PHYS 1441 – Section 001
Lecture # 9

Wednesday, June 11, 2008

Dr. Jaehoon Yu

- Scalar Product
- Work-Kinetic Energy Theorem
- Work and Energy Involving Kinetic Friction
- Potential Energy
- Gravitational Potential Energy
- Conservative and Non-conservative Forces
- Conservation of Mechanical Energy

Announcements

- Quiz results
  - Class Average: 5.4/8
    - Equivalent to 68/100
  - Top score: 8/8
- Second term exam
  - 8 – 10am, Tuesday, June 17, in SH103
  - Covers CH4.1 – What we finish tomorrow, June 12 (CH7?)
  - Practice tests have been posted on the class web page
  - Dr. Satyanand will conduct a help session 8 – 10am, Monday, June 16 in class
- Reading assignment: CH6.9

Special Project for Extra Credit

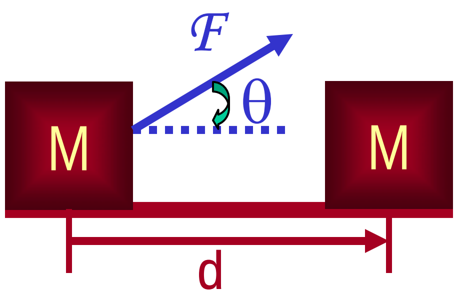
- Prove the following equivalence mathematically

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = \left|\vec{A}\right|\left|\vec{B}\right| \cos\theta$$

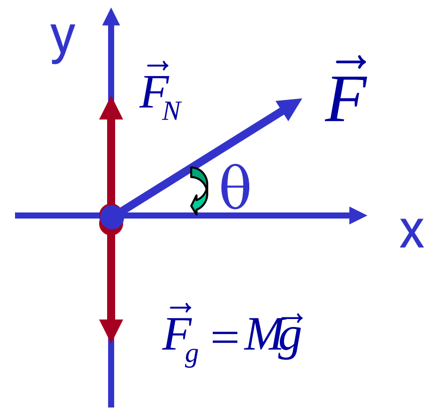
  - where $\vec{A} = A_x \hat{i} + A_y \hat{j}$
  - and $\vec{B} = B_x \hat{i} + B_y \hat{j}$
  - 20 point extra credit
  - Due: Wednesday, June 18

Work Done by a Constant Force

A meaningful work in physics is done only when a sum of forces exerted on an object made a motion to the object.



Free Body Diagram



Which force did the work? Force $\vec{F}$ Why?

How much work did it do? $W = \left(\sum \vec{F}\right) \cdot \vec{d} = Fd\cos\theta$ Unit? $N \cdot m$ $= J$ (for Joule)

What does this mean?

Physically meaningful work is done only by the component of the force along the movement of the object.

Work is an <u>energy transfer</u>!!

Scalar Product of Two Vectors

- Product of magnitude of the two vectors and the cosine of the angle between them $\vec{A}\cdot\vec{B} \equiv |\vec{A}||\vec{B}|\cos\theta$
- Operation is commutative $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta = |\vec{B}||\vec{A}|\cos\theta = \vec{B}\cdot\vec{A}$
- Operation follows the distribution law of multiplication $\vec{A}\cdot\left(\vec{B}+\vec{C}\right) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
- Scalar products of Unit Vectors $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \quad \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
- How does scalar product look in terms of components?

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \qquad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

