PHYS 1441 – Section 001
Lecture #4

Monday, June 2, 2008

Dr. Jaehoon Yu

- One Dimensional Motion:
  - Free Fall
- Two dimensional Motion
  - Motion under constant acceleration
  - Vector recap
  - Projectile Motion
  - Maximum ranges and heights

Today's homework is homework #3, due 9pm, Friday, June 6!!

Announcements

- E-mail Distribution list
  - 40 out of 55 registered as of this morning!!
  - 3 points extra credit if done by next Monday, June 2
- First term exam
  - 8 – 10am, this Wednesday, June 4, in SH103
  - Covers CH1 – what we finish next Tomorrow + appendices
- No after-class office hour today

Reminder: Special Problems for Extra Credit

- Derive the quadratic equation for $yx^2-zx+v=0$
  ➔ 5 points
- Derive the kinematic equation $v^2 = v_0^2 + 2a(x-x_0)$ from first principles and the known kinematic equations ➔ 10 points
- You must <u>show your work in detail</u> to obtain the full credit
- Due tomorrow, Tuesday, June 3

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v(t) = v_0 + at$$

Velocity as a function of time

$$x - x_0 = \frac{1}{2}\bar{v}t = \frac{1}{2}(v + v_0)t$$

Displacement as a function of velocities and time

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Displacement as a function of time, velocity, and acceleration

$$v^2 = v_0^2 + 2a(x - x_0)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinematic equations, depending on the information given to you in specific physical problems!!

Free Fall

- Free fall is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
  - A motion under constant acceleration
  - All kinematic formulae we learned can be used to solve for falling motions.

Down to the center of the Earth!

- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The magnitude of the gravitational acceleration is $|g|=9.80\text{m/s}^2$ on the surface of the Earth, most of the time.
- The direction of gravitational acceleration is <u>ALWAYS</u> toward the center of the earth, which we normally call (-y); when the vertical directions are indicated with the variable "y"
- Thus the correct denotation of the gravitational acceleration on the surface of the earth is $g=-9.80\text{m/s}^2$ $g = -32.2\,\text{ft}/\text{s}^2$

Note the negative sign!!

1D Motion Under Gravitational Acceleration

$$\boxed{v(t) = v_0 + gt}$$

Velocity as a function of time

$$\boxed{y - y_0 = \frac{1}{2}\bar{v}t = \frac{1}{2}(v + v_0)t}$$

Displacement as a function of velocities and time

$$\boxed{y = y_0 + v_0 t + \frac{1}{2}gt^2}$$

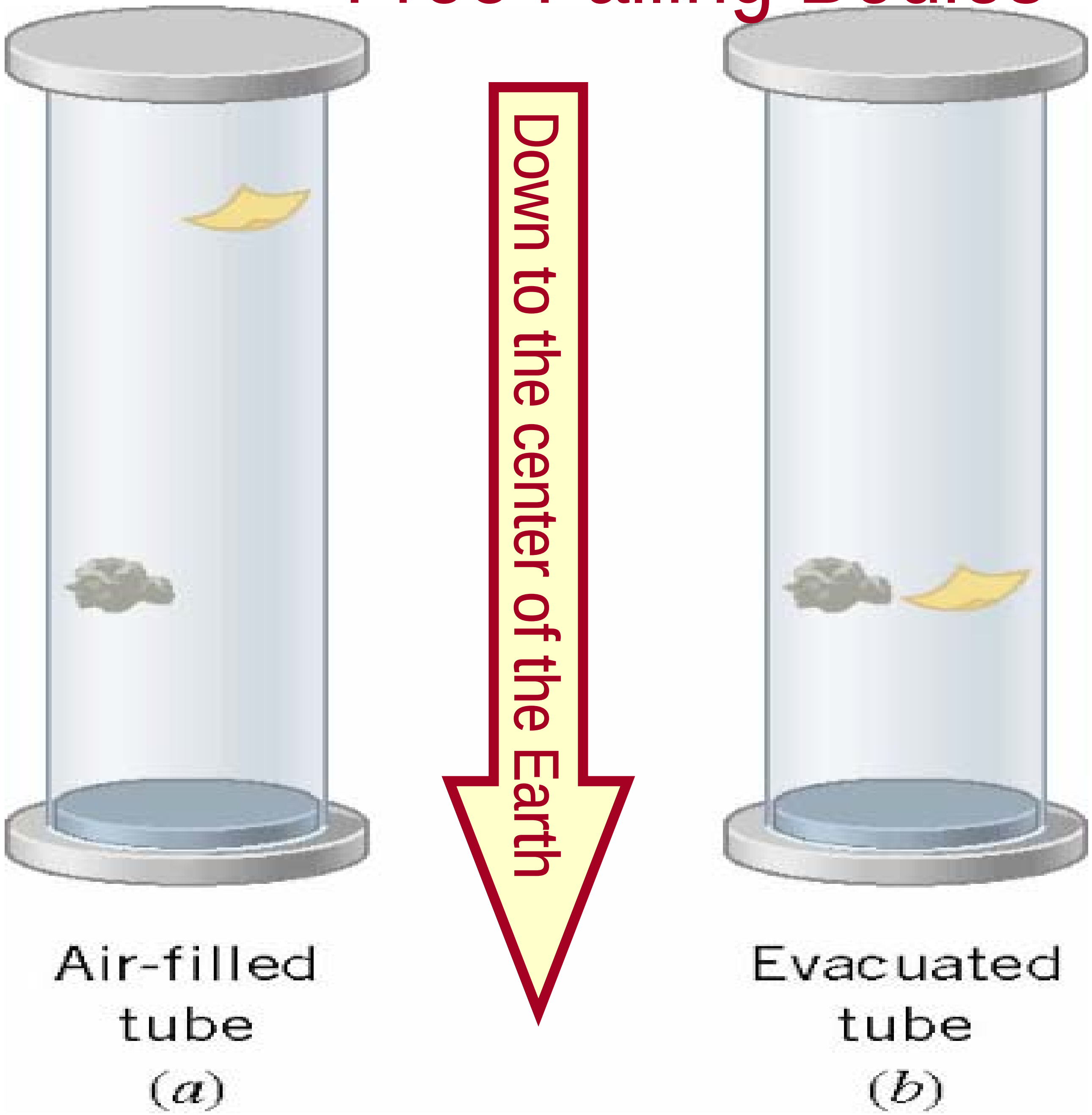
Displacement as a function of time, velocity, and acceleration

$$\boxed{v^2 = v_0^2 + 2g(y - y_0)}$$

Velocity as a function of Displacement and acceleration

All you need to do is to replace a with g=-9.8m/s^2.

