PHYS 1441 – Section 001
Lecture # 10

Thursday, June 12, 2008

Dr. Jaehoon Yu

- Potential Energy
- Gravitational Potential Energy
- Conservative and Non-conservative Forces
- Conservation of Mechanical Energy
- Power

. Today's homework is homework #6, due 9pm, Friday, June 20!!

Announcements

- Second term exam
  - 8 – 10am, Tuesday, June 17, in SH103
  - Covers CH4.1 – CH 6.6
  - Practice tests have been posted on the class web page
  - Dr. Satyanand will conduct a help session 8 – 10am, Monday, June 16 in class
  - Good luck with the exam!
- Reading assignment: CH6.9

Summary for Work and Kinetic Energy

A meaningful work in physics is done only when the sum of the forces exerted on an object made a motion to the object.

What does this mean?

However much tired your arms feel, if you were just holding an object without moving it you have not done any physically meaningful work to the object.

Mathematically, the work is written as the product of magnitudes of the net force vector, the magnitude of the displacement vector and cosine of the angle between them.

$$W = \sum\left(\vec{F}_i\right)\bullet\vec{d} = \left|\sum\left(\vec{F}_i\right)\right|\left|\vec{d}\right|\cos\theta$$

Kinetic Energy is the energy associated with the motion and capacity to perform work.
Work causes change of KE after the completion ← Work-Kinetic energy theorem

$$K = \frac{1}{2}mv^2$$

$$\sum W = K_f - K_i = \Delta K$$

Nm=Joule

Potential Energy

Energy associated with a system of objects → Stored energy which has the potential or the possibility to work or to convert to kinetic energy

What does this mean?

In order to describe potential energy, PE, a system must be defined.

The concept of potential energy can only be used under the special class of forces called the conservative force which results in the principle of conservation of mechanical energy.

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

What are other forms of energies in the universe?

Mechanical Energy *Chemical Energy* *Biological Energy*

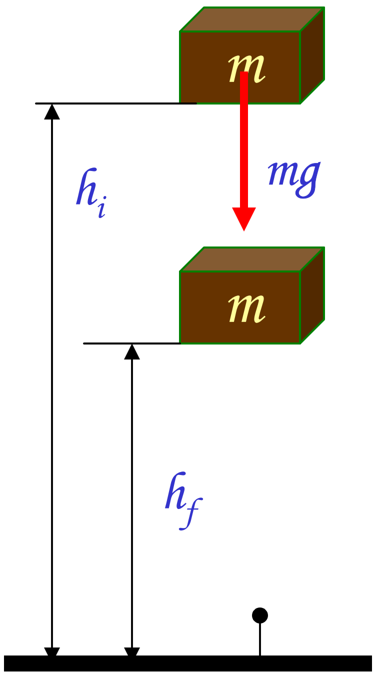
Electromagnetic Energy *Nuclear Energy*

These different types of energies are stored in the universe in many different forms!!!

If one takes into account ALL forms of energy, the total energy in the entire universe is conserved. It just transforms from one form to another.

Gravitational Potential Energy

The potential energy is given to an object by the gravitational field in the system of Earth by virtue of the object's height from an arbitrary zero level



When an object is falling, the gravitational force, Mg, performs the work on the object, increasing the object's kinetic energy. So the potential energy of an object at a height y, which is the potential to do work is expressed as

$$PE = \vec{F}_g \cdot \vec{y} = \left|\vec{F}_g\right|\left|\vec{y}\right|\cos\theta = \left|\vec{F}_g\right|\left|\vec{y}\right| = mgh \qquad PE \equiv mgh$$

The work done on the object by the gravitational force as the brick drops from h_i to h_f is:

