# PHYS 1441 – Section 001
# Lecture #5

*Tuesday, June 3, 2008*
*Dr. Jaehoon Yu*

- Motion in Two Dimensions
  - Projectile Motion
  - Maximum ranges and heights
- Newton's Laws of Motion
  - Force
  - Newton's first law: Inertia & Mass
  - Newton's second law
  - Newton's third law of motion

# Announcements

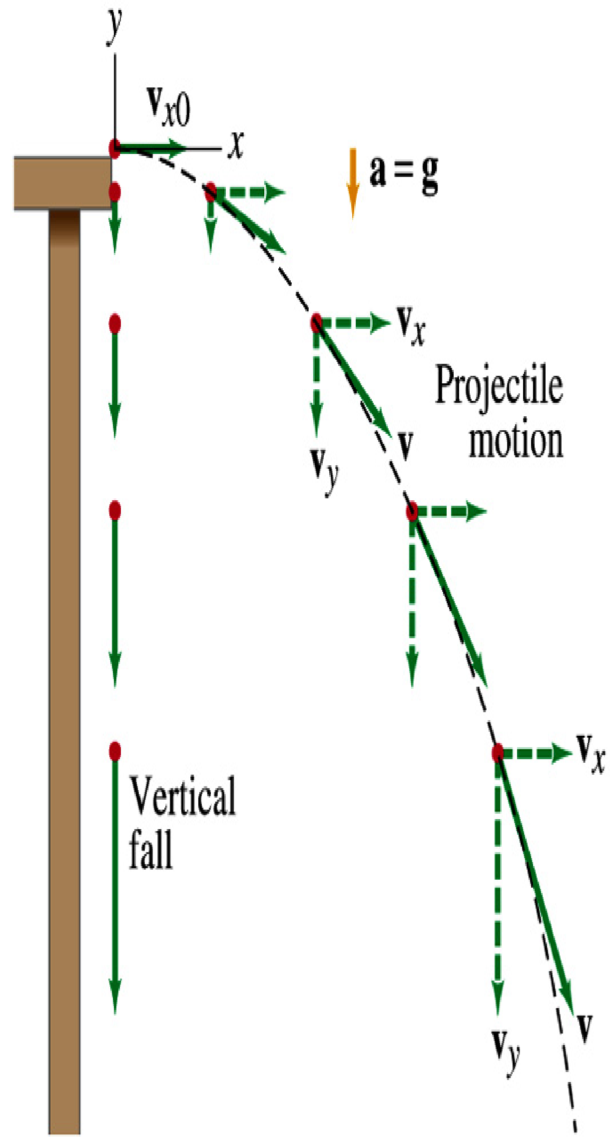
- E-mail Distribution list
  - 42 out of 55 registered as of this morning!!
- Quiz results
  - Class average: 6.5/11
    - Equivalent to 59.1/100
  - Top score: 11/11
- First term exam
  - 8 – 10am, tomorrow, Wednesday, June 4, in SH103
  - Covers CH1 – CH4.4 + appendices
  - Practice test posted on the class web page
    - No answer keys will be posted
- Quiz next Monday, June 9

# Special Project for Extra Credit

- Show that a projectile motion's trajectory is a parabola!!
  - 20 points
  - Due: Monday, June 9
  - You MUST show full details of computations to obtain any credit
    - Beyond what is included in this lecture!!

# Projectile Motion

- A 2-dim motion of an object under the gravitational acceleration with the following assumptions
  - Free fall acceleration, $g$, is constant over the range of the motion
    - $\vec{g} = -9.8\vec{j}\left(m/s^2\right)$
    - $a_x = 0\,m/s^2$ and $a_y = -9.8\,m/s^2$
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity ( <u>no acceleration</u> ) $v_{xf} = v_{x0}$
  - Vertical motion under constant acceleration ( $g$ ) $v_{yf} = v_{y0} + a_y t = v_{y0} + (-9.8)t$



# Kinematic Equations in 2-Dim

| x-component | y-component |
| --- | --- |
| $v_x = v_{xo} + a_x t$ | $v_y = v_{yo} + a_y t$ |
| $x = \frac{1}{2}\left(v_{xo} + v_x\right) t$ | $y = \frac{1}{2}\left(v_{yo} + v_y\right) t$ |
| $v_x^2 = v_{xo}^2 + 2a_x x$ | $v_y^2 = v_{yo}^2 + 2a_y y$ |
| $x = v_{xo} t + \frac{1}{2} a_x t^2$ | $y = v_{yo} t + \frac{1}{2} a_y t^2$ |

# *Show that a projectile motion is a parabola!!!*

x-component $v_{xi} = v_i \cos\theta_i$

y-component $v_{yi} = v_i \sin\theta_i$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = -g\vec{j}$$