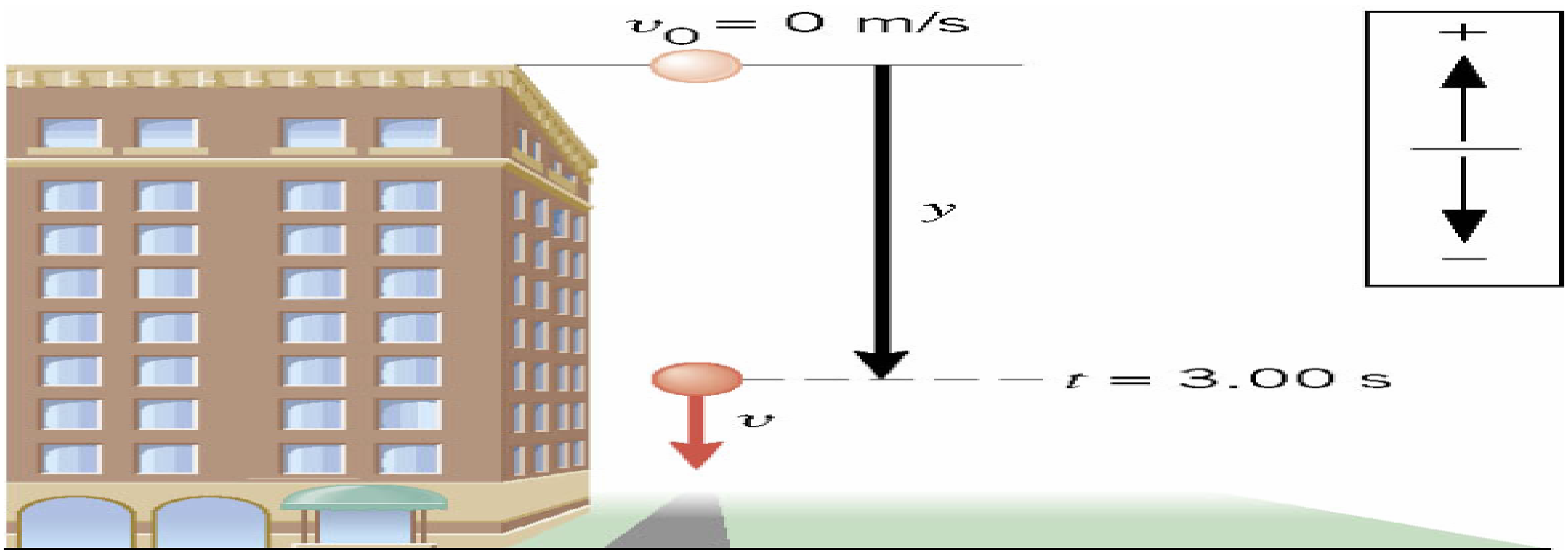
Free Falling Bodies



$$|\vec{g}| = 9.80\,\mathrm{m/s^2}$$

Example 10

A stone is dropped from the top of a tall building. After 3.00s of free fall, what is the displacement *y* of the stone?



y	a	v	v_o	t
?	-9.8 m/s^2		0 m/s	3.00 s

y	a	v	v_o	t
?	-9.80 m/s^2		0 m/s	3.00 s

Which formula would you like to use?

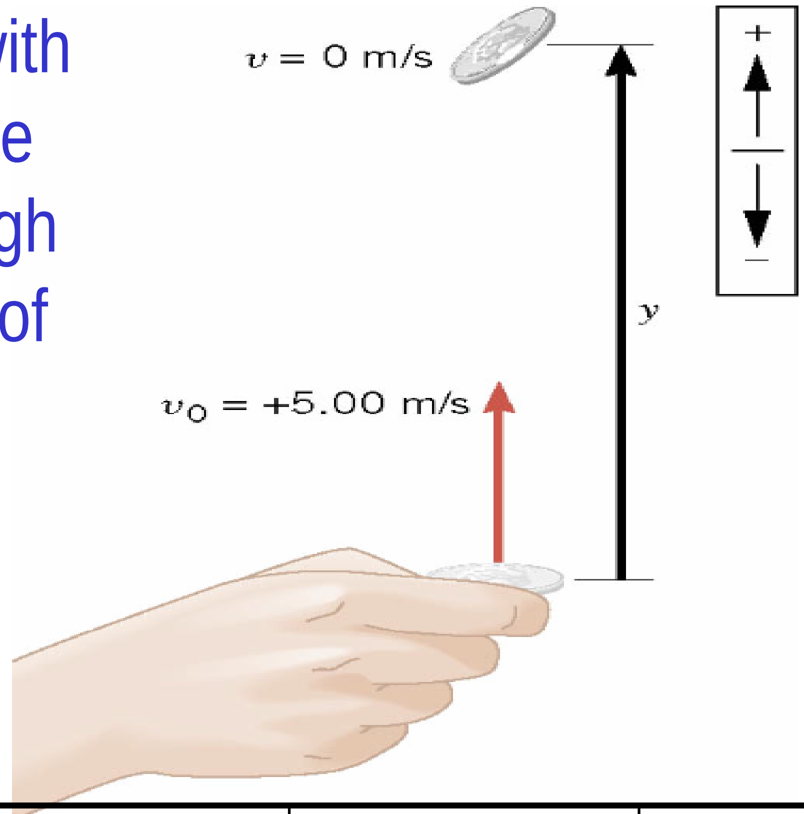
$$y = v_o t + \tfrac{1}{2} a t^2 = v_o t + \tfrac{1}{2} g t^2$$

$$= (0\,\mathrm{m/s})(3.00\ \mathrm{s}) + \tfrac{1}{2}\left(-9.80\,\mathrm{m/s^2}\right)(3.00\ \mathrm{s})^2$$

$$= -44.1\,\mathrm{m}$$

Example 12 How high does it go?

The referee tosses the coin up with an initial speed of 5.00m/s. In the absence if air resistance, how high does the coin go above its point of release?



y	a	v	v_o	t
?	-9.8 m/s^2	0 m/s	+ 5.00 m/s	

y	a	v	v_o	t
?	-9.80 m/s^2	0 m/s	+5.00 m/s	

$$v^2 = v_o^2 + 2ay$$

Solve for y

$$y = \frac{v^2 - v_o^2}{2a}$$

$$y = \frac{v^2 - v_o^2}{2a} = \frac{(0\,\mathrm{m/s})^2 - (5.00\,\mathrm{m/s})^2}{2(-9.80\,\mathrm{m/s^2})} = 1.28\ \mathrm{m}$$

Conceptual Ex. 14 Acceleration vs Velocity

- There are three parts to the motion of the coin.
  - On the way up, the coin has a vector velocity that is directed upward and has decreasing magnitude.
  - At the top of its path, the coin momentarily has zero velocity.
  - On the way down, the coin has downward-pointing velocity with an increasing magnitude.
- In the absence of air resistance, does the acceleration of the coin, like the velocity, change from one part to another?

Nope!

Example for Using 1D Kinematic Equations on a Falling object

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

What is the acceleration in this motion? $\boxed{g=-9.80m/s^2}$

(a) Find the time the stone reaches at the maximum height.

What is so special about the maximum height? V=0

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00 m/s$$ Solve for t $$t = \frac{20.0}{9.80} = 2.04s$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

(e) Find the velocity and position of the stone at t=5.00s.

