PHYS 1441 – Section 001
Lecture #3

Thursday, May 29, 2008
Dr. Jaehoon Yu

- One Dimensional Motion:
- Average Velocity
- Average Acceleration
- Motion under constant acceleration
- Free Fall
- Two dimensional Motion

Today's homework is homework #2, due 9pm, Monday, June 2!!

Announcements

- E-mail Distribution list
  - 30 out of 55 registered as of this morning!!
  - 5 points extra credit if done by Friday, May 30
  - 3 points extra credit if done by next Monday, June 2
- Extra credit opportunity
  - Special department colloquium at 4pm, today Thursday, in SH103
    - Refreshment at 3:30pm in SH108
  - Dr. Sandip Chakrabarti
  - Copies of extra fliers will be available for you to get the signature of the speaker
- There will be a quiz in class Monday, June 2
  - Appendices + CH1 – CH2
- First term exam
  - 8 – 10am, Next Wednesday, June 4, in SH103
  - Covers CH1 – what we finish next Tuesday + appendices

Special Problems for Extra Credit

- Derive the quadratic equation for $yx^2-zx+v=0$ ➔ 5 points
- Derive the kinematic equation $v^2 = v_0^2 + 2a\left(x - x_0\right)$ from first principles and the known kinematic equations ➔ 10 points
- You must <u>show your work in detail</u> to obtain the full credit
- Due next Tuesday, June 3

Displacement, Velocity and Speed

One dimensional displacement is defined as:

$$\Delta x \equiv x_f - x_i$$

A vector quantity

Displacement is the difference between initial and final potions of the motion and is a vector quantity. How is this different than distance?

Unit? m

The average velocity is defined as: $v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} \equiv \frac{\text{Displacement}}{\text{Elapsed Time}}$

Unit? m/s

A vector quantity

Displacement per unit time in the period throughout the motion

The average speed is defined as: $v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Elapsed Time}}$

Unit? m/s

A scalar quantity

Example 1 Distance Run by a Jogger

How far does a jogger run in 1.5 hours (5400 s) if his average speed is 2.22 m/s?

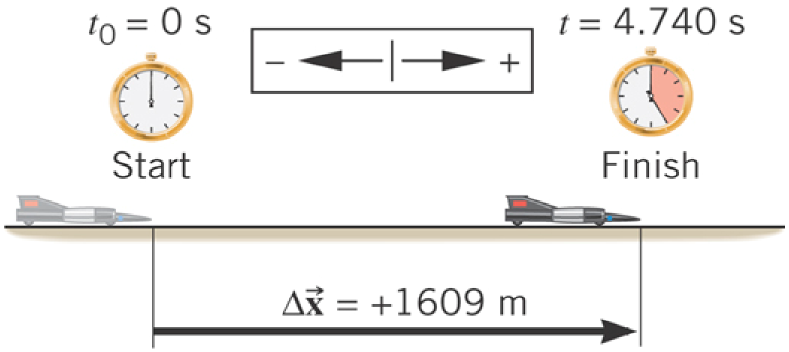
$$\text{Average speed} = \frac{\text{Distance}}{\text{Elapsed time}}$$

$$\text{Distance} = (\text{Average speed})(\text{Elapsed time}) =$$

$$= (2.22 \text{ m/s})(5400 \text{ s}) = 12000 \text{ m}$$

Example 2 The World's Fastest Jet-Engine Car

Andy Green in the car *ThrustSSC* set a world record of 341.1 m/s in 1997. To establish such a record, the driver makes two runs through the course, one in each direction to nullify wind effects. From the data, determine the average velocity for each run.



$$\bar{\vec{v}} = \frac{\Delta\vec{x}}{\Delta t} = \frac{+1609\ \text{m}}{4.740\ \text{s}} = +339.5\,\text{m/s}$$

What is the speed? $v = \left|\bar{\vec{v}}\right| = 339.5\,\text{m/s}$

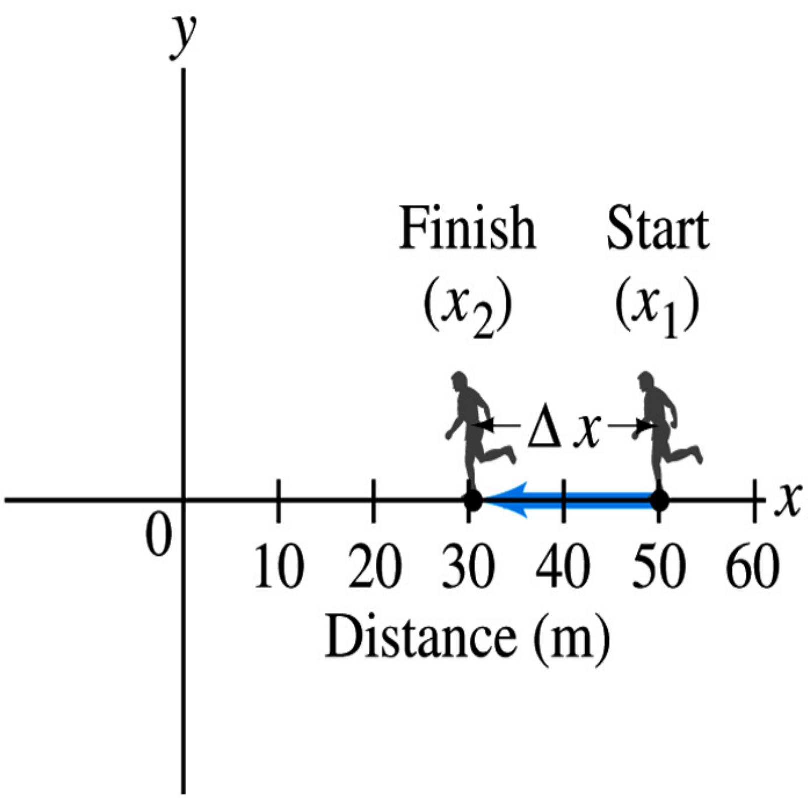
$$\bar{\vec{v}} = \frac{\Delta\vec{x}}{\Delta t} = \frac{-1609\ \text{m}}{4.695\ \text{s}} = -342.7\,\text{m/s}$$