$$W_g = PE_i - PE_f$$
$$= mgh_i - mgh_f = -\Delta PE$$

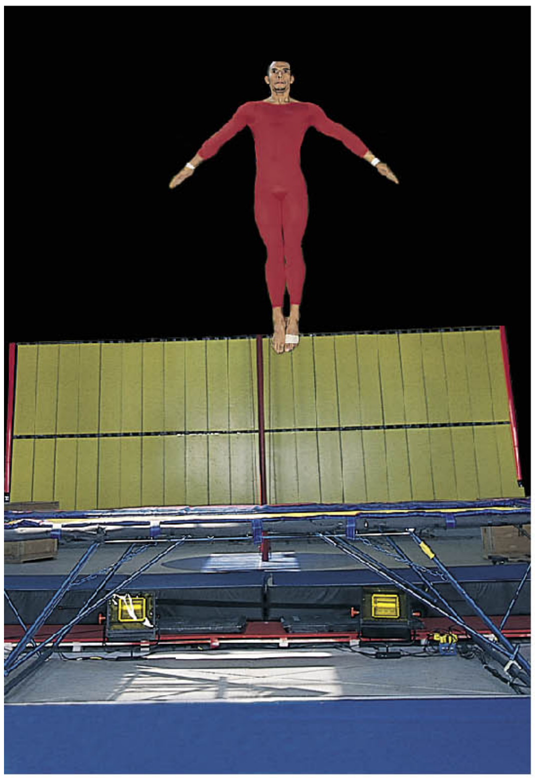
What does this mean?

Work by the gravitational force as the brick drops from h_i to h_f is the negative change of the system's potential energy

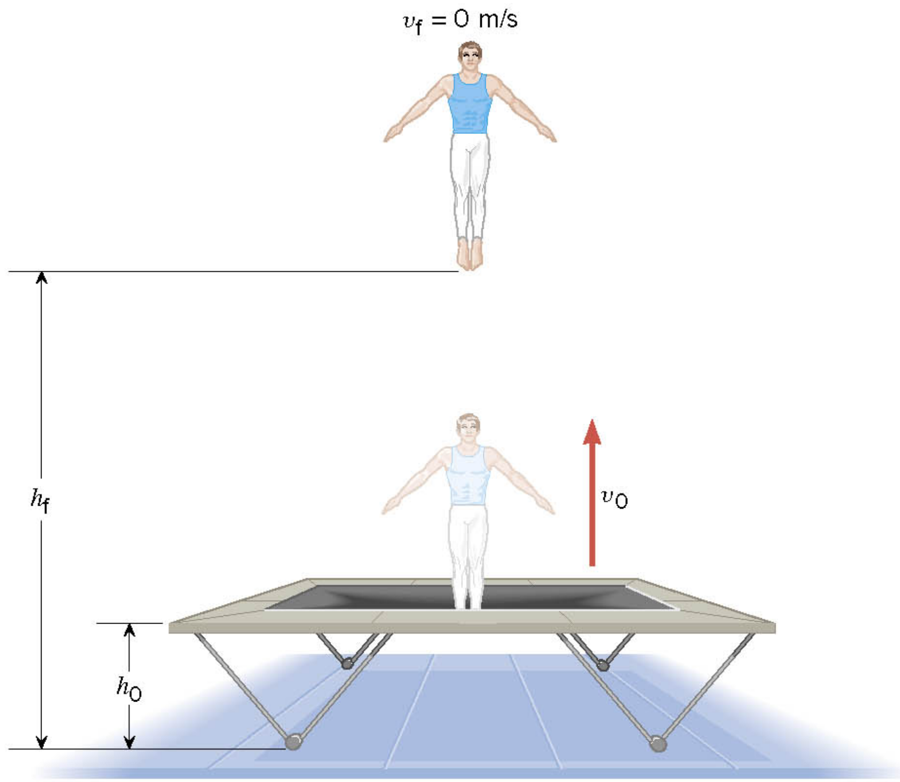
➔ Potential energy was spent in order for the gravitational force to increase the brick's kinetic energy.

Ex. 7 A Gymnast on a Trampoline

A gymnast leaves the trampoline at an initial height of 1.20 m and reaches a maximum height of 4.80 m before falling back down. What was the initial speed of the gymnast?



(a)