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

$a_x=0$

$$x_f = v_{xi}t = v_i\cos\theta_i t$$

$$t = \frac{x_f}{v_i\cos\theta_i}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)t^2 = v_i\sin\theta_i t - \frac{1}{2}gt^2$$

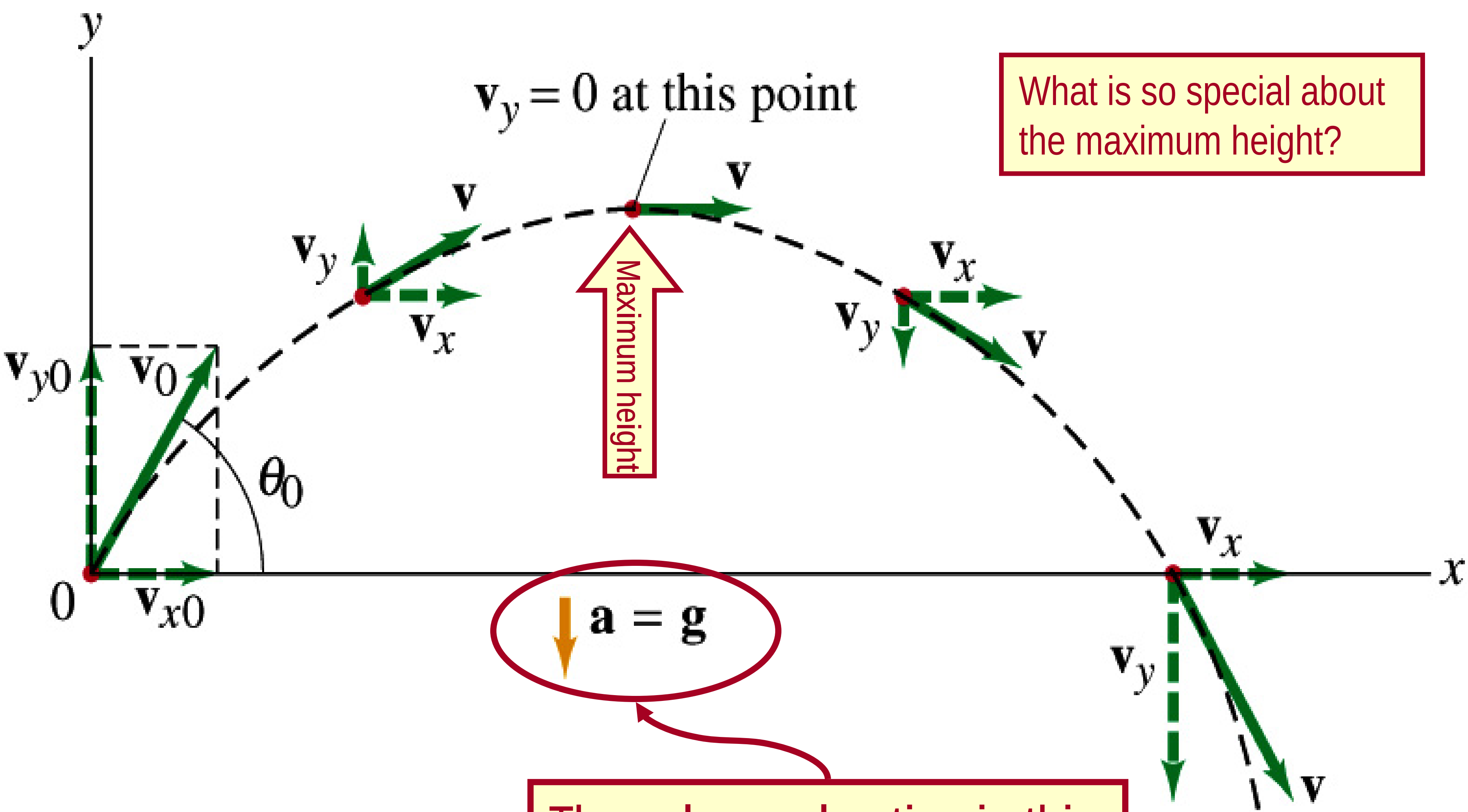
Plug t into the above

$$y_f = v_i\sin\theta_i\left(\frac{x_f}{v_i\cos\theta_i}\right) - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta_i}\right)^2$$

$$y_f = x_f\tan\theta_i - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2$$

What kind of parabola is this?