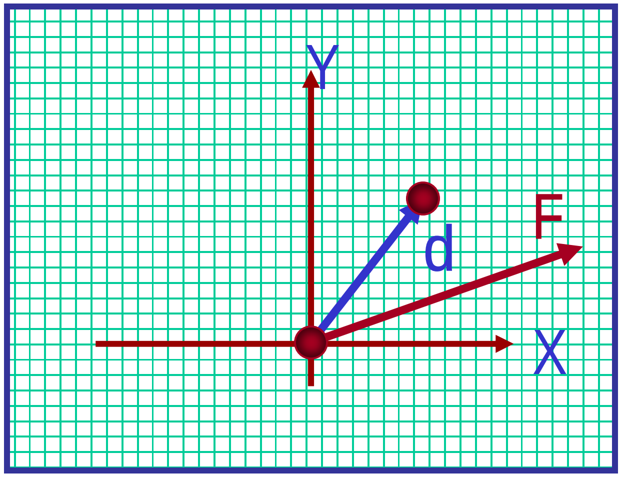
$$\vec{A}\cdot\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) = \left(A_xB_x\hat{i}\cdot\hat{i} + A_yB_y\hat{j}\cdot\hat{j} + A_zB_z\hat{k}\cdot\hat{k}\right) + cross\ terms$$

=0

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Example of Work by Scalar Product

A particle moving on the xy plane undergoes a displacement **d**=(2.0**i**+3.0**j**)m as a constant force F=(5.0**i**+2.0**j**) N acts on the particle.



a) Calculate the magnitude of the displacement and that of the force.

$$\left|\vec{d}\right| = \sqrt{d_x^2 + d_y^2} = \sqrt{(2.0)^2 + (3.0)^2} = 3.6m$$

$$\left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = 5.4N$$

b) Calculate the work done by the force F.

$$W = \vec{F}\cdot\vec{d} = \left(2.0\hat{i}+3.0\hat{j}\right)\cdot\left(5.0\hat{i}+2.0\hat{j}\right) = 2.0\times5.0\,\hat{i}\cdot\hat{i}+3.0\times2.0\,\hat{j}\cdot\hat{j} = 10+6 = 16(J)$$

Can you do this using the magnitudes and the angle between **d** and F?

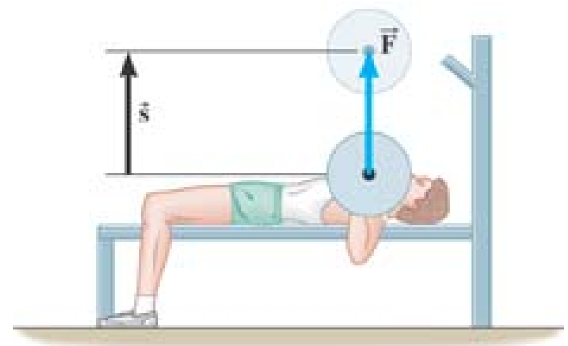
$$W = \vec{F}\cdot\vec{d} = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta$$

Ex. 2 Bench Pressing and The Concept of Negative Work

A weight lifter is bench-pressing a barbell whose weight is 710N a distance of 0.65m above his chest. Then he lowers it the same distance. The weight is raised and lowered at a constant velocity. Determine the work in the two cases.

(a)

What is the angle between the force and the displacement?



(b)

Lifting up

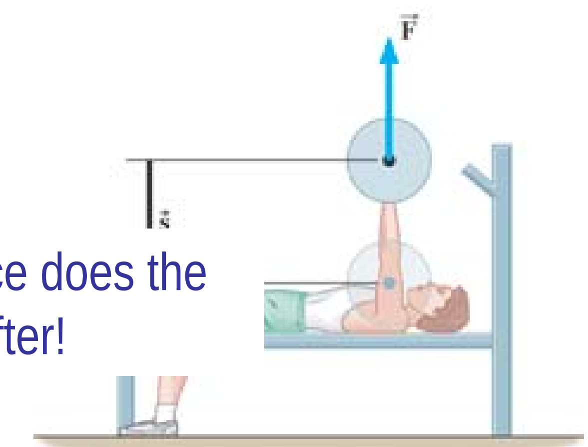
$$W = (F\cos 0)s = Fs$$

$$= 710 \cdot 0.65 = +460(J)$$

Lowering down

$$W = (F\cos 180)s = -Fs$$

$$= -710 \cdot 0.65 = -460(J)$$

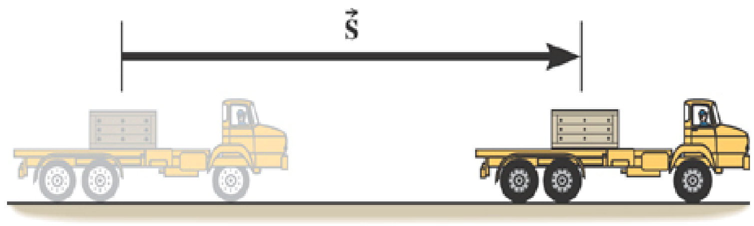


(c)

What does the negative work mean? The gravitational force does the work on the weight lifter!