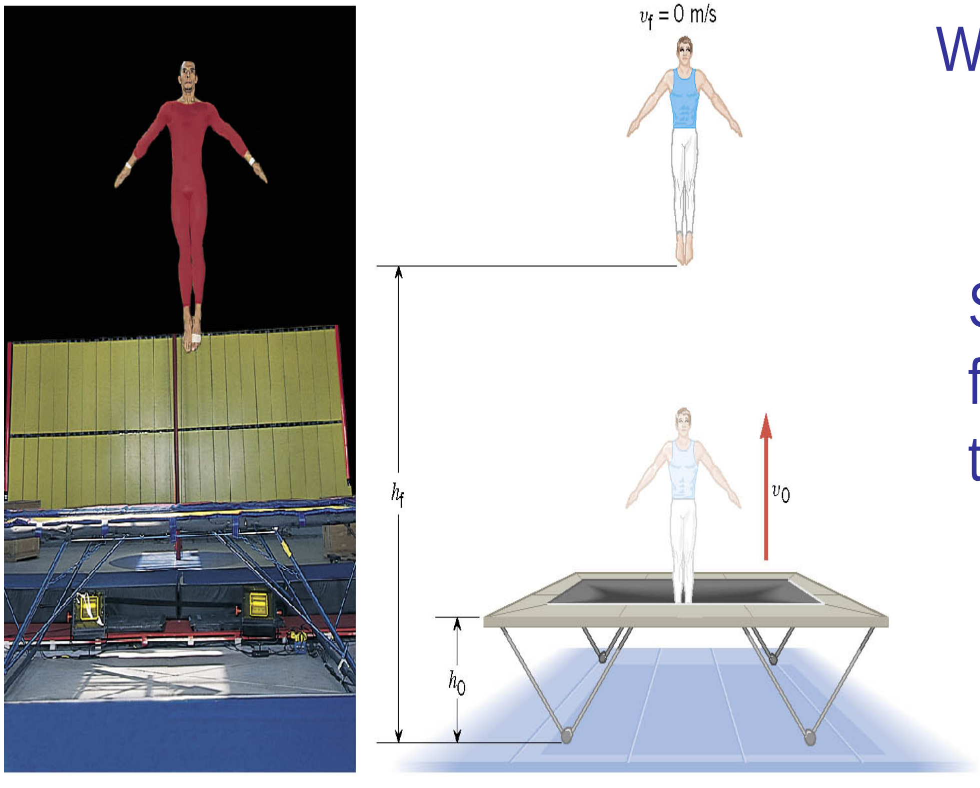


(b)

Ex. 7 Continued

From the work-kinetic energy theorem

$$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$



(a) (b)

Work done by the gravitational force

$$W_{\text{gravity}} = mg\left(h_o - h_f\right)$$

Since at the maximum height, the final speed is 0. Using work-KE theorem, we obtain

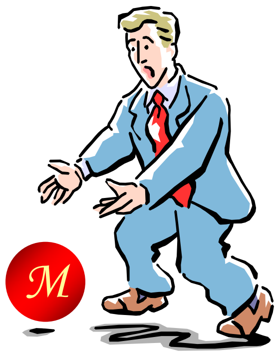
$$\not{m}g\left(h_o - h_f\right) = -\tfrac{1}{2}\not{m}v_o^2$$

$$v_o = \sqrt{-2g\left(h_o - h_f\right)}$$

$$\therefore v_o = \sqrt{-2\left(9.80\,\text{m/s}^2\right)\left(1.20\text{ m} - 4.80\text{ m}\right)} = 8.40\,\text{m/s}$$

Example for Potential Energy

A bowler drops bowling ball of mass 7kg on his toe. Choosing the floor level as y=0, estimate the total work done on the ball by the gravitational force as the ball falls on the toe.



Let's assume the top of the toe is 0.03m from the floor and the hand was 0.5m above the floor.

$$U_i = mgy_i = 7 \times 9.8 \times 0.5 = 34.3J \quad U_f = mgy_f = 7 \times 9.8 \times 0.03 = 2.06J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.24J \cong 30J$$

b) Perform the same calculation using the top of the bowler's head as the origin.

What has to change?

First we must re-compute the positions of the ball in his hand and on his toe.

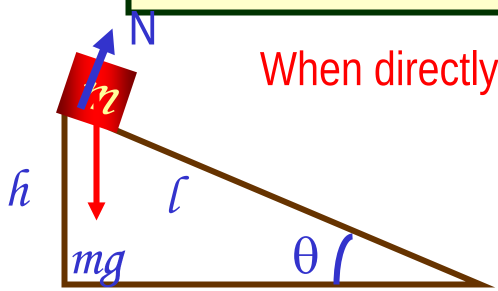
Assuming the bowler's height is 1.8m, the ball's original position is –1.3m, and the toe is at –1.77m.

$$U_i = mgy_i = 7 \times 9.8 \times \left(-1.3\right) = -89.2J \quad U_f = mgy_f = 7 \times 9.8 \times \left(-1.77\right) = -121.4J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.2J \cong 30J$$

Conservative and Non-conservative Forces

The work done on an object by the gravitational force does not depend on the object's path in the absence of a retardation force.



When directly falls, the work done on the object by the gravitation force is $W_g = mgh$

When sliding down the hill of length l, the work is

$$W_g = F_{g-incline} \times l = mg\sin\theta \times l$$
$$= mg(l\sin\theta) = mgh$$

How about if we lengthen the incline by a factor of 2, keeping the height the same??