# Projectile Motion

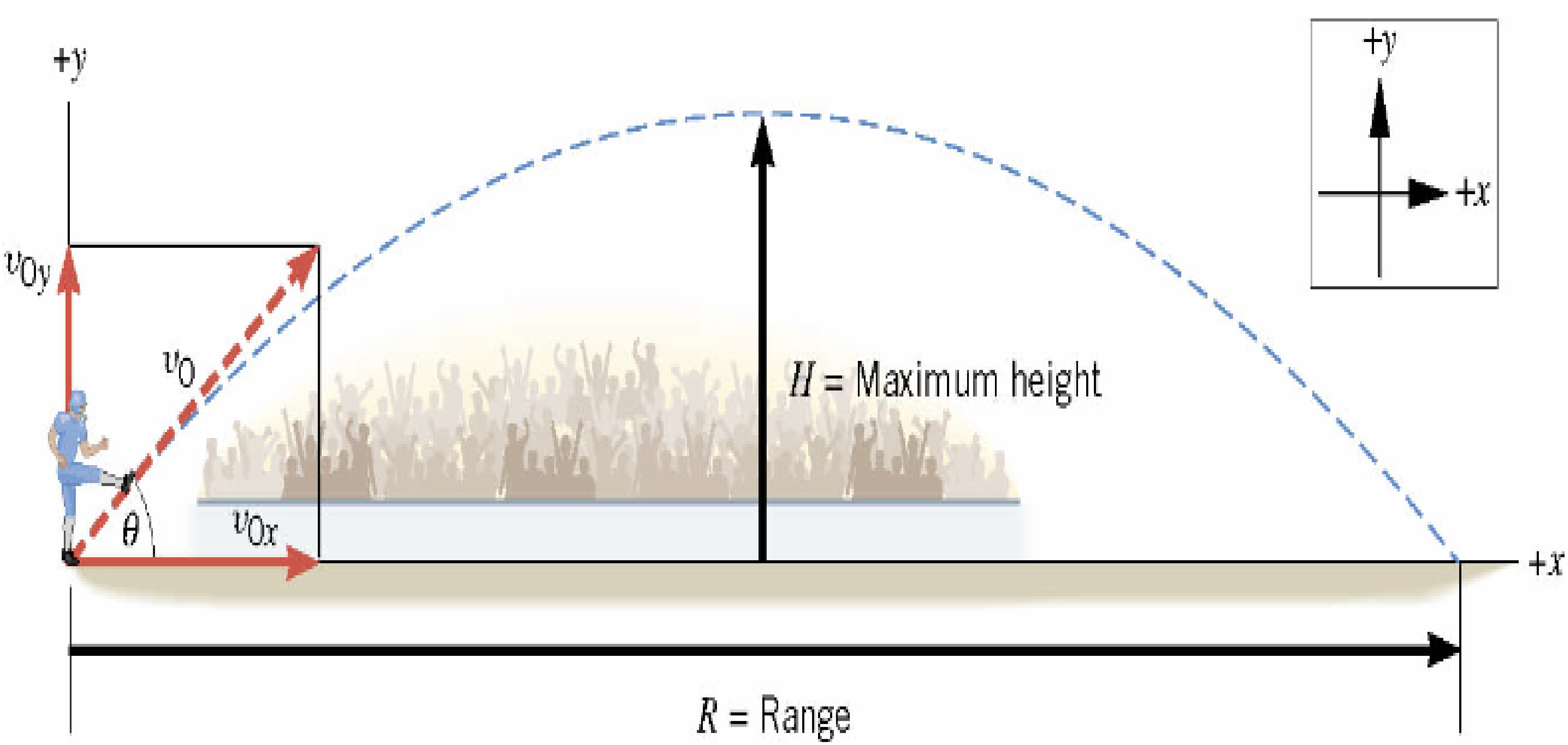



What is so special about the maximum height?

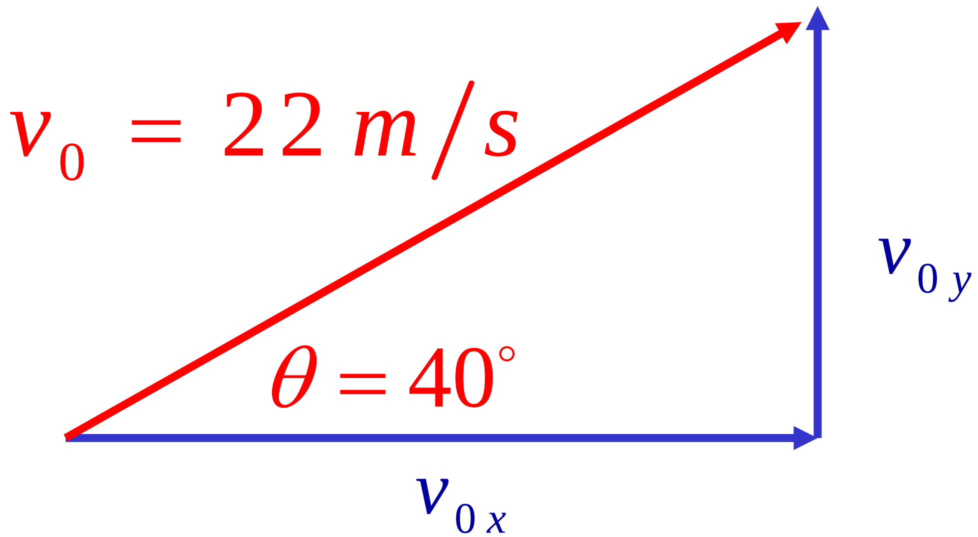
The only acceleration in this motion. It is a constant!!