Ex. 3 Accelerating Crate

A truck is accelerating at a rate of +1.50 m/s^2. The mass of the crate is 120-kg and it does not slip. The magnitude of the displacement is 65 m. What is the total work done on the crate by all of the forces acting on it?



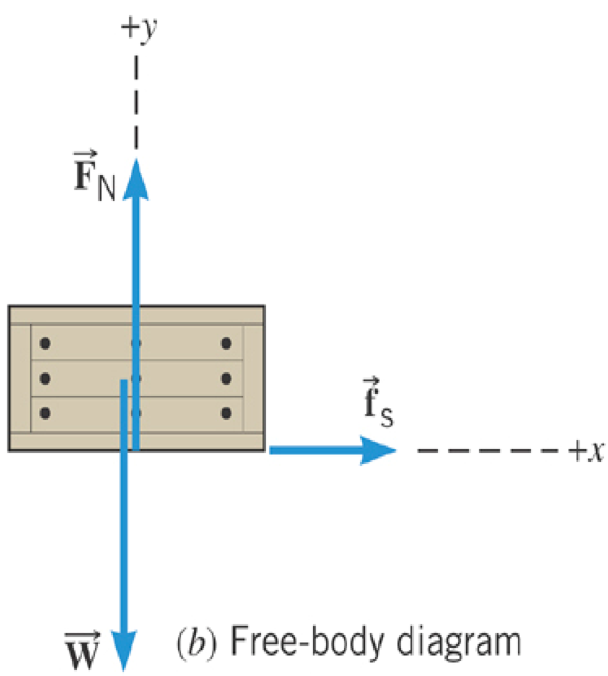
(a)

What are the forces acting in this motion?

Gravitational force on the crate, weight, W or F_g

Normal force force on the crate, F_N

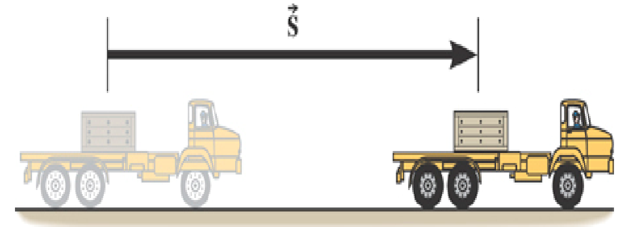
Static frictional force on the crate, f_s



(b) Free-body diagram for the crate

Ex. 3 Continued…

Let's figure out what the work done by each force in this motion is.



(a)

Work done by the gravitational force on the crate, W or F_g

$$W = \left(F_g \cos\left(-90^o\right)\right)s = 0$$

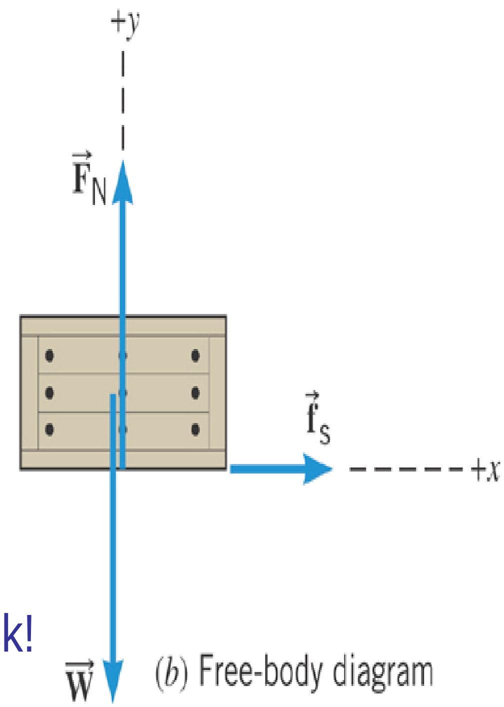
Work done by Normal force force on the crate, F_N