What is the speed? $v = \left|\bar{\vec{v}}\right| = \left|-342.7\,\text{m/s}\right|$

$= 342.7\,\text{m/s}$

Example for displacement, velocity and speed

The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from x_1=50.0m to x_2=30.5 m, as shown in the figure. What was the runner's average velocity? What was the average speed?



- Displacement:

$$\Delta x \equiv x_f - x_i = x_2 - x_1 = 30.5 - 50.0 = -19.5(m)$$

- Average Velocity:

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{-19.5}{3.00} = -6.50(m/s)$$

- Average Speed:

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}}$$

$$= \frac{50.0 - 30.5}{3.00} = \frac{+19.5}{3.00} = +6.50(m/s)$$

Instantaneous Velocity and Speed

- Can average quantities tell you the detailed story of the whole motion?
- Instantaneous velocity is defined as:
  - What does this mean?
    - Displacement in an infinitesimal time interval
    - Velocity at any given moment

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

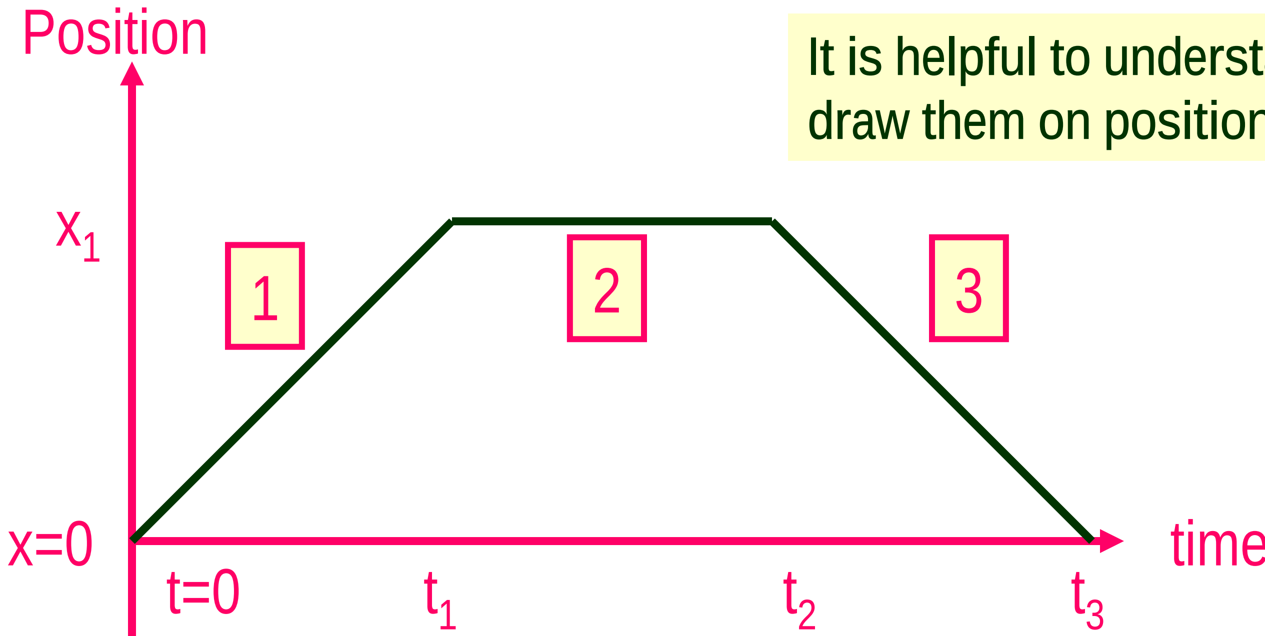
•Instantaneous speed is the size (magnitude) of the velocity vector: Speed at any given moment