# *Example 6* The Height of a Kickoff

A placekicker kicks a football at and angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.



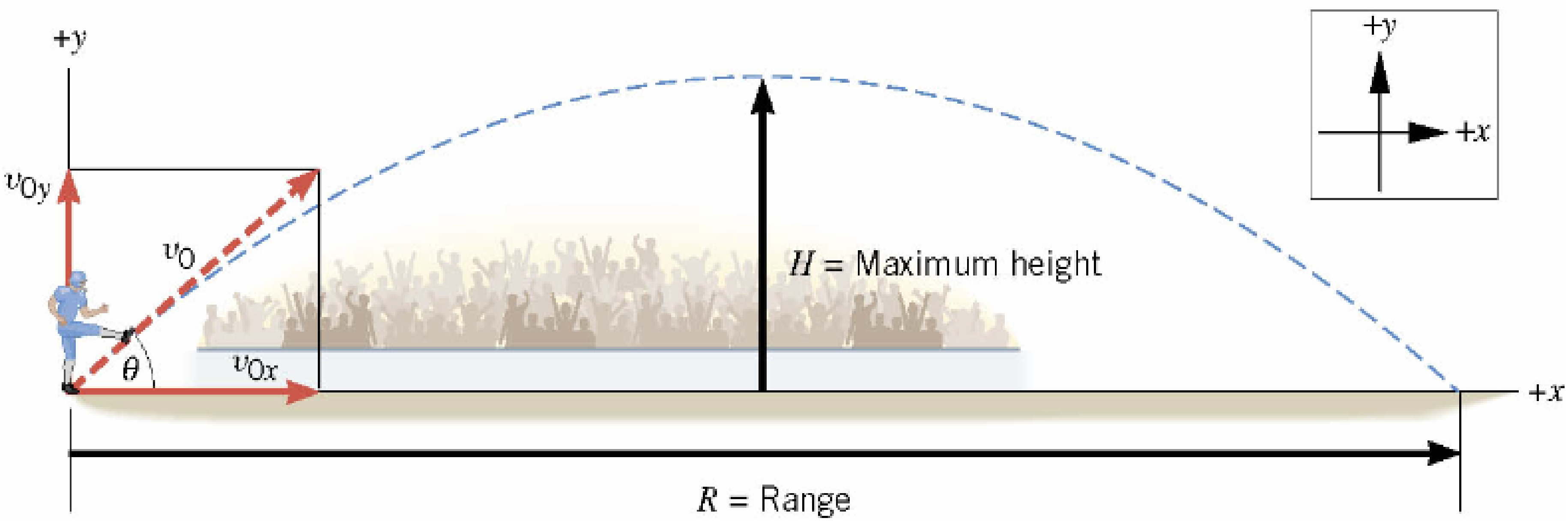
# First, the initial velocity components




$$v_{ox} = v_o \cos\theta = \left(22\,\mathrm{m/s}\right)\cos 40^\circ = 17\,\mathrm{m/s}$$

$$v_{oy} = v_o \sin\theta = \left(22\,\mathrm{m/s}\right)\sin 40^\circ = 14\,\mathrm{m/s}$$

# Motion in y-direction is of the interest..




| $y$ | $a_y$ | $v_y$ | $v_{oy}$ | $t$ |
|---|---|---|---|---|
| ? | -9.8 m/s$^2$ | 0 m/s | +14 m/s | |

# Now the nitty, gritty calculations…

| $y$ | $a_y$ | $v_y$ | $v_{oy}$ | $t$ |
|---|---|---|---|---|
| ? | -9.80 m/s$^2$ | 0 | 14 m/s | |

What happens at the maximum height?

The ball's velocity in y-direction becomes 0!!

And the ball's velocity in x-direction? Stays the same!! Why?

Because there is no acceleration in x-direction!!