Velocity

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0(m/s)$$

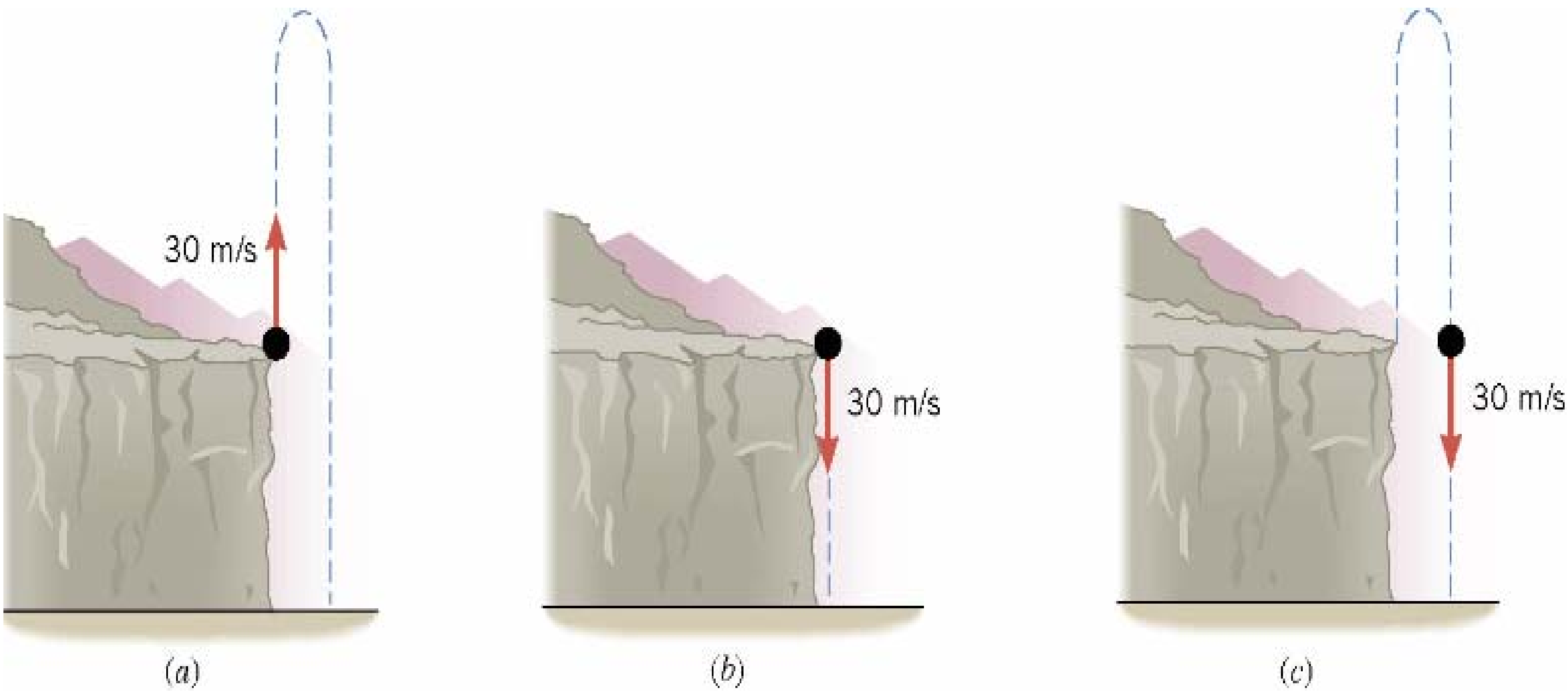
$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

Position

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$

Conceptual Example 15 Taking Advantage of Symmetry

Does the pellet in part *b* strike the ground beneath the cliff with a smaller, greater, or the same speed as the pellet in part *a*?



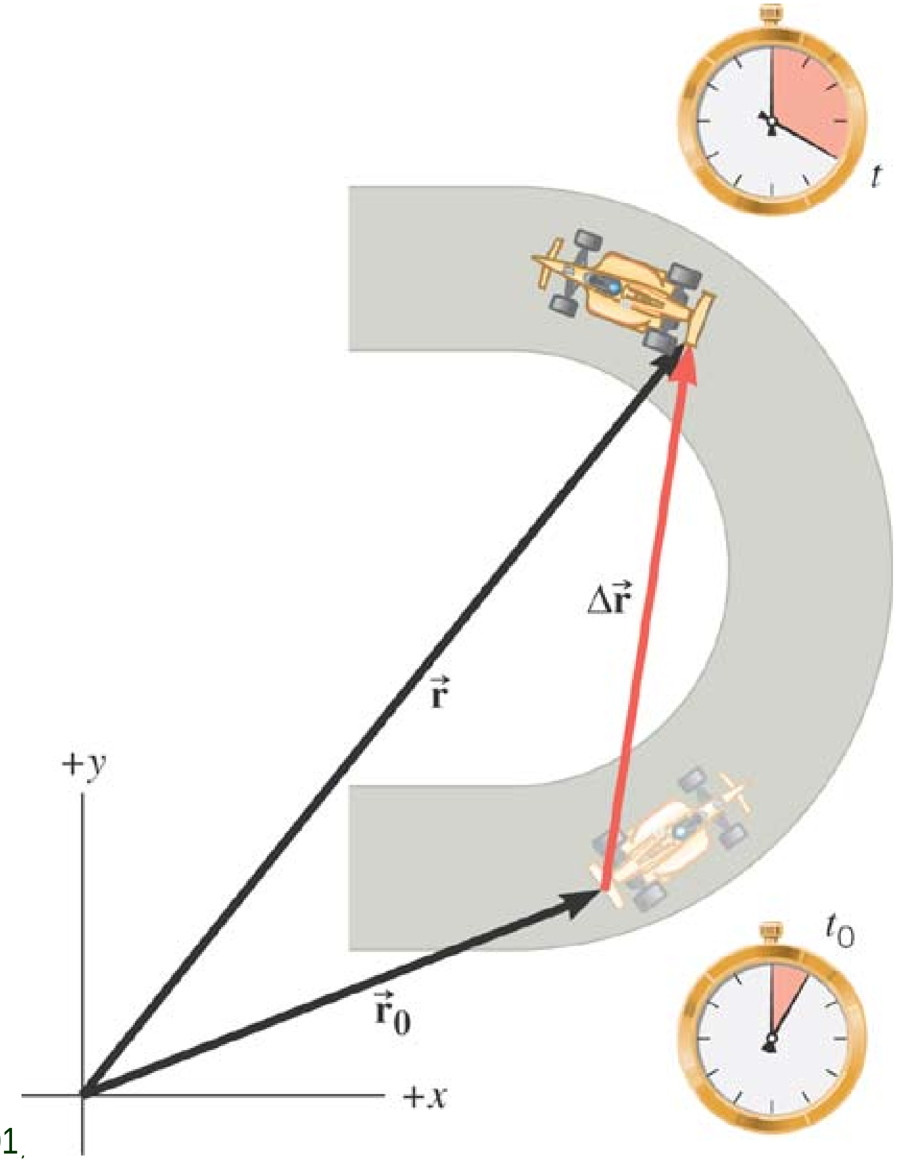
The same!!