$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right|$$

*Magnitude of Vectors are expressed in absolute values

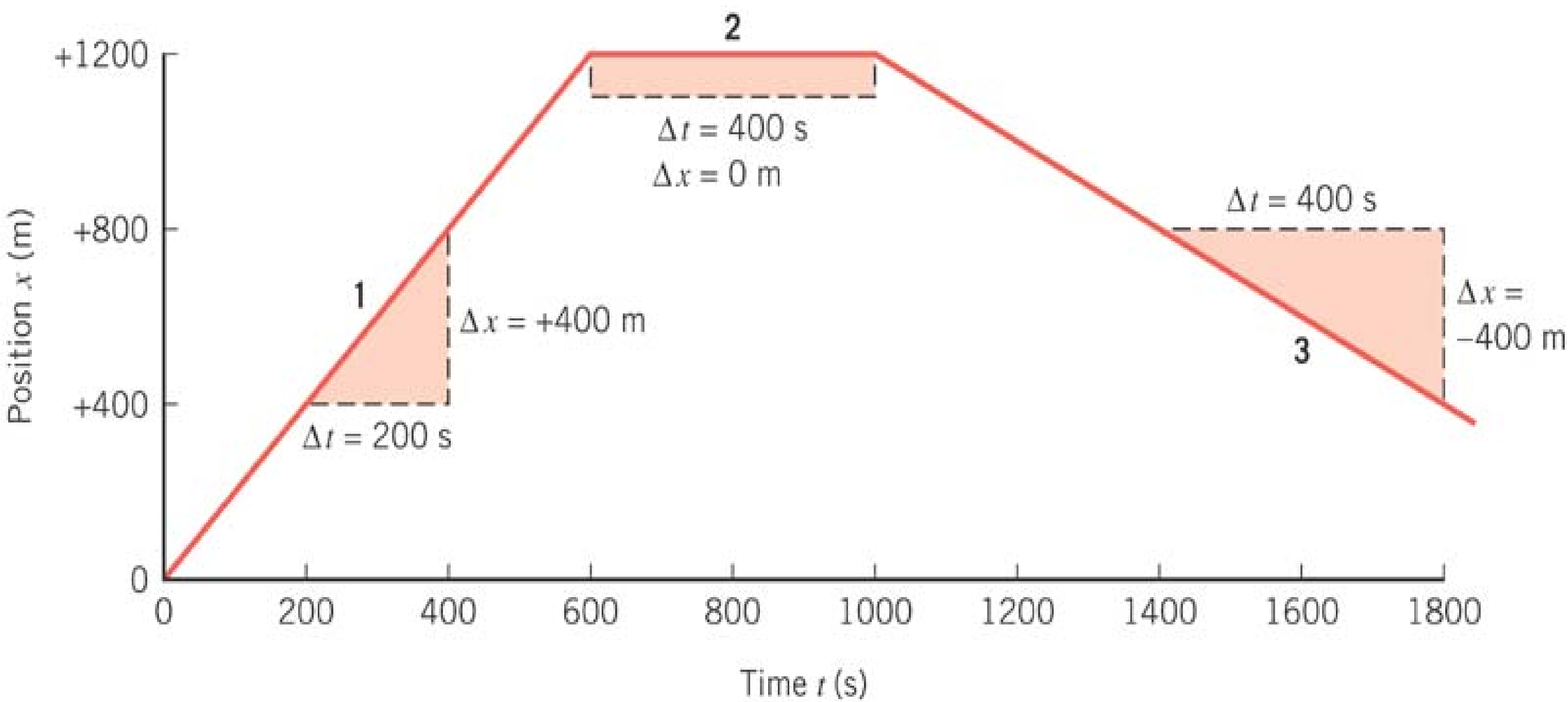
Position vs Time Plot

It is helpful to understand motions to draw them on position vs time plots.



1. Running at a constant velocity (go from $x=0$ to $x=x_1$ in t_1, Displacement is $+ x_1$ in t_1 time interval)
2. Velocity is 0 (go from x_1 to x_1 no matter how much time changes)
3. Running at a constant velocity but in the reverse direction as the step 1 above. (go from x_1 to $x=0$ in t_3-t_2 time interval, Displacement is $- x_1$ in t_3-t_2 time interval)

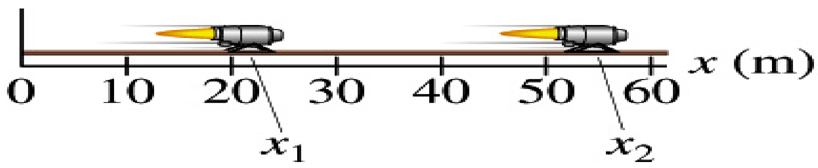
Does this motion physically make sense?

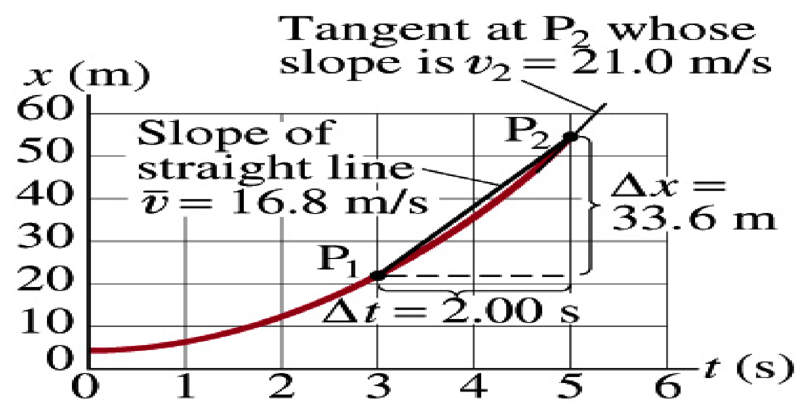


2
+1200
Δt = 400 s
Δx = 0 m
Δt = 400 s
+800
1
Δx = +400 m
Δx =
−400 m
3
+400
Δt = 200 s
Position x (m)
0
0
200
400
600
800
1000
1200
1400
1600
1800
Time t (s)

Example

A jet engine moves along a track. Its position as a function of time is given by the equation $x = At^2 + B$ where A=2.10m/s^2 and B=2.80m.





(a) Determine the displacement of the engine during the interval from t_1=3.00s to t_2=5.00s.

$$x_1 = x_{t_1=3.00} = 2.10 \times (3.00)^2 + 2.80 = 21.7m \qquad x_2 = x_{t_2=5.00} = 2.10 \times (5.00)^2 + 2.80 = 55.3m$$

Displacement is, therefore:

$$\Delta x = x_2 - x_1 = 55.3 - 21.7 = +33.6(m)$$

(b) Determine the average velocity during this time interval.