Which kinematic formula would you like to use?

$$v_y^2 = v_{oy}^2 + 2a_y y$$

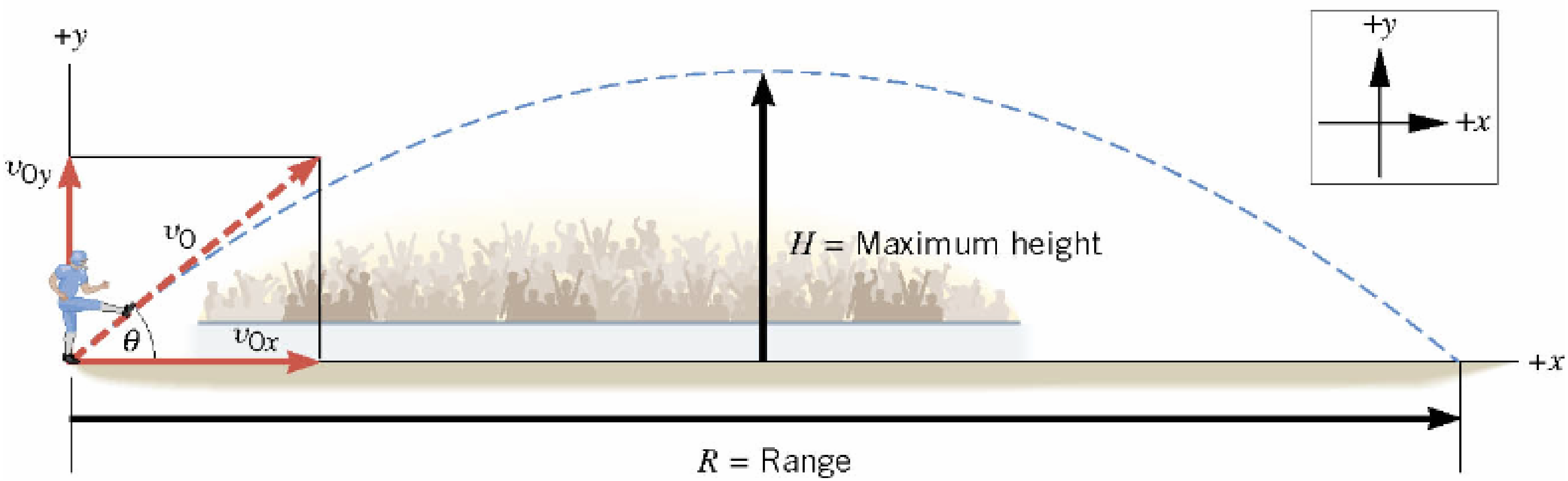
Solve for y

$$y = \frac{v_y^2 - v_{oy}^2}{2a_y}$$

$$y = \frac{0 - (14\,\mathrm{m/s})^2}{2(-9.8\,\mathrm{m/s^2})} = +10\ \mathrm{m}$$

# *Example 7* The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?




What is y when the ball reaches the maximum range?

| $y$ | $a_y$ | $v_y$ | $v_{oy}$ | $t$ |
|---|---|---|---|---|
| 0 m | -9.80 m/s$^2$ | | 14 m/s | ? |

# Now solve the kinematic equations in y direction!!

| $y$ | $a_y$ | $v_y$ | $v_{oy}$ | $t$ |
|---|---|---|---|---|
| 0 | -9.80 m/s$^2$ | | 14 m/s | ? |

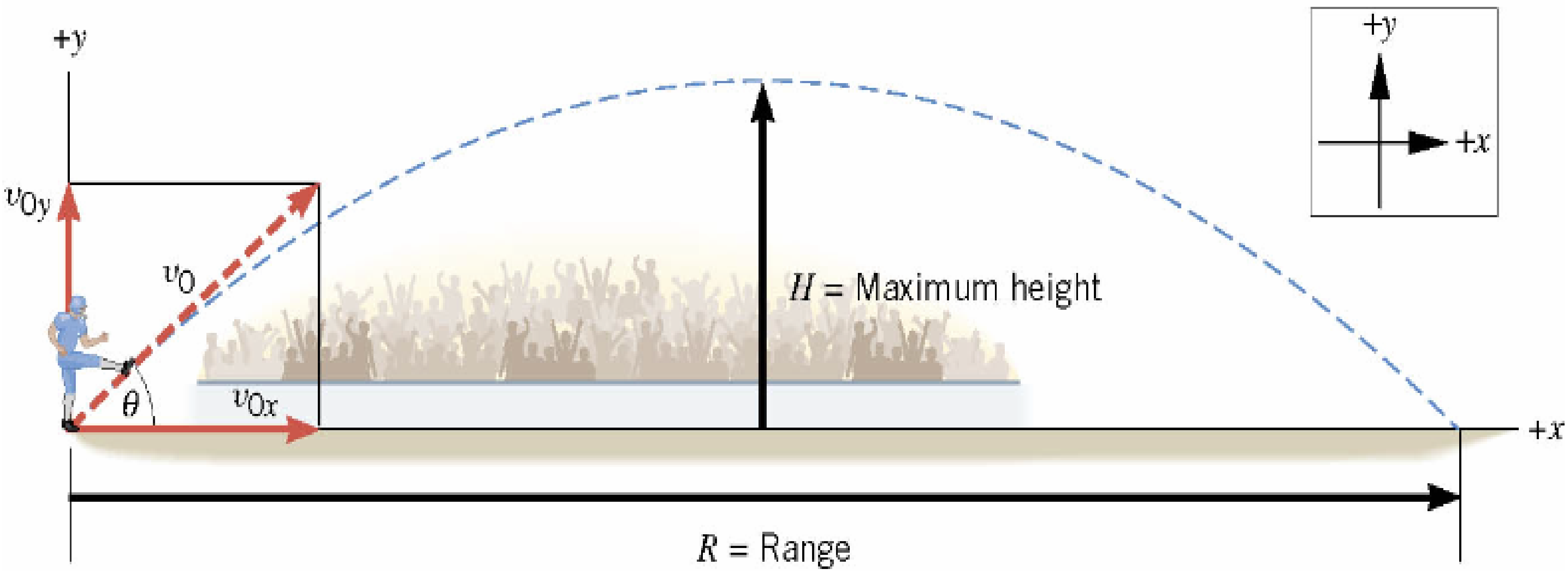
$$y = v_{oy}t + \tfrac{1}{2}a_y t^2 \xrightarrow{\text{Since } y=0} 0 = v_{oy}t + \tfrac{1}{2}a_y t^2 = t\left(v_{oy} + \tfrac{1}{2}a_y t\right)$$