2 Dimensional Kinematic Quantities

$\vec{\mathbf{r}}_o =$ initial position

$\vec{\mathbf{r}} =$ final position

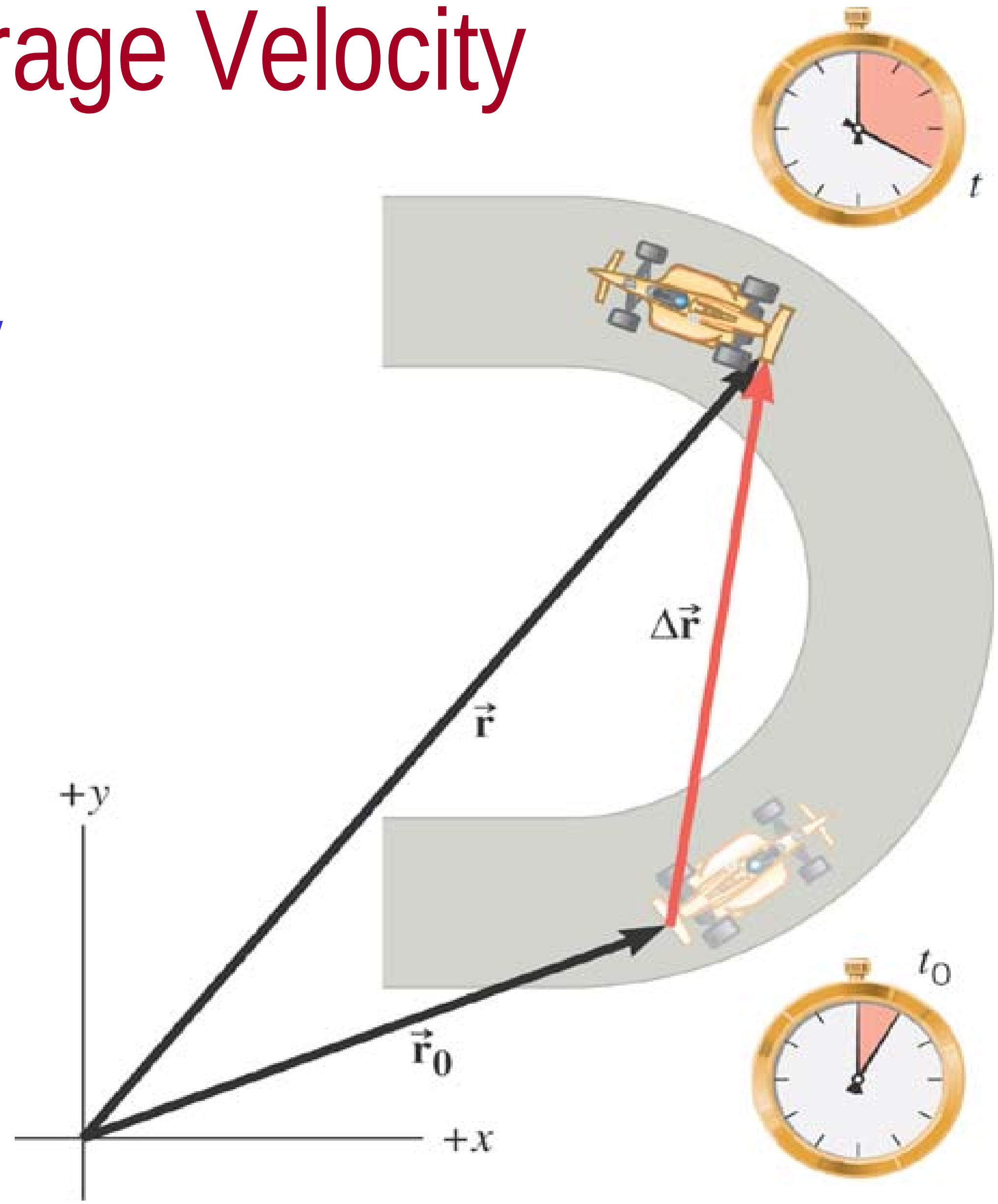
$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_o =$ displacement



2D Average Velocity

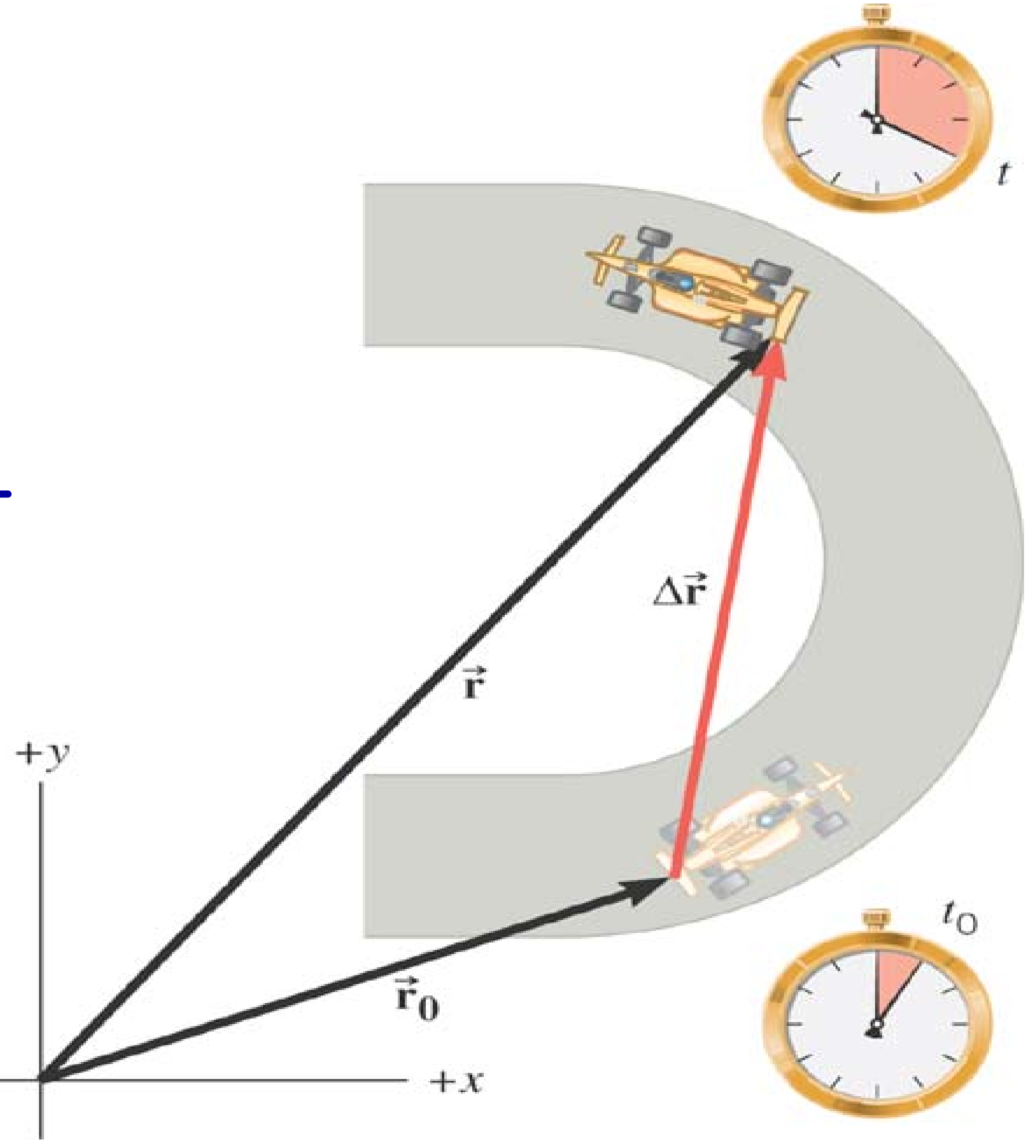
Average velocity is the displacement divided by the elapsed time.

$$\overline{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$



The *instantaneous velocity* indicates how fast the car moves and the direction of motion at each instant of time.