$$W = \left(F_g \cos\left(+90^o\right)\right)s = 0$$

Work done by the static frictional force on the crate, f_s

$$f_s = ma = (120\ \text{kg})\left(1.5\,\text{m/s}^2\right) = 180\text{N}$$

$$W = f_s \cdot s = \left[(180\text{N})\cos 0\right](65\ \text{m}) = 1.2\times10^4\,\text{J}$$



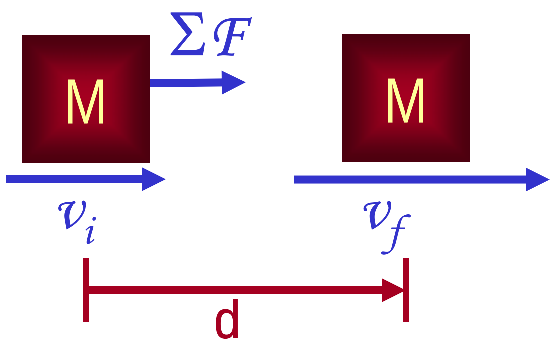
(b) Free-body diagram for the crate

Which force did the work? Static frictional force on the crate, f_s

How? By holding on to the crate so that it moves with the truck!

Kinetic Energy and Work-Kinetic Energy Theorem

- Some problems are hard to solve using Newton's second law
  - If forces exerting on an object during the motion are complicated
  - Relate the work done on the object by the net force to the change of the speed of the object



Suppose net force ΣF was exerted on an object for displacement d to increase its speed from v_i to v_f.

The work on the object by the net force ΣF is

$$W = \left(\sum \vec{F}\right)\cdot\vec{d} = (ma\cos 0)\,d = (ma)\,d$$

Using the kinematic equation of motion

$$2ad = v_f^2 - v_0^2 \quad\Rightarrow\quad ad = \frac{v_f^2 - v_0^2}{2}$$

Kinetic Energy

$$KE \equiv \frac{1}{2}mv^2$$

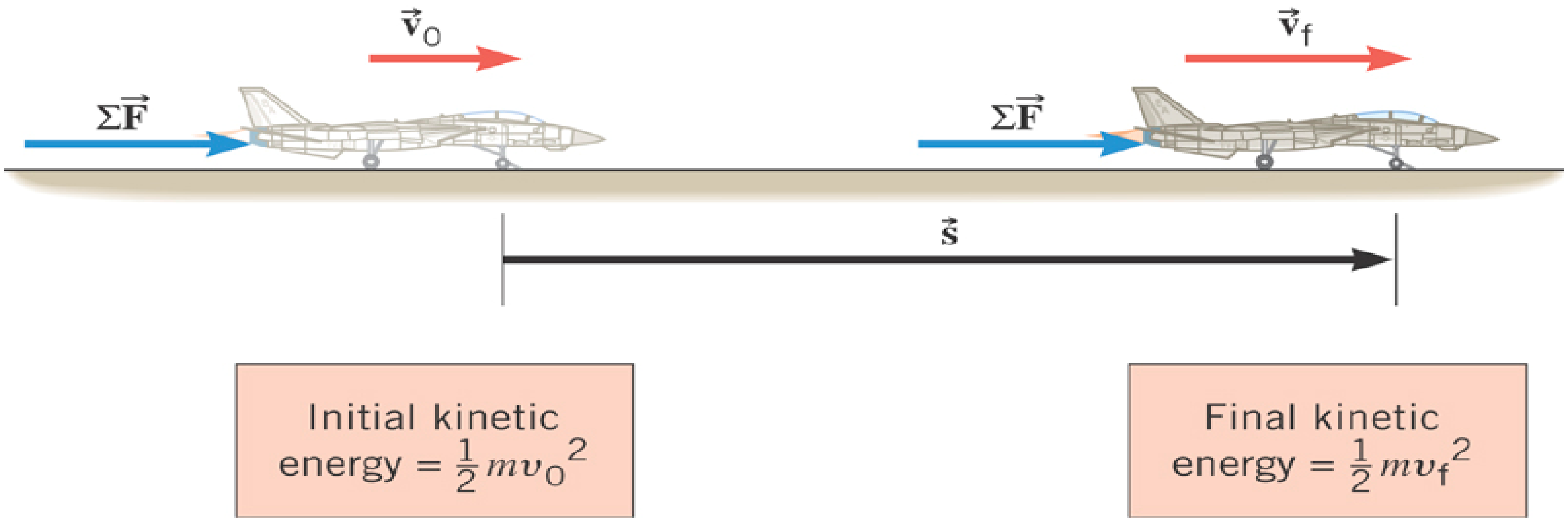
Work $W = (ma)d = \frac{1}{2}m\left(v_f^2 - v_0^2\right) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