Two soultions $t = 0$ or

$$v_{oy} + \tfrac{1}{2}a_y t = 0 \xrightarrow{\text{Solve for t}} t = \frac{-v_{oy}}{\frac{1}{2}a_y} = \frac{-2v_{oy}}{a_y} = \frac{-2\cdot 14}{-9.8} = 2.9s$$

# Ex. 8 The Range of a Kickoff

Calculate the range R of the projectile.




$$x = v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t = (17\,\text{m/s})(2.9\ \text{s}) = +49\ \text{m}$$

# Horizontal Range and Max Height

- Based on what we have learned in the previous lecture, one can analyze a projectile motion in more detail
  - Maximum height an object can reach
  - Maximum range

What happens at the maximum height?

At the maximum height the object's vertical motion stops to turn around!!

$$v_{yf} = v_{0y} + a_y t = v_0 \sin\theta_0 - g t_A = 0$$

Solve for $t_A$

$$\therefore t_A = \frac{v_0 \sin\theta_0}{g}$$

Time to reach to the maximum height!!

# Horizontal Range and Max Height

Since no acceleration is in x direction, the object still flies even if $v_y$=0.

$$R = v_{0x}t = v_{0x}\left(2t_A\right) = v_0 \cos\theta_0 \left(2 \cdot \frac{v_0 \sin\theta_0}{g}\right)$$

Range

$$R = \left(\frac{v_0^2 \sin 2\theta_0}{g}\right)$$

$$y_f = h = v_{0y}t + \frac{1}{2}(-g)t^2 = v_0 \sin\theta_0\left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\theta_0}{g}\right)^2$$

Height

$$y_f = h = \left(\frac{v_0^2 \sin^2\theta_0}{2g}\right)$$

# Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left( \frac{v_0^{\,2} \sin^2 \theta_0}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_i = 90^o$!!!

$$R = \left( \frac{v_0^{\,2} \sin 2\theta_0}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i = 90^o$, i.e., $\theta_i = 45^o$!!!

# Force

We've been learning kinematics; describing motion without understanding what the cause of the motion is. Now we are going to learn dynamics!!

*Can someone tell me what FORCE is?*

*FORCE is what causes an object to move.*

*The above statement is not entirely correct. Why?*

*Because when an object is moving with a constant velocity no force is exerted on the object!!!*

*FORCEs are what cause changes to the velocity of an object!!*

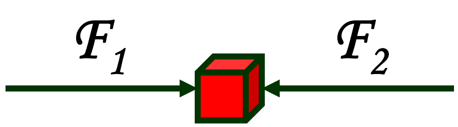
*What does this statement mean?*

*When there is force, there is change of velocity!!*

*What does force cause? It causes an acceleration.!!*

*What happens if there are several forces being exerted on an object?*

*Forces are vector quantities, so vector sum of all forces, the NET FORCE, determines the direction of the acceleration of the object.*




*NET FORCE,* $F = F_1 + F_2$

*When the net force on an object is* **0**, *it has constant velocity and is at its equilibrium!!*

# Newton's First Law

Aristotle (384-322BC): *A natural state of a body is rest. Thus force is required to move an object. To move faster, ones needs larger forces.*

Galileo's statement on natural states of matter: *Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed!!*

Galileo's statement is formulated by Newton into the 1st law of motion (Law of Inertia): *In the absence of net external force, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.*

# Newton's First Law and Inertial Frame

Newton's 1st law of motion (Law of Inertia): *In the absence of net external force, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.*

What does this statement tell us?