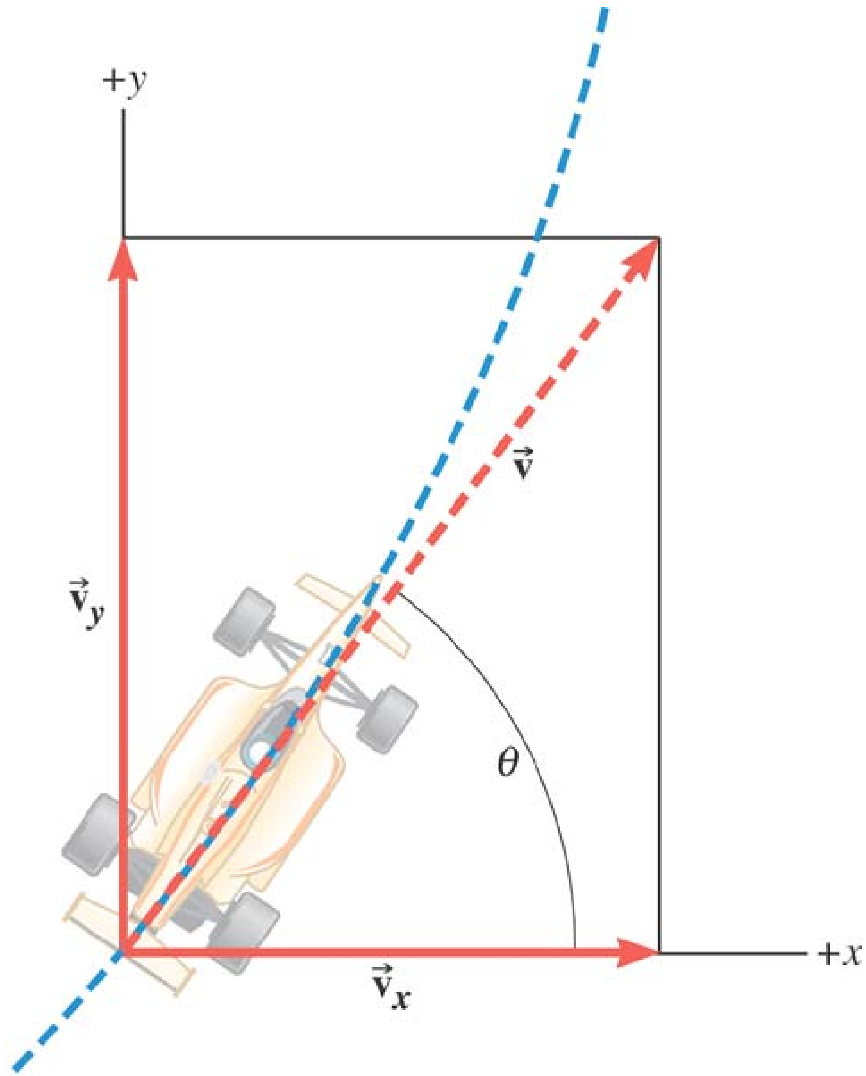
$$\overline{\vec{\mathbf{v}}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$



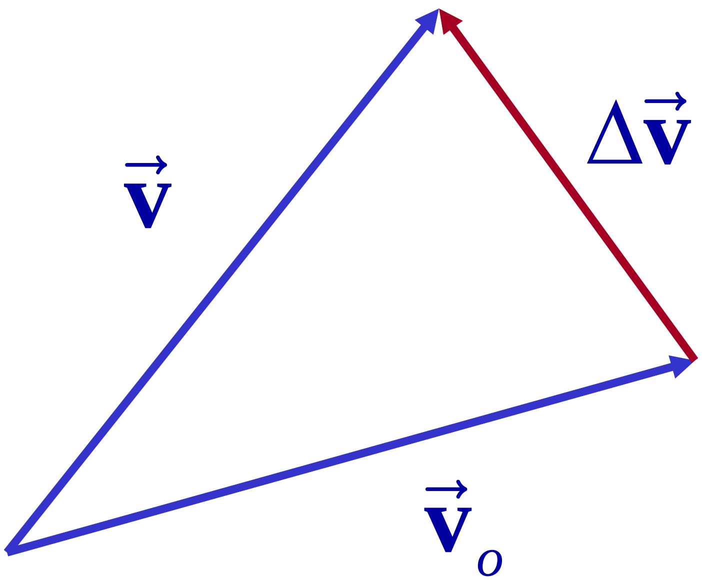
2D Instantaneous Velocity

$$\overline{\vec{\mathbf{v}}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$

Slope of the trajectory as a function of time!!



2D Average Acceleration



$$\overline{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$
- Average Acceleration $$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

How is each of these quantities defined in 1-D?

Kinematic Quantities in 1D and 2D

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$

What is the difference between 1D and 2D quantities?