Work $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$

Work done by the net force causes change in the object's kinetic energy.

Work-Kinetic Energy Theorem

Work-Kinetic Energy Theorem

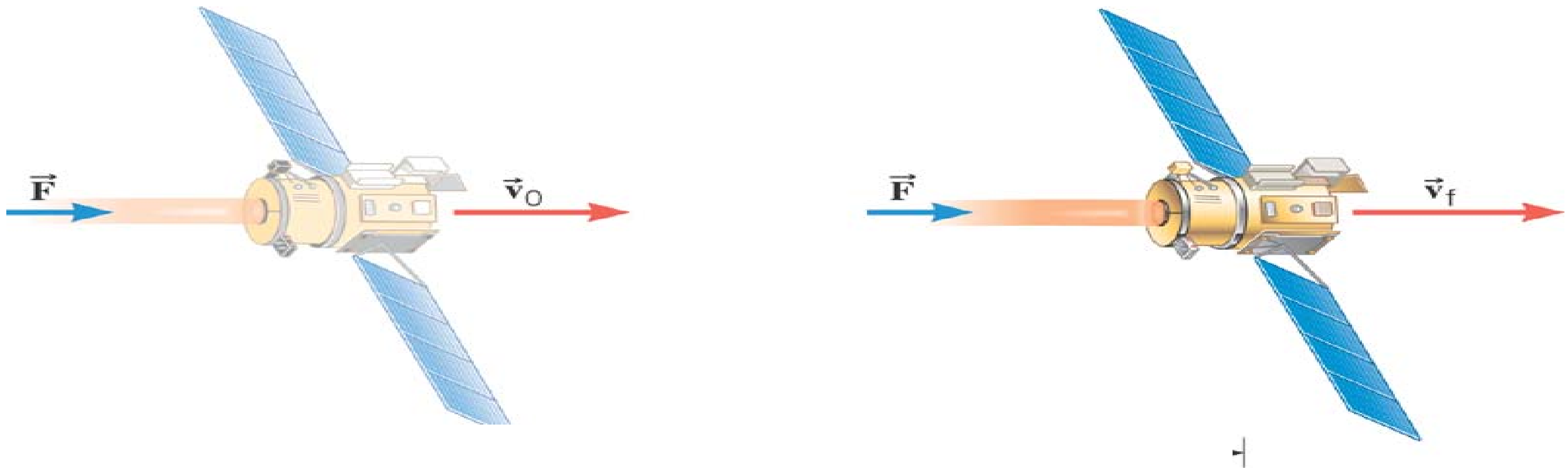


When a net external force by the jet engine does work on and object, the kinetic energy of the object changes according to

$$W = \mathrm{KE_f} - \mathrm{KE_o} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

Ex. 4 Deep Space 1

The mass of the space probe is 474-kg and its initial velocity is 275 m/s. If the 56.0-mN force acts on the probe parallel through a displacement of 2.42×10^{9}m, what is its final speed?