- When no force is exerted on an object, the acceleration of the object is 0.
- Any isolated object, the object that do not interact with its surroundings, is either at rest or moving at a constant velocity.
- Objects would like to keep its current state of motion, as long as there are no forces that interfere with the motion. This tendency is called the Inertia.

A frame of reference that is moving at a constant velocity is called the ***Inertial Frame***

Is a frame of reference with an acceleration an ***Inertial Frame?***

***NO!***

# Mass

Mass: *A measure of the inertia of a body* or *quantity of matter*

- Independent of the object's surroundings: The same no matter where you go.
- Independent of the method of measurement: The same no matter how you measure it.

*The heavier the object, the bigger the inertia !!*

*It is harder to make changes of motion of a heavier object than a lighter one.*

*The same forces applied to two different masses result in different acceleration depending on the mass.*

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1}$$

*Note that the mass and the weight of an object are two different quantities!!*

*Weight of an object is the magnitude of the gravitational force exerted on the object.*

*Not an inherent property of an object!!!*

*Weight will change if you measure on the Earth or on the moon but the mass won't!!*

Unit of mass? $kg$

# Newton's Second Law of Motion

*The acceleration of an object is directly proportional to the net force exerted on it and is inversely proportional to the object's mass.*

*How do we write the above statement in a mathematical expression?*

$$\vec{a} = \frac{\sum_i \vec{F}_i}{m}$$

From this we obtain

$$\sum_i \vec{F}_i = m\vec{a}$$

Newton's 2nd Law of Motion

*Since it's a vector expression, each component must also satisfy:*

$$\sum_i F_{ix} = ma_x \qquad \sum_i F_{iy} = ma_y \qquad \sum_i F_{iz} = ma_z$$

# Unit of the Force

*From the vector expression in the previous page, what do you conclude the dimension and the unit of the force are?*

$$\sum_i \vec{F_i} = m\vec{a}$$

*The dimension of force is* $[m][a] = [M][LT^{-2}]$

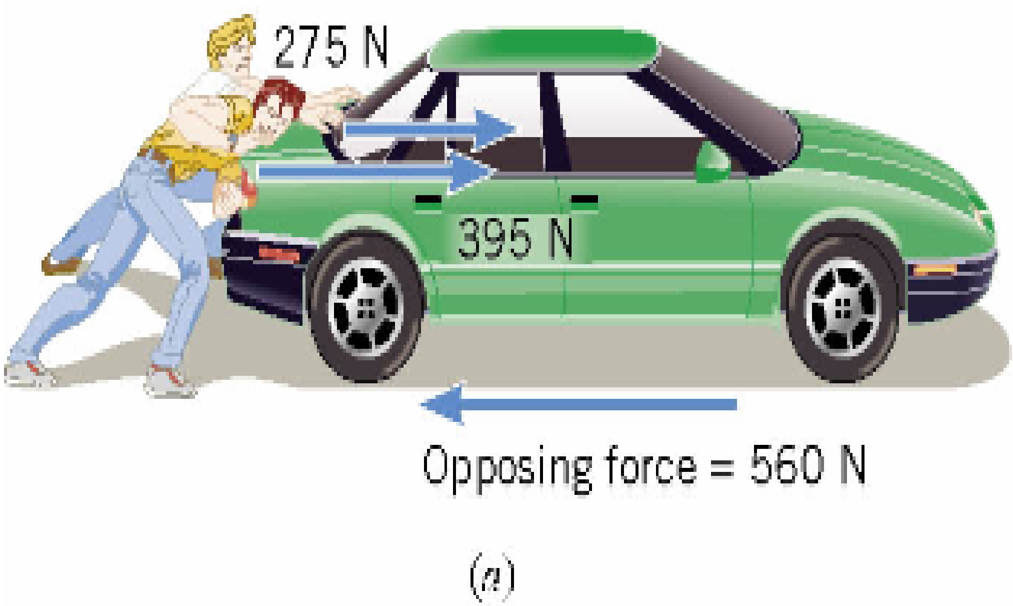
*The unit of force in SI is* $[Force] = [m][a] = [M][LT^{-2}] = (\mathrm{kg})\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right) = kg \cdot m/s^2$