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{33.6}{5.00 - 3.00} = \frac{33.6}{2.00} = 16.8(m/s)$$

Displacement, Velocity and Speed

Displacement

$$\Delta x \equiv x_f - x_i$$

Average velocity

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Average speed

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

Instantaneous velocity

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

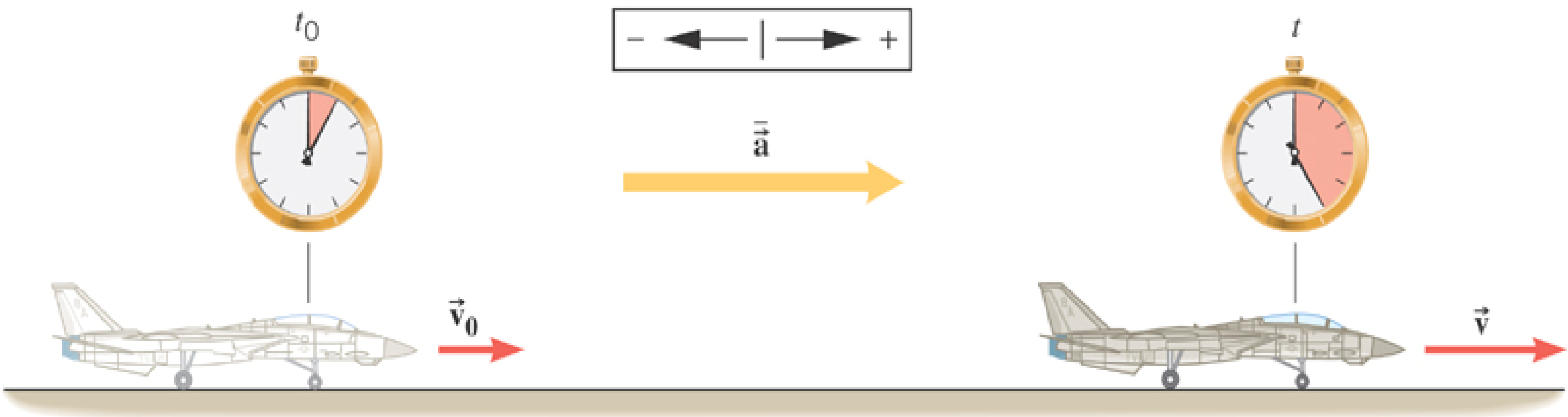
Instantaneous speed

$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right|$$

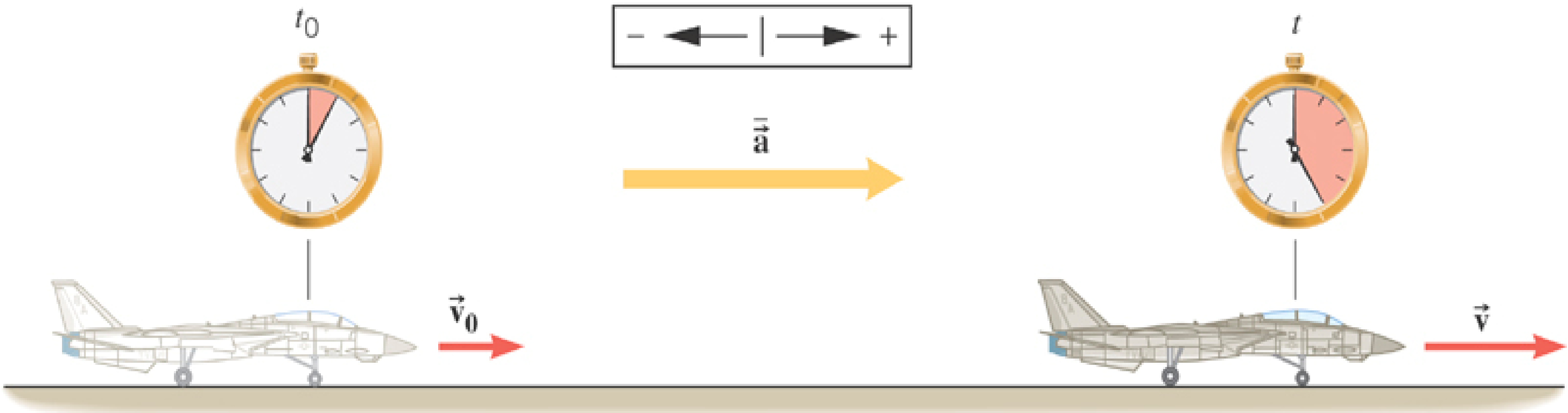
Acceleration

Change of velocity in time (what kind of quantity is this?) **Vector!!**

The notion of ***acceleration*** emerges when a change in velocity is combined with the time during which the change occurs.