$$\left[\left(\sum F\right)\cos\theta\right]s = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

$$\left(5.60\times10^{-2}\,\mathrm{N}\right)\cos 0^\circ\left(2.42\times10^{9}\,\mathrm{m}\right) = \tfrac{1}{2}\left(474\ \mathrm{kg}\right)v_f^2 - \tfrac{1}{2}\left(474\ \mathrm{kg}\right)\left(275\,\mathrm{m/s}\right)^2$$

$$v_f = 805\,\mathrm{m/s}$$

Ex. 6 Satellite Motion and Work By the Gravity

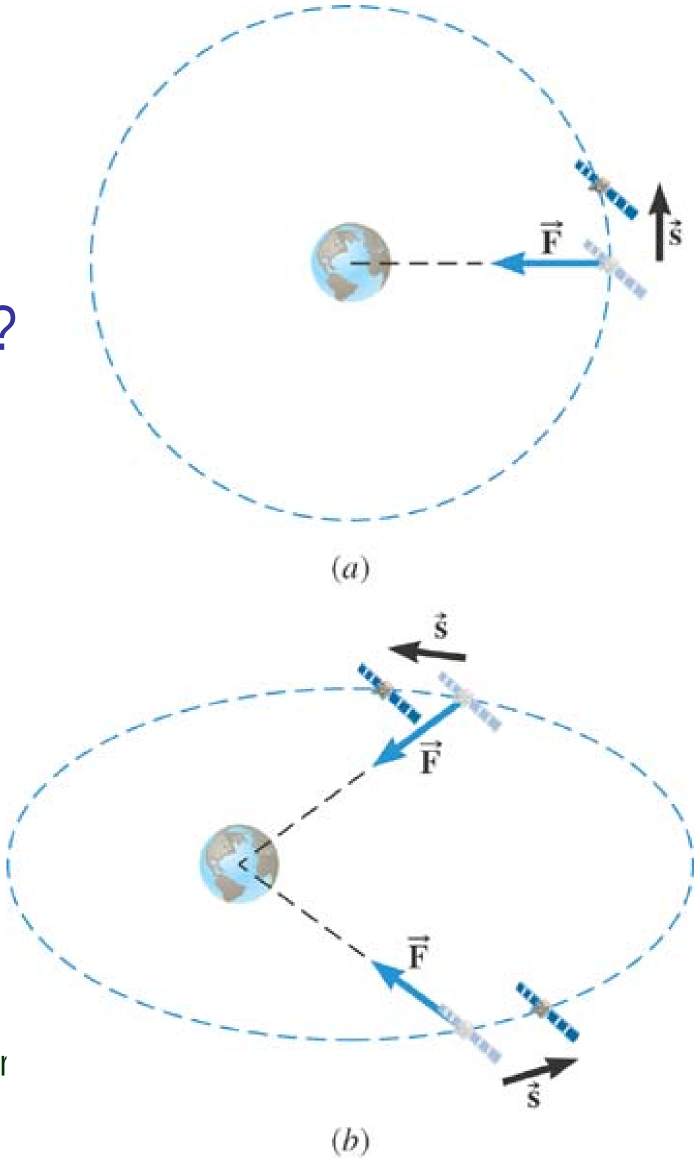
A satellite is moving about the earth in a circular orbit and an elliptical orbit. For these two orbits, determine whether the kinetic energy of the satellite changes during the motion.

For a circular orbit No change! Why not?

Gravitational force is the only external force but it is perpendicular to the displacement. So no work.

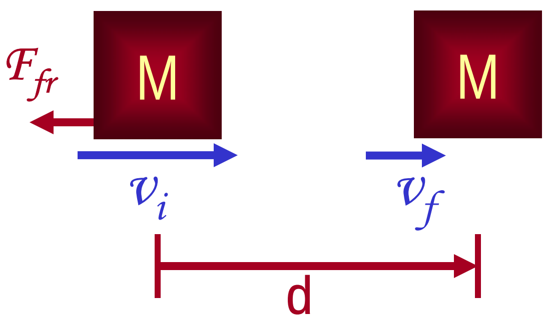
For an elliptical orbit Changes! Why?

