exerted by the scale on the object is the weight of the crown

$$T_{air} = mg = 7.84N$$

In the water the tension exerted by the scale on the object is

$$T_{water} = mg - B = 6.86N$$

Therefore the buoyant force B is

$$B = T_{air} - T_{water} = 0.98N$$

Since the buoyant force B is

$$B = \rho_w V_w g = \rho_w V_c g = 0.98N$$

The volume of the displaced water by the crown is

$$V_c = V_w = \frac{0.98N}{\rho_w g} = \frac{0.98}{1000 \times 9.8} = 1.0 \times 10^{-4} m^3$$

Therefore the density of the crown is

$$\rho_c = \frac{m_c}{V_c} = \frac{m_c g}{V_c g} = \frac{7.84}{V_c g} = \frac{7.84}{1.0 \times 10^{-4} \times 9.8} = 8.3 \times 10^3 kg/m^3$$

Since the density of pure gold is $19.3 \times 10^3 kg/m^3$, this crown is not made of pure gold.

# Example for Buoyant Force

What fraction of an iceberg is submerged in the sea water?

Let's assume that the total volume of the iceberg is $V_i$. Then the weight of the iceberg $F_{gi}$ is

$$F_{gi} = \rho_i V_i g$$

Let's then assume that the volume of the iceberg submerged in the sea water is $V_w$. The buoyant force B caused by the displaced water becomes

$$B = \rho_w V_w g$$

Since the whole system is at its static equilibrium, we obtain

$$\rho_i V_i g = \rho_w V_w g$$

Therefore the fraction of the volume of the iceberg submerged under the surface of the sea water is

$$\frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{917\,kg/m^3}{1030\,kg/m^3} = 0.890$$

About 90% of the entire iceberg is submerged in the water!!!

# Flow Rate and the Equation of Continuity

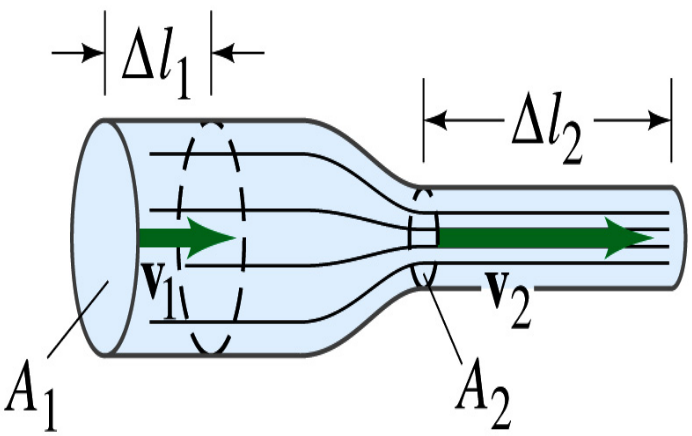
**Study of fluid in motion: Fluid Dynamics**

If the fluid is water: Water dynamics?? Hydro-dynamics

Two main types of flow

- **Streamline or Laminar flow**: Each particle of the fluid follows a smooth path, a streamline
- **Turbulent flow**: Erratic, small, whirlpool-like circles called eddy current or eddies which absorbs a lot of energy

Flow rate: the mass of fluid that passes a given point per unit time $\Delta m / \Delta t$




$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta l_1}{\Delta t} = \rho_1 A_1 v_1$$

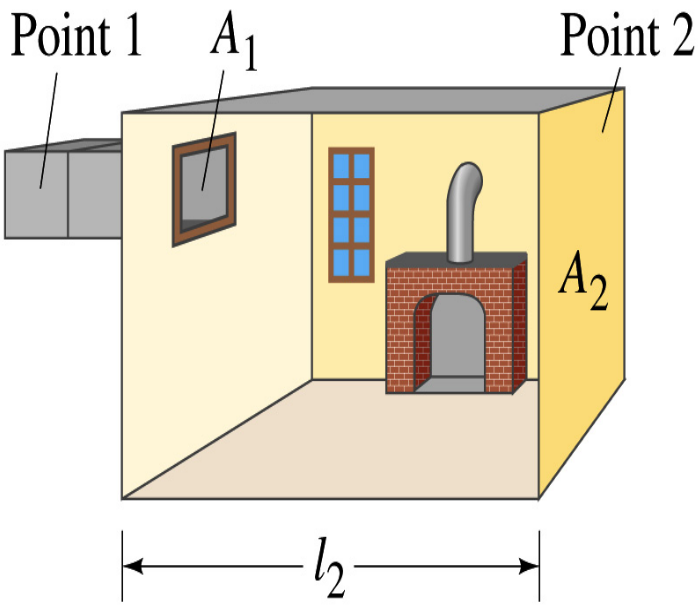
since the total flow must be conserved

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \Rightarrow \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

**Equation of Continuity**

# Example for Equation of Continuity

How large must a heating duct be if air moving at 3.0m/s along it can replenish the air every 15 minutes, in a room of 300m$^3$ volume? Assume the air's density remains constant.