*For ease of use, we define a new derived unit called, Newton (N)*

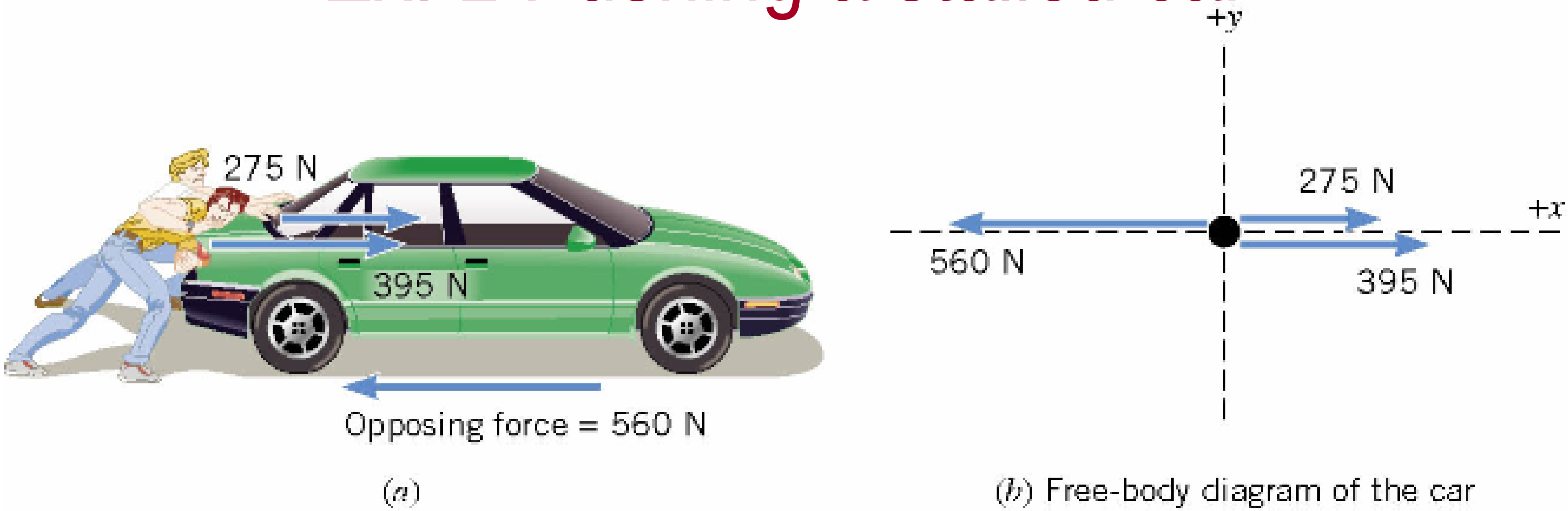
$$1N \equiv 1kg \cdot m/s^2 \approx \frac{1}{4} lbs$$

# Free Body Diagram

A *free-body-diagram* is a diagram that represents the object and the forces that act on it.



# Ex. 1 Pushing a stalled car




What is the net force in this example?

$$F = 275\ N + 395\ N - 560\ N = +110\ N$$

Which direction? The + x axis of the coordinate system.

# What is the acceleration the car receives?

If the mass of the car is 1850 kg then, by Newton's second law, the acceleration is

$$\sum \vec{F} = m\vec{a}$$

Since the motion is in 1 dimension

$$\sum F = ma$$

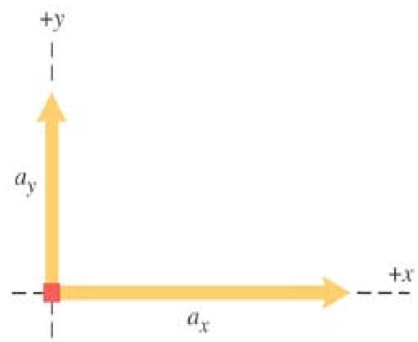
Now we solve this equation for a

$$a = \frac{\sum F}{m} = \frac{+110\ \mathrm{N}}{1850\ \mathrm{kg}} = +0.059\ \mathrm{m/s^2}$$

# Ex. 2 A stranded man on a raft

A man is stranded on a raft (mass of man and raft = 1300kg)m as shown in the figure. By paddling, he causes an average force P of 17N to be applied to the raft in a direction due east (the +x direction). The wind also exerts a force A on the raft. This force has a magnitude of 15N and points 67° north of east. Ignoring any resistance from the water, find the x and y components of the rafts acceleration.

(a)




(c)

First, let's compute the net force on the raft as follows:

| Force | *x* component | *y* component |
|---|---|---|
| $\vec{\mathbf{P}}$ | +17 N | 0 N |
| $\vec{\mathbf{A}}$ | $+(15N)\cos 67^{o}$ | $+(15N)\sin 67^{o}$ |
| $\vec{F} = \vec{P} + \vec{A}$ | $+17+15\cos 67^{o} =$ +23(N) | $+15\sin 67^{o} =$ +14(N) |

*Now compute the acceleration components in x and y directions!!*

$$a_x = \frac{\sum F_x}{m} = \frac{+23\ \mathrm{N}}{1300\ \mathrm{kg}} = +0.018\ \mathrm{m/s^2}$$

$$a_y = \frac{\sum F_y}{m} = \frac{+14\ \mathrm{N}}{1300\ \mathrm{kg}} = +0.011\ \mathrm{m/s^2}$$

*The overall acceleration is*

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = \left(0.018\vec{i} + 0.011\vec{j}\right) m/s^2$$