Kinematic Equations

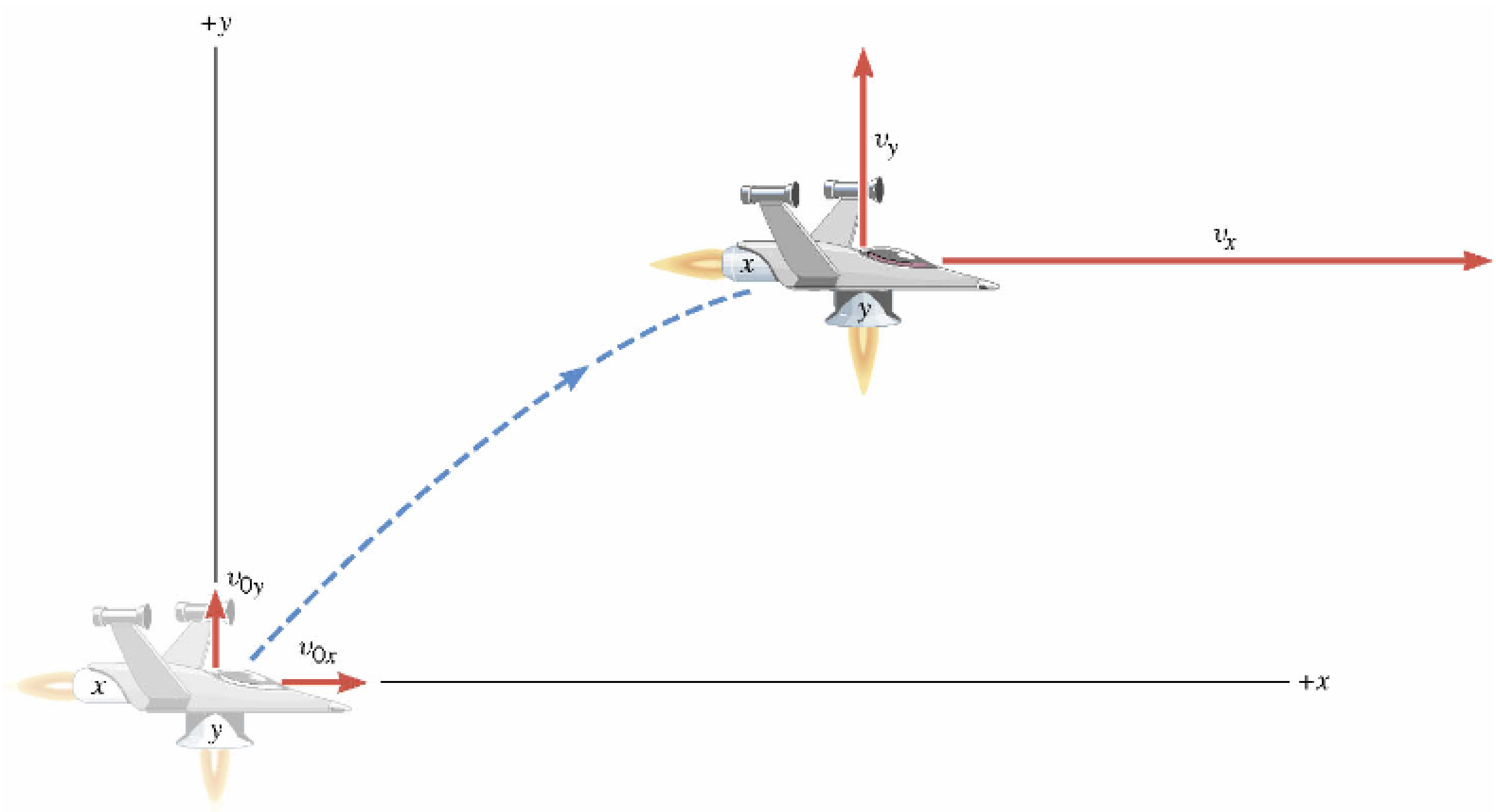
$$v = v_o + at$$

$$x = \tfrac{1}{2}\left(v_o + v\right)t$$

$$v^2 = v_o^2 + 2ax$$

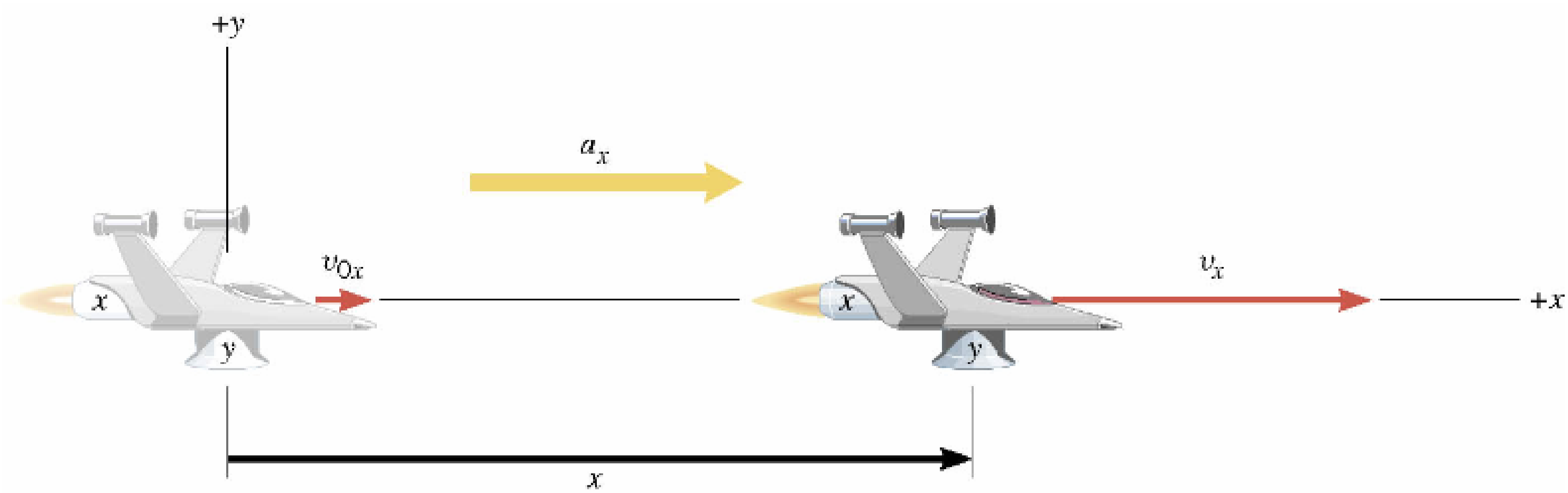
$$x = v_o t + \tfrac{1}{2}at^2$$

A Motion in 2 Dimension



This is a motion that could be viewed as two motions combined into one.

Motion in horizontal direction (x)