Still the same amount of work☺

$$W_g = mgh$$

So the work done by the gravitational force on an object is independent of the path of the object's movements. It only depends on the difference of the object's initial and final position in the direction of the force.

Forces like gravitational and elastic forces are called the conservative force

1. If the work performed by the force does not depend on the path.
2. If the work performed on a closed path is 0.

Total mechanical energy is conserved!!

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

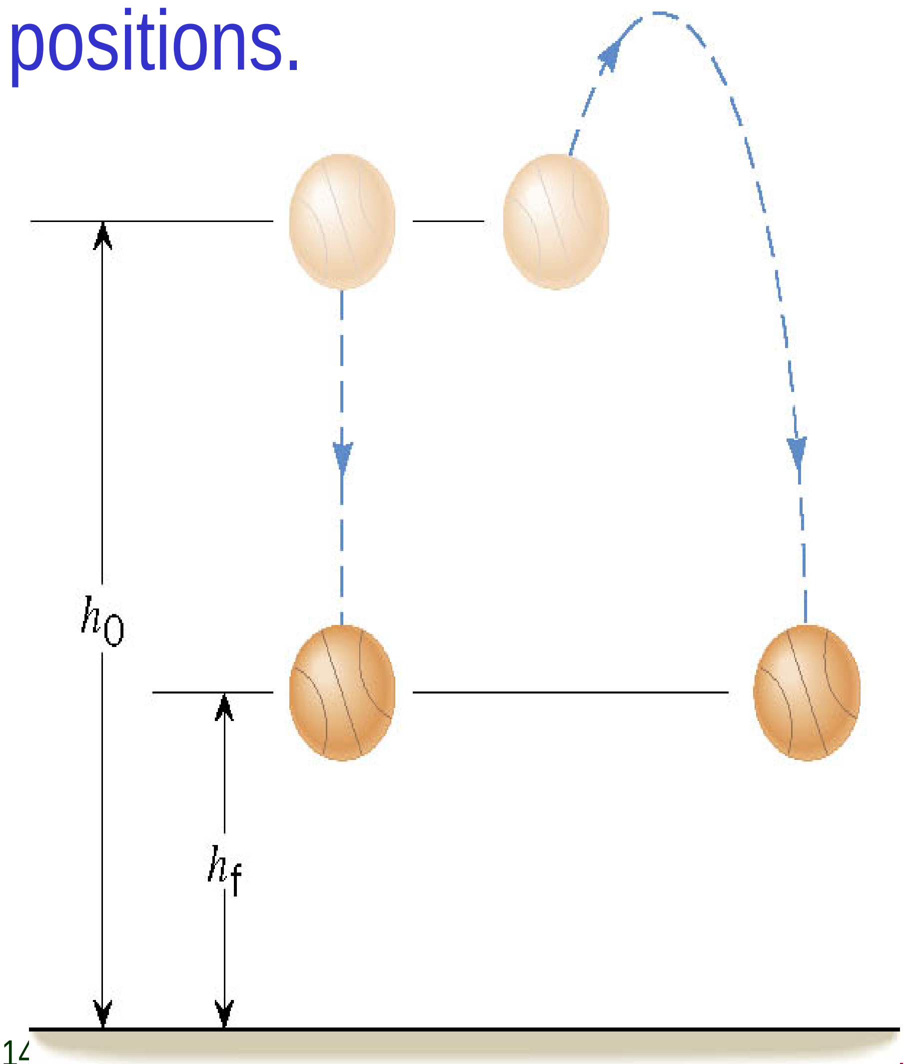
A force is conservative when the work it does on a moving object is independent of the path between the object's initial and final positions.