Gravitational force is the only external force but its angle with respect to the displacement varies. So it performs work.



Work and Energy Involving Kinetic Friction

- What do you think the work looks like if there is friction?
  - Static friction does not matter! Why? **It isn't there when the object is moving.**
  - Then which friction matters? **Kinetic Friction**



Friction force $\mathcal{F}_{fr}$ works on the object to slow down

The work on the object by the friction $\mathcal{F}_{fr}$ is

$$W_{fr} = F_{fr} d \cos(180) = -F_{fr} d \quad \Delta KE = -F_{fr} d$$

The negative sign means that the work is done on the friction!!

The final kinetic energy of an object, taking into account its initial kinetic energy, friction force and other source of work, is

$$KE_f = KE_i + \sum W - F_{fr} d$$

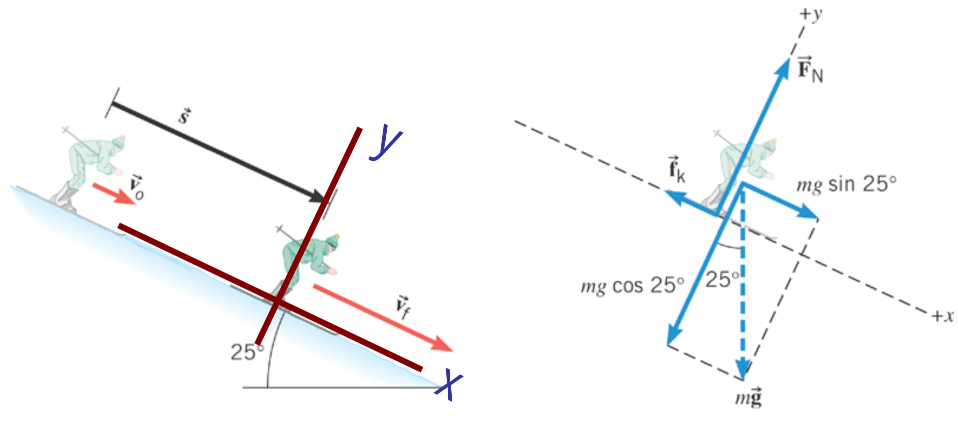
t=0, KE_i

Friction, Engine work

t=T, KE_f

Ex. 5 Downhill Skiing

A 58kg skier is coasting down a 25° slope. A kinetic frictional force of magnitude f_k=70N opposes her motion. Neat the top of the slope, the skier's speed is v_0=3.6m/s. Ignoring air resistance, determine the speed v_f at the point that is displaced 57m downhill.



(a)
(b) Free-body diagram for the skier

What are the forces in this motion?

Gravitational force: F_g Normal force: F_N Kinetic frictional force: f_k

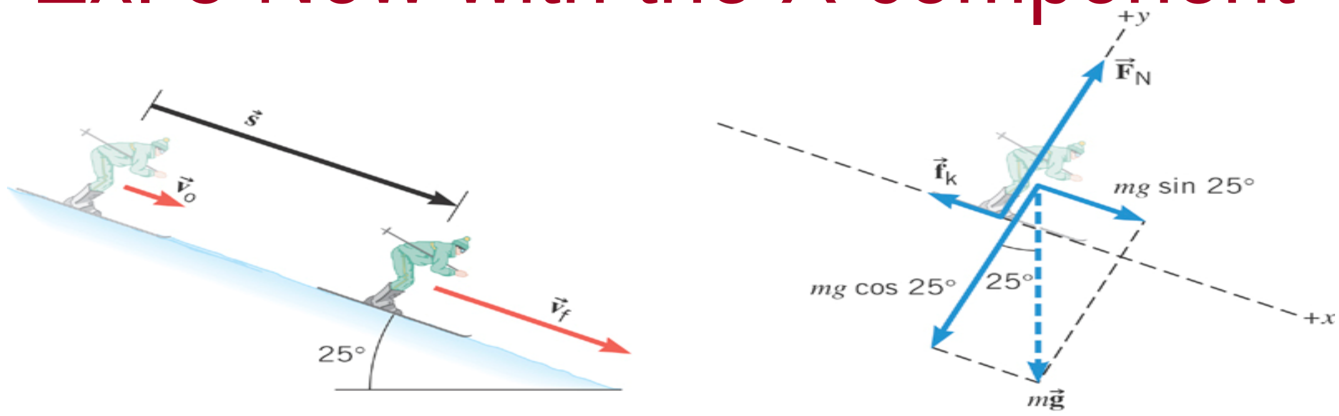
What are the X and Y component of the net force in this motion?