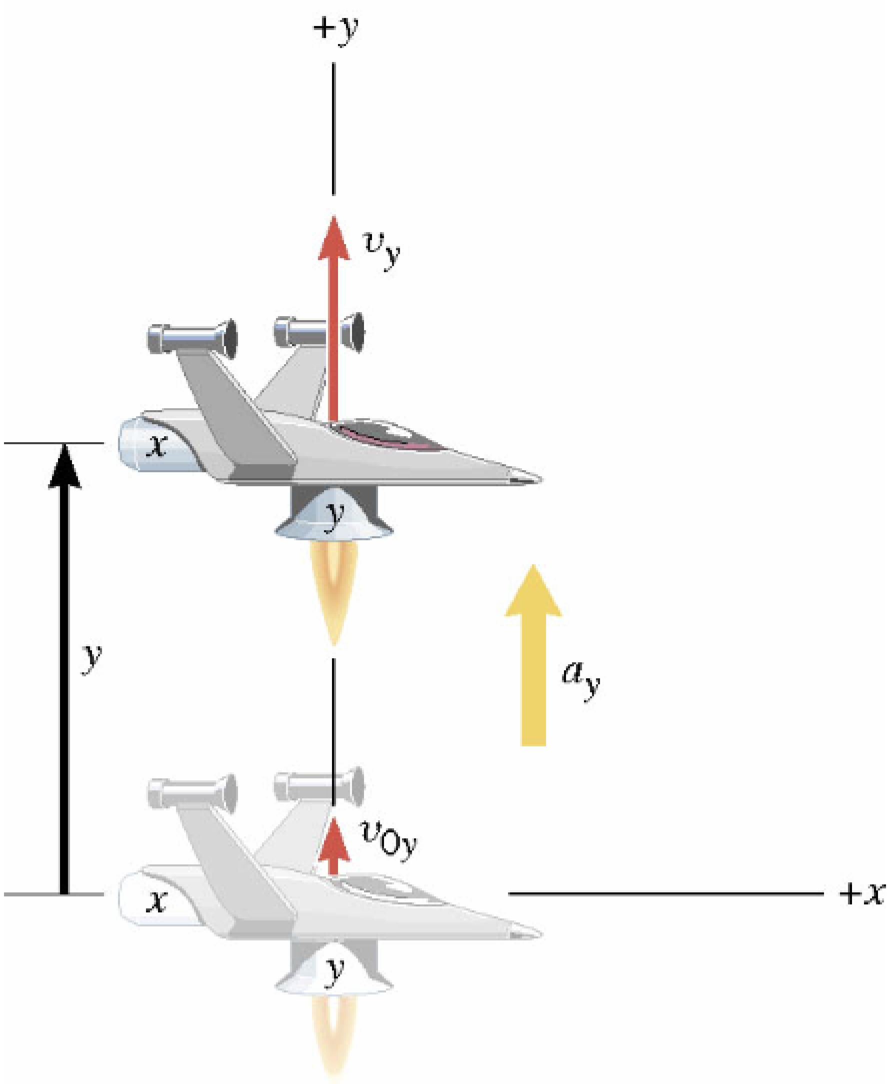
$$v_x = v_{xo} + a_x t$$

$$x = \tfrac{1}{2}\left(v_{xo} + v_x\right) t$$

$$v_x^2 = v_{xo}^2 + 2a_x x$$

$$x = v_{xo} t + \tfrac{1}{2} a_x t^2$$

Motion in vertical direction (y)



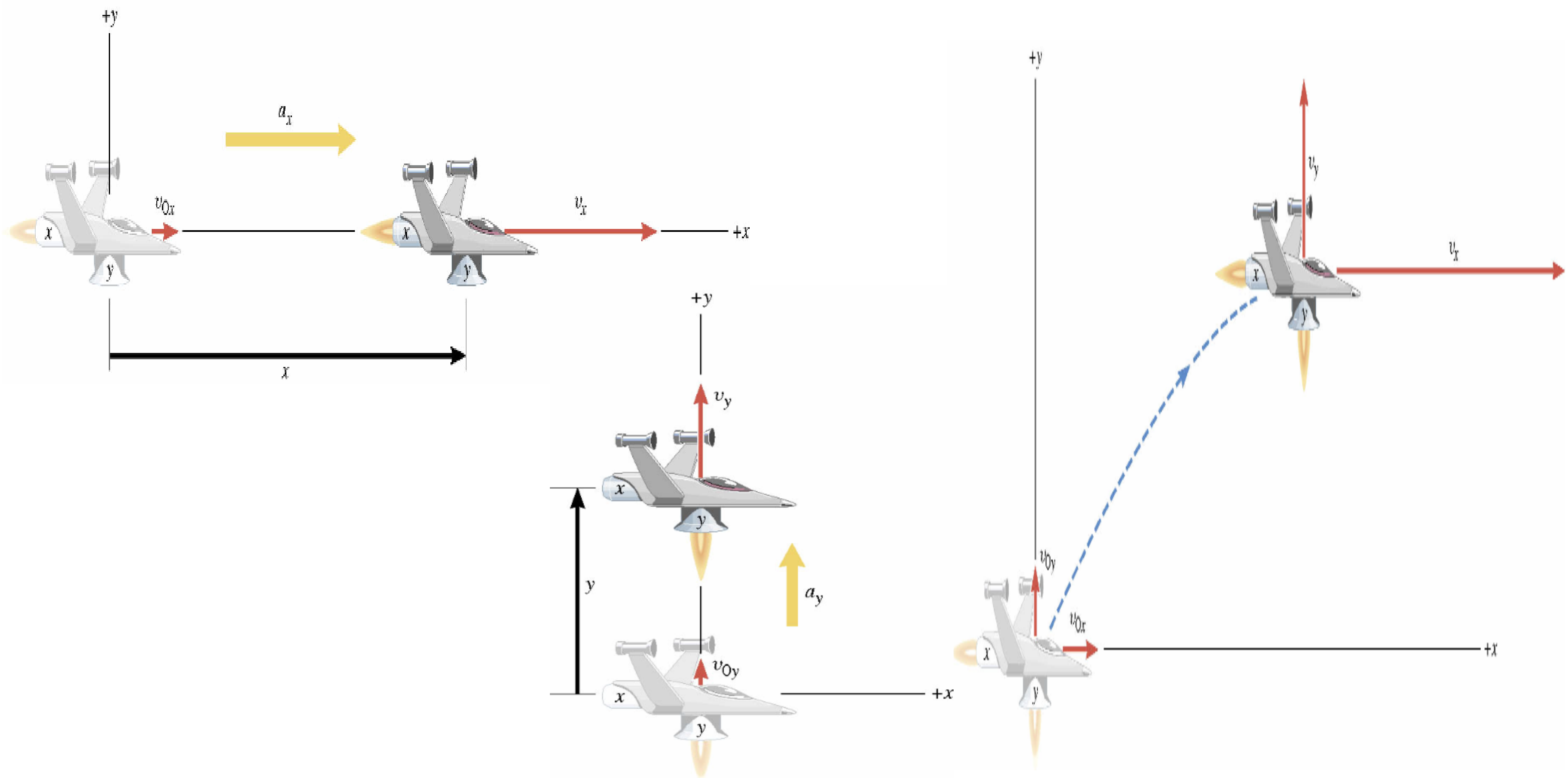
$$v_y = v_{yo} + a_y t$$

$$y = \tfrac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$y = v_{yo}t + \tfrac{1}{2}a_y t^2$$

Motion in 2 Dimension



Imagine you add the two 1 dimensional motions on the left. It would make up a one 2 dimensional motion on the right.