Using equation of continuity

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Since the air density is constant

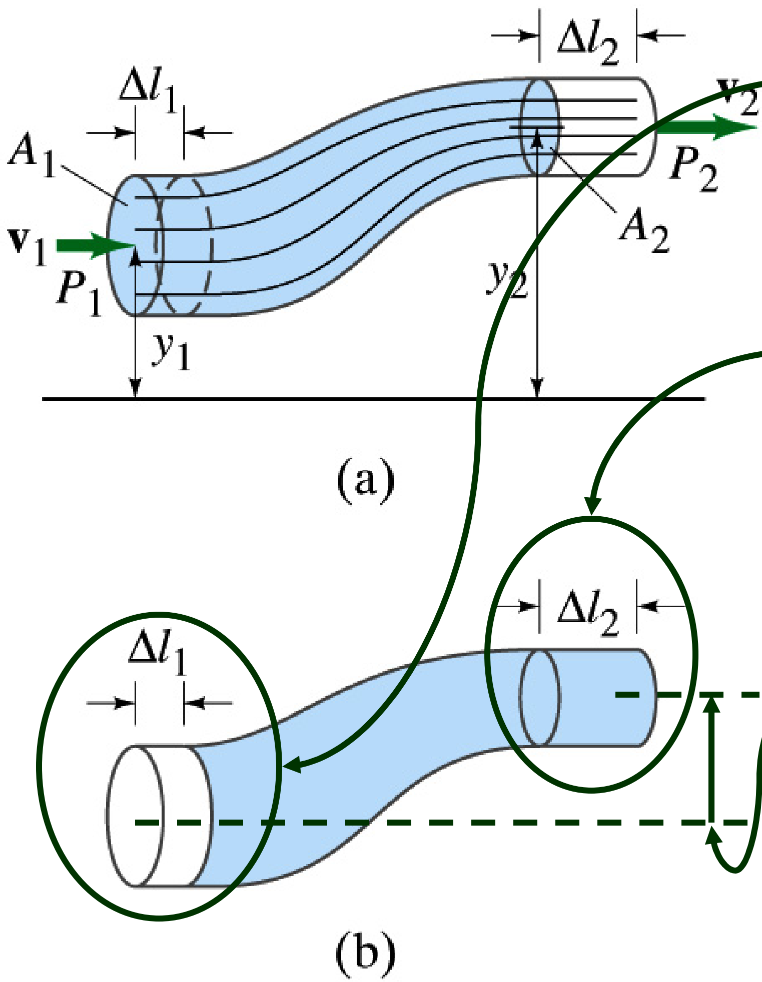
$$A_1 v_1 = A_2 v_2$$

Now let's imagine the room as the large section of the duct

$$A_1 = \frac{A_2 v_2}{v_1} = \frac{A_2 l_2 / t}{v_1} = \frac{V_2}{v_1 \cdot t} = \frac{300}{3.0 \times 900} = 0.11 m^2$$

# Bernoulli's Principle

**Bernoulli's Principle:** Where the velocity of fluid is high, the pressure is low, and where the velocity is low, the pressure is high.




Amount of the work done by the force, $F_1$, that exerts pressure, $P_1$, at point 1

$$W_1 = F_1 \Delta l_1 = P_1 A_1 \Delta l_1$$

Amount of the work done on the other section of the fluid is

$$W_2 = -P_2 A_2 \Delta l_2$$

Work done by the gravitational force to move the fluid mass, m, from $y_1$ to $y_2$ is

$$W_3 = -mg\left(y_2 - y_1\right)$$

# Bernoulli's Equation cont'd

The total amount of the work done on the fluid is

$$W = W_1 + W_2 + W_3 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mgy_2 + mgy_1$$

From the work-energy principle

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mgy_2 + mgy_1$$

Since mass, m, is contained in the volume that flowed in the motion

$$A_1 \Delta l_1 = A_2 \Delta l_2 \quad \text{and} \quad m = \rho A_1 \Delta l_1 = \rho A_2 \Delta l_2$$

Thus,

$$\frac{1}{2}\rho A_2 \Delta l_2 v_2^2 - \frac{1}{2}\rho A_1 \Delta l_1 v_1^2$$

$$= P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - \rho A_2 \Delta l_2 g y_2 + \rho A_1 \Delta l_1 g y_1$$

# Bernoulli's Equation cont'd

Since

$$\frac{1}{2}\rho \cancel{A_2 \Delta l_2} v_2^2 - \frac{1}{2}\rho \cancel{A_1 \Delta l_1} v_1^2 = P_1 \cancel{A_1 \Delta l_1} - P_2 \cancel{A_2 \Delta l_2} - \rho \cancel{A_2 \Delta l_2} g y_2 + \rho \cancel{A_1 \Delta l_1} g y_1$$

We obtain

$$\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_1 - P_2 - \rho g y_2 + \rho g y_1$$

Re-organize

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Bernoulli's Equation

Thus, for any two points in the flow

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = \ const.$$

Result of Energy conservation!

For static fluid

$$P_2 = P_1 + \rho g\left(y_1 - y_2\right) = P_1 + \rho g h$$

Pascal's Law

For the same heights

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

The pressure at the faster section of the fluid is smaller than slower section.

# Example for Bernoulli's Equation

Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.5m/s through a 4.0cm diameter pipe in the basement under a pressure of 3.0atm, what will be the flow speed and pressure in a 2.6cm diameter pipe on the second 5.0m above? Assume the pipes do not divide into branches.

Using the equation of continuity, flow speed on the second floor is

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi r_1^2 v_1}{\pi r_2^2} = 0.5 \times \left(\frac{0.020}{0.013}\right)^2 = 1.2 m/s$$

Using Bernoulli's equation, the pressure in the pipe on the second floor is

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g\left(y_1 - y_2\right)$$

$$= 3.0\times10^5 + \frac{1}{2}1\times10^3\left(0.5^2 - 1.2^2\right) + 1\times10^3\times9.8\times(-5)$$

$$= 2.5\times10^5 N/m^2$$

# *Congratulations!!!!*

*You all have done very well!!!*

*I certainly had a lot of fun with ya'll and am truly proud of you!*

*Good luck with your exam!!!*

*Have a safe summer!!*