Y component $$\sum F_y = F_{gy} + F_N = -mg\cos 25^o + F_N = 0$$

From this we obtain $$F_N = mg\cos 25^o = 58 \cdot 9.8 \cdot \cos 25^o = 515N$$

What is the coefficient of kinetic friction? $$f_k = \mu_k F_N \Rightarrow \mu_k = \frac{f_k}{F_N} = \frac{70}{515} = 0.14$$

Ex. 5 Now with the X component



(a) (b) Free-body diagram for the skier

X component $\sum F_x = F_{gx} - f_k = mg\sin 25^o - f_k = \left(58 \cdot 9.8 \cdot \sin 25^o - 70\right) = 170N = ma$

Total work by this force $W = \left(\sum F_x\right) \cdot s = \left(mg\sin 25^o - f_k\right) \cdot s = \left(58 \cdot 9.8 \cdot \sin 25^o - 70\right) \cdot 57 = 9700J$

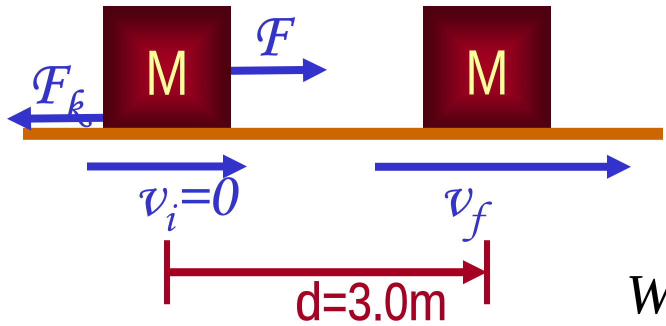
From work-kinetic energy theorem $W = KE_f - KE_i$ ⇒ $KE_f = \frac{1}{2}mv_f^2 = W + KE_i = W + \frac{1}{2}mv_0^2$

Solving for v_f $v_f^2 = \frac{2W + mv_0^2}{m}$ ⇒ $v_f = \sqrt{\frac{2W + mv_0^2}{m}} = \sqrt{\frac{2 \cdot 9700 + 58 \cdot (3.6)^2}{58}} = 19\,m/s$

What is her acceleration? $\sum F_x = ma$ ⇒ $a = \frac{\sum F_x}{m} = \frac{170}{58} = 2.93\,m/s^2$

Example of Work Under Friction

A 6.0kg block initially at rest is pulled to East along a horizontal surface with coefficient of kinetic friction μ_k=0.15 by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W_F = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36\left(J\right)$$

Work done by friction $\mathcal{F}_k$ is

$$W_k = \vec{F}_k \bullet \vec{d} = \left|\vec{F}_k\right|\left|\vec{d}\right|\cos\theta = \left|\mu_k m g\right|\left|\vec{d}\right|\cos\theta$$

$$= 0.15\times 6.0\times 9.8\times 3.0\cos 180 = -26\left(J\right)$$

Thus the total work is

$$W = W_F + W_k = 36 - 26 = 10(J)$$

Using work-kinetic energy theorem and the fact that initial speed is 0, we obtain

$$W = W_F + W_k = \frac{1}{2}mv_f^2$$

Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 10}{6.0}} = 1.8m/s$$