Kinematic Equations in 2-Dim

x-component

$$v_x = v_{xo} + a_x t$$

$$x = \tfrac{1}{2}\left(v_{xo} + v_x\right)t$$

$$v_x^2 = v_{xo}^2 + 2a_x x$$

$$x = v_{xo}t + \tfrac{1}{2}a_x t^2$$

y-component

$$v_y = v_{yo} + a_y t$$

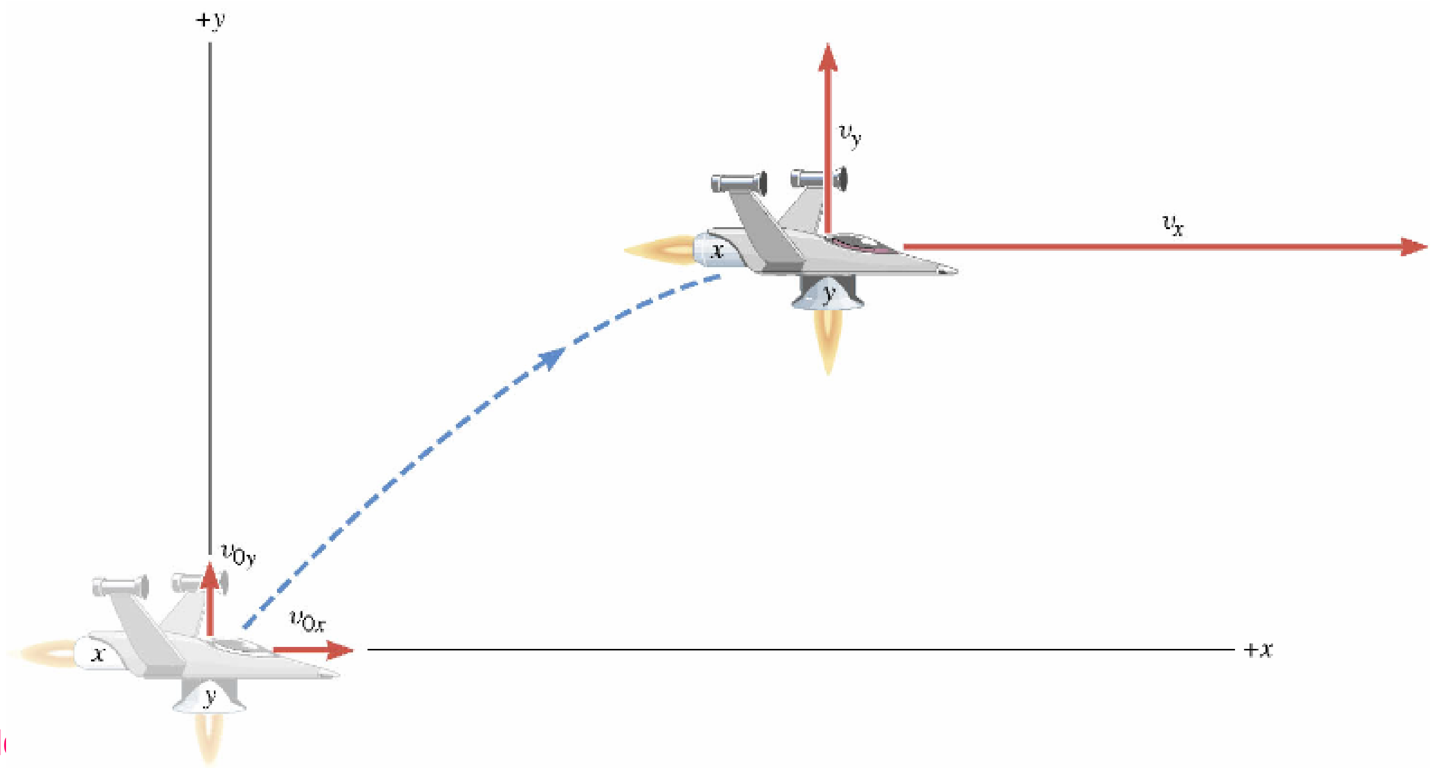
$$y = \tfrac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$y = v_{yo}t + \tfrac{1}{2}a_y t^2$$

Ex.1 A Moving Spacecraft

In the *x* direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.



How do we solve this problem?

1. Visualize the problem ➔ Make a drawing.
2. Decide which directions are to be called positive (+) and negative (-).
3. Write down the values that are given for any of the five kinematic variables associated with each direction.
4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y separately.* Select the appropriate equation.
5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.
6. Keep in mind that there may be two possible answers to a kinematics problem.

Ex.1 continued

In the x direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the y direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) x and v_x, (b) y and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22.0 m/s	7.0 s

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14.0 m/s	7.0 s

First, the motion in x-direciton…

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$x = v_{ox}t + \tfrac{1}{2}a_x t^2$$

$$= (22\,\mathrm{m/s})(7.0\ \mathrm{s}) + \tfrac{1}{2}\left(24\,\mathrm{m/s^2}\right)(7.0\ \mathrm{s})^2 = +740\ \mathrm{m}$$

$$v_x = v_{ox} + a_x t$$

$$= (22\,\mathrm{m/s}) + \left(24\,\mathrm{m/s^2}\right)(7.0\ \mathrm{s}) = +190\,\mathrm{m/s}$$

Now, the motion in y-direction…

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

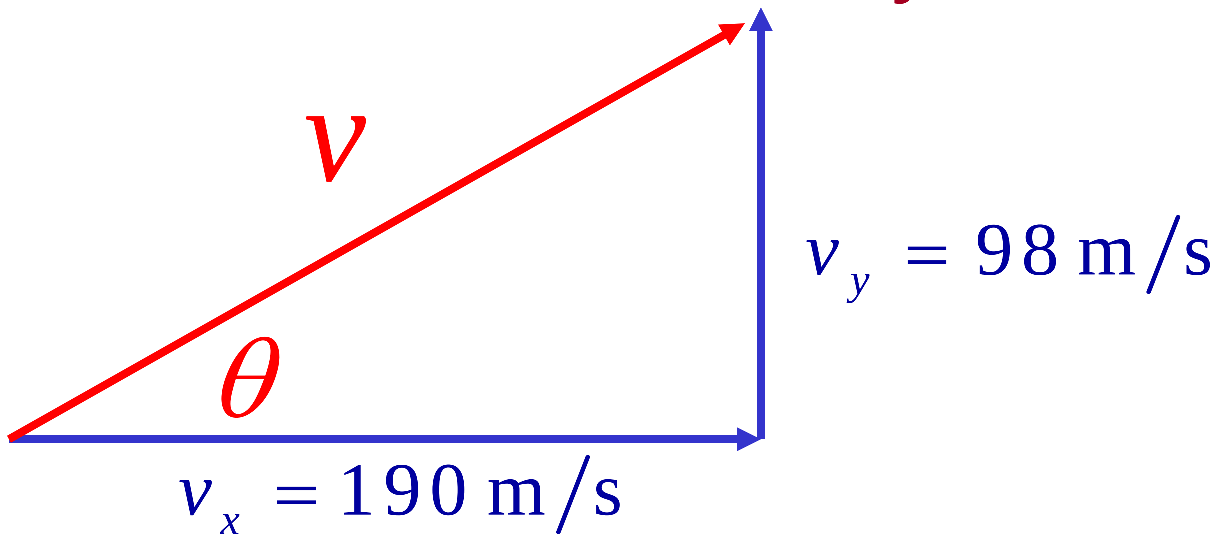
$$y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

$$= (14\,\text{m/s})(7.0\text{ s}) + \tfrac{1}{2}(12\,\text{m/s}^2)(7.0\text{ s})^2 = +390\text{ m}$$

$$v_y = v_{oy} + a_y t$$

$$= (14\,\text{m/s}) + (12\,\text{m/s}^2)(7.0\text{ s}) = +98\,\text{m/s}$$

The final velocity...



$$v = \sqrt{(190\,\mathrm{m/s})^2 + (98\,\mathrm{m/s})^2} = 210\,\mathrm{m/s}$$

$$\theta = \tan^{-1}(98/190) = 27^\circ$$

A vector can be fully described when the magnitude and the direction are given. Any other way to describe it?

Yes, you are right! Using components and unit vectors!!

$$\vec{v} = v_x\vec{i} + v_y\vec{j} = \left(190\vec{i} + 98\vec{j}\right) m/s$$

If we visualize the motion…