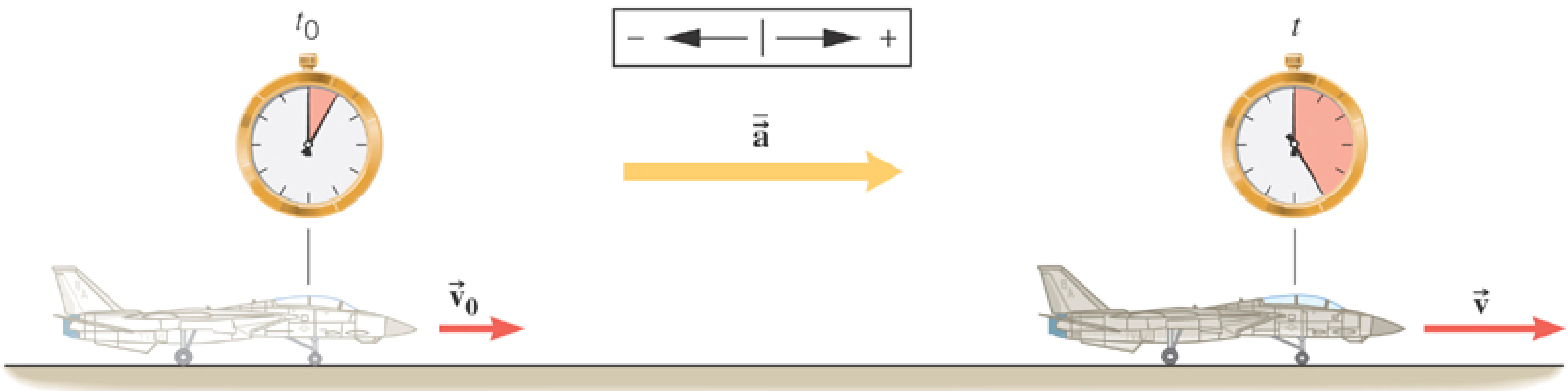
Definition of Average Acceleration



$$\overline{\vec{a}} \equiv \frac{\vec{v} - \vec{v}_0}{t - t_0} = \frac{\Delta \vec{v}}{\Delta t}$$

Ex. 3: Acceleration and Increasing Velocity

Determine the average acceleration of the plane.



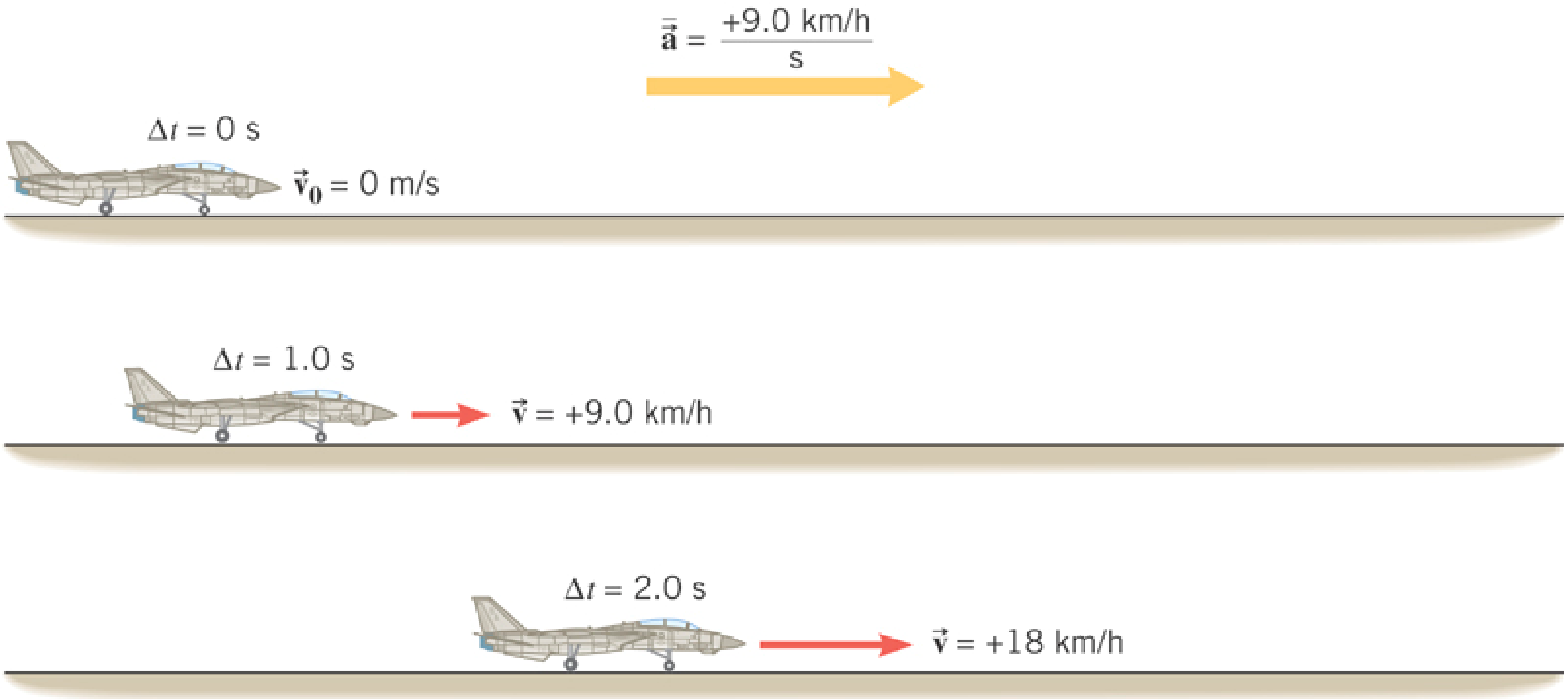
$t_o = 0\ \text{s} \quad \vec{\mathbf{v}}_o = 0\,\text{m/s}$

$t = 29\ \text{s} \quad \vec{\mathbf{v}} = 260\,\text{km/h}$

$$\bar{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{260\,\text{km/h} - 0\,\text{km/h}}{29\ \text{s} - 0\ \text{s}} = +9.0\frac{\text{km/h}}{\text{s}} =$$

$$+\frac{9 \cdot 1000m}{3600s \cdot s} = 2.5\,m/s^2$$

2.3 Acceleration



Ex.4 Acceleration and Decreasing Velocity

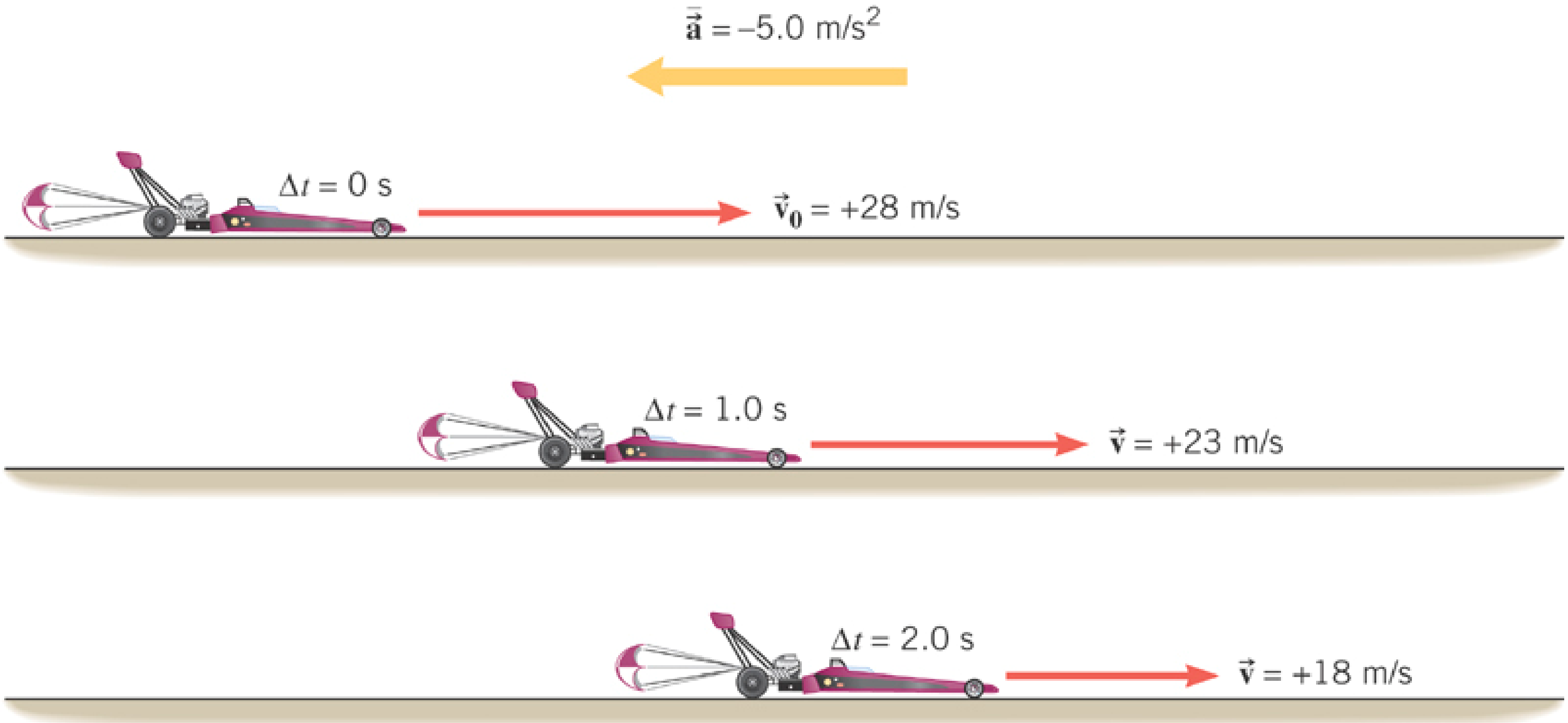
A drag racer crosses the finish line, and the driver deploys a parachute and applies the brakes to slow down. The driver begins slowing down when t_0=9.0s and the car's velocity is v_0=+28m/s. When t=12.0s, the velocity has been reduced to v=+13m/s. What is the average acceleration of the dragster?

$$\overline{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} =$$

$$\frac{13\,\mathrm{m/s} - 28\,\mathrm{m/s}}{12\ \mathrm{s} - 9\ \mathrm{s}} =$$

$$-5.0\,\mathrm{m/s}^2$$

(b)



Deceleration is an Acceleration in the opposite direction!!

Acceleration Summary

Change of velocity in time (what kind of quantity is this?) **Vector!!**

- Average acceleration:

$$\bar{\vec{a}} \equiv \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta \vec{v}}{\Delta t} \quad \text{analogous to} \quad \bar{\vec{v}} \equiv \frac{\vec{x}_f - \vec{x}_i}{t_f - t_i} = \frac{\Delta \vec{x}}{\Delta t}$$

- Instantaneous acceleration:

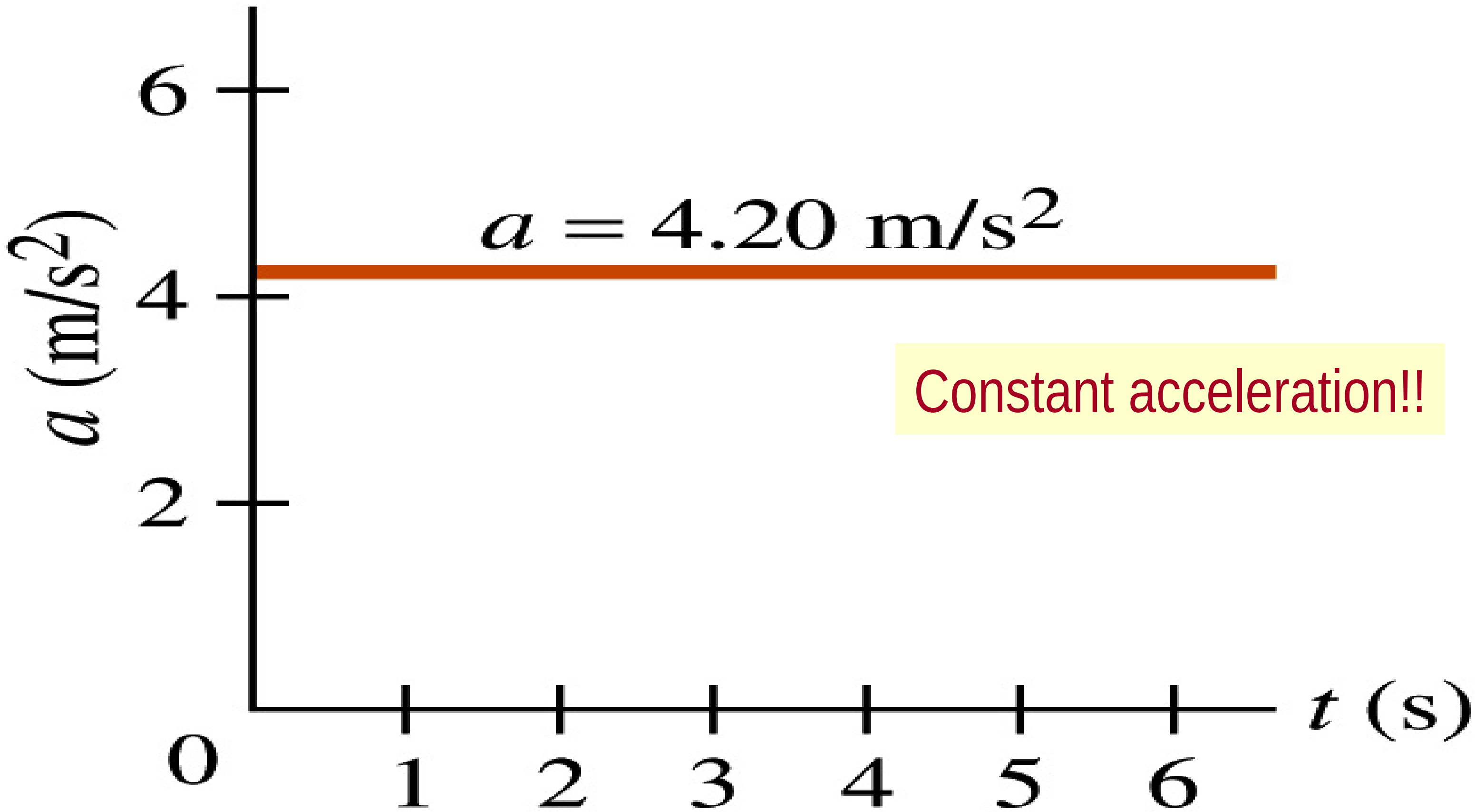
$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \quad \text{analogous to} \quad v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Meanings of the Acceleration

- When an object is moving at a constant velocity ($v=v_0$), there is no acceleration ($a=0$)
  - Is there any net acceleration when an object is not moving?
- When an object speeds up as time goes on, ($v=v(t)$), acceleration has the same sign as v.
- When an object slows down as time goes on, ($v=v(t)$), acceleration has the opposite sign as v.
- Is there acceleration if an object moves in a constant speed but changes direction? YES!!

One Dimensional Motion

- Let's start with the simplest case: the acceleration is constant ($a=a_0$)



Equations of Kinematics

- Five kinematic variables
  - displacement, x
  - acceleration (constant), a
  - final velocity (at time t), v
  - initial velocity, v_o
  - elapsed time, t

One Dimensional Kinematic Equation

- Let's start with the simplest case: the acceleration is constant ($a=a_0$)
- Using definitions of average acceleration and velocity, we can derive equation of motion (description of motion, position *wrt* time)

$$\bar{a} = \frac{v - v_0}{t - t_0} = a$$

Let $t=t$ and $t_0=0$

$$a = \frac{v - v_0}{t}$$

Solve for v

$$v = v_0 + at$$

One Dimensional Kinematic Equation

For constant acceleration, simple numeric average

$$\bar{v} = \frac{v + v_0}{2} = \frac{2v_0 + at}{2} = v_0 + \frac{1}{2}at$$

$$\bar{v} = \frac{x - x_0}{t - t_0}$$

Let $t=t$ and $t_0=0$

$$\bar{v} = \frac{x - x_0}{t}$$

Solve for x

$$x = x_0 + \bar{v}t$$

Resulting Equation of Motion becomes

$$x = x_0 + \bar{v}t = x_0 + v_0 t + \frac{1}{2}at^2$$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v(t) = v_0 + at$$

Velocity as a function of time

$$x - x_0 = \frac{1}{2}\bar{v}t = \frac{1}{2}(v + v_0)t$$

Displacement as a function of velocities and time

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Displacement as a function of time, velocity, and acceleration

$$v^2 = v_0^2 + 2a(x - x_0)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinematic equations, depending on the information given to you in specific physical problems!!

How do we solve a problem using a kinematic formula under constant acceleration?

- Identify what information is given in the problem.
  - Initial and final velocity?
  - Acceleration?
  - Distance, initial position or final position?
  - Time?
- Convert the units of all quantities to SI units to be consistent.
- Identify what the problem wants
- Identify which kinematic formula is appropriate and easiest to solve for what the problem wants.
  - Frequently multiple formulae can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that can be easiest to solve.
- Solve the equations for the quantity or quantities wanted.

Example 8 An Accelerating Spacecraft

A spacecraft is traveling with a velocity of +3250 m/s. Suddenly the retrorockets are fired, and the spacecraft begins to slow down with an acceleration whose magnitude is 10.0 m/s^2. What is the velocity of the spacecraft when the displacement of the craft is +215 km, relative to the point where the retrorockets began firing?

x	a	v	v_o	t
+215000 m	-10.0 m/s^2	?	+3250 m/s	

x	a	v	v_o	t
+215000 m	-10.0 m/s^2	?	+3250 m/s	

$$v^2 = v_o^2 + 2ax$$

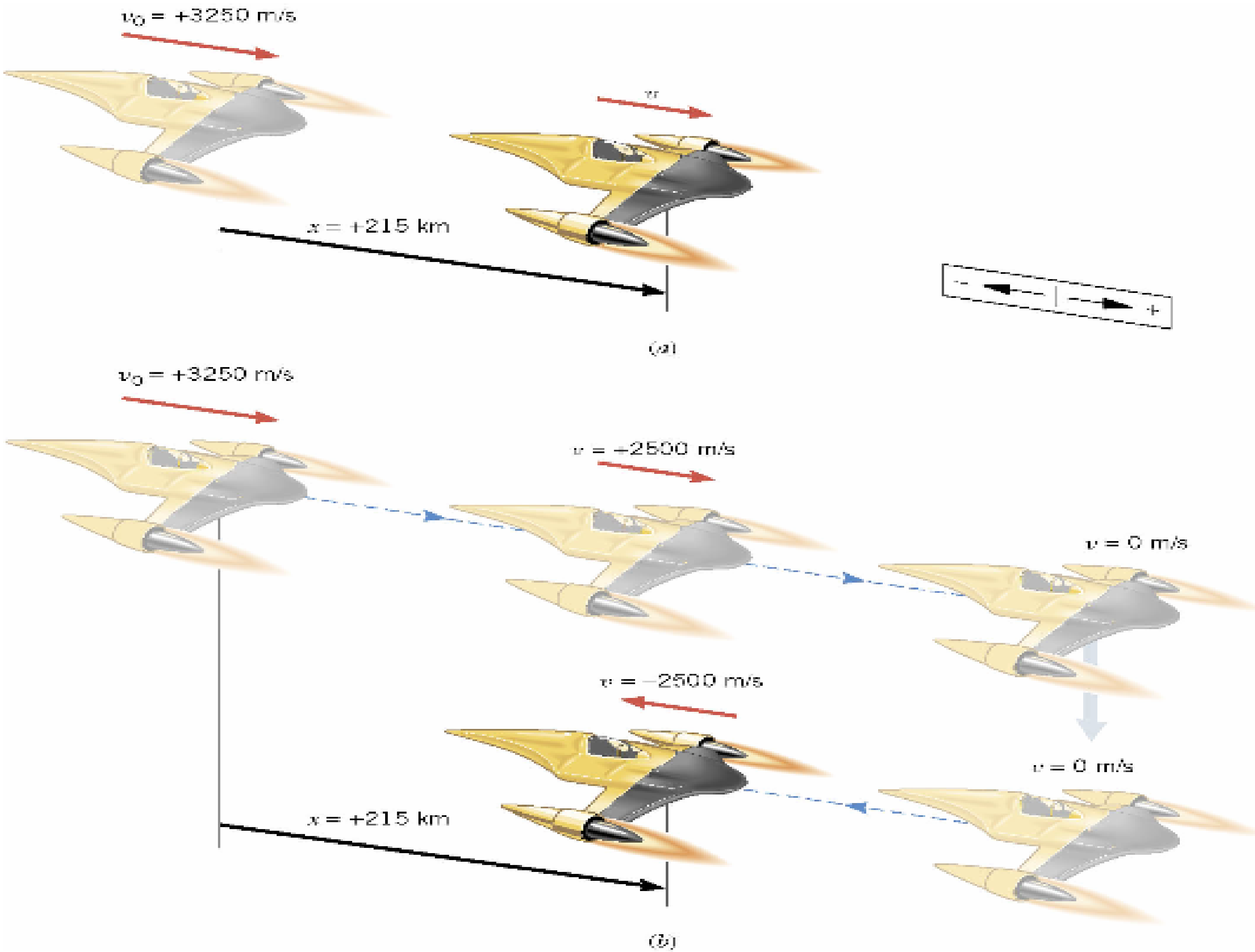
Solve for v

$$v = \pm\sqrt{v_o^2 + 2ax}$$

$$v = \pm\sqrt{(3250\,\mathrm{m/s})^2 + 2\left(-10.0\,\mathrm{m/s^2}\right)(215000\ \mathrm{m})}$$

$$= \pm 2500\,\mathrm{m/s}$$

What do two opposite signs mean?



v0 = +3250 m/s
v
x = +215 km
(a)
v0 = +3250 m/s
v = +2500 m/s
v = 0 m/s
v = −2500 m/s
v = 0 m/s
x = +215 km
(b)

Example

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? ⇨ As long as it takes for it to crumple.

The initial speed of the car is $$v_{xi} = 100km/h = \frac{100000m}{3600s} = 28m/s$$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinematic formula $$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

The acceleration is $$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2\left(x_f - x_i\right)} = \frac{0-\left(28m/s\right)^2}{2\times1m} = -390m/s^2$$

Thus the time for air-bag to deploy is $$t = \frac{v_{xf} - v_{xi}}{a} = \frac{0-28m/s}{-390m/s^2} = 0.07s$$