The work done by the gravitational force on the object is

$$W_{\text{gravity}} =$$

$$= mgh_o - mgh_f$$

$$= mg\left(h_o - h_f\right)$$



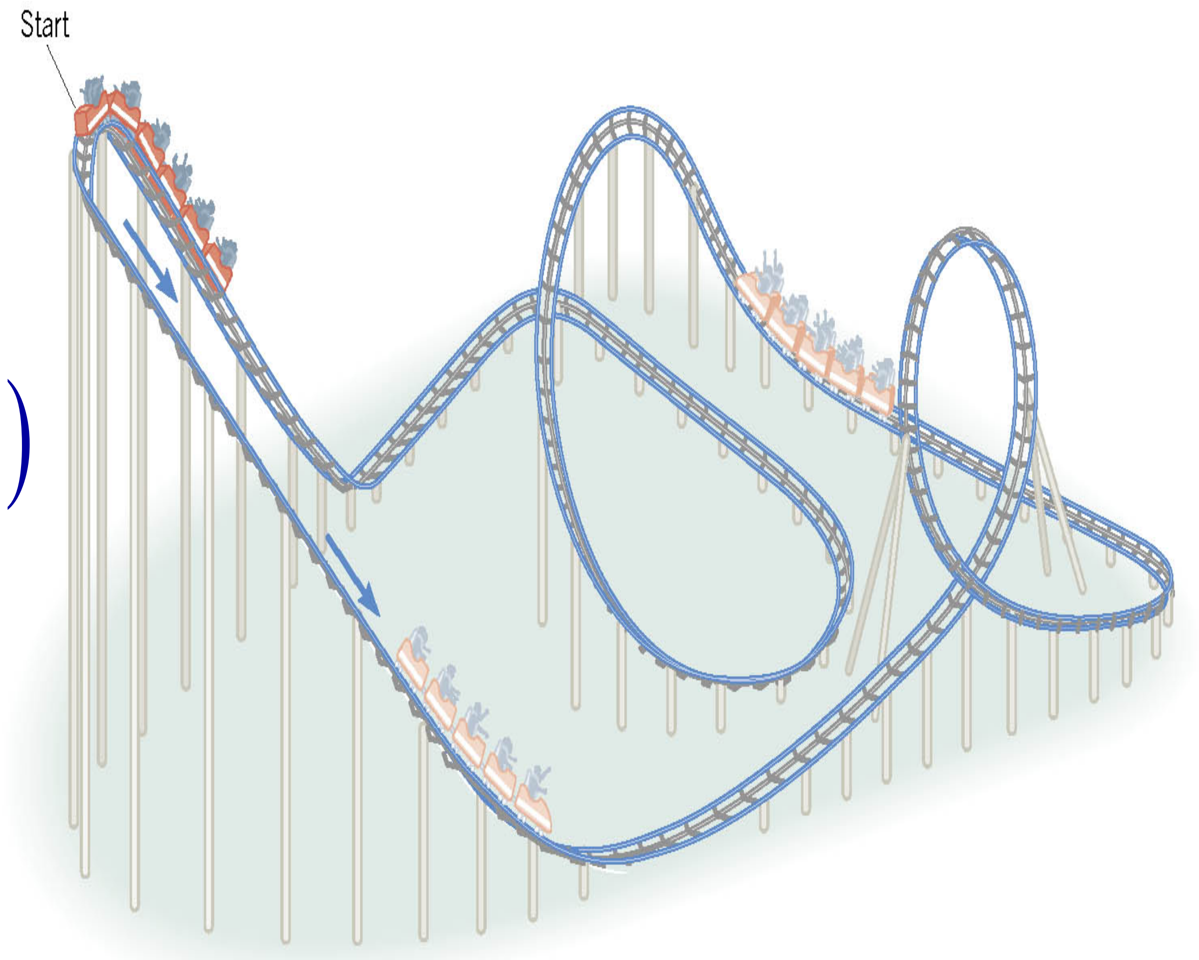
A force is conservative when it does no work on an object moving around a closed path, starting and finishing at the same point.

The work done by the gravitational force on the object is

$$W_{\text{gravity}} = mg\left(h_o - h_f\right)$$

since $h_o = h_f$

$$W_{\text{gravity}} = 0$$



So what is the conservative force again?

- A force is conservative when the work it does on a moving object is independent of the path between the object's initial and final positions.
- A force is conservative when it does no work on an object moving around a closed path, starting and finishing at the same point.
- The work is done by a conservative force, the total mechanical energy of the system is conserved!

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

Some examples of conservative and non-conservative forces

Table 6.2 Some Conservative and Nonconservative Forces

Conservative Forces

Gravitational force (Ch. 4)

Elastic spring force (Ch. 10)

Electric force (Ch. 18, 19)

Nonconservative Forces

Static and kinetic frictional forces

Air resistance

Tension

Normal force

Propulsion force of a rocket

Non-conservative force

An example of a non-conservative force is the kinetic frictional force.

$$W = (F\cos\theta)s = f_k \cos 180^{o} s = -f_k s$$

The work done by the kinetic frictional force is always negative. Thus, it is impossible for the work it does on an object that moves around a closed path to be zero.

The concept of potential energy is not defined for a non-conservative force.

Work-Energy Theorem

In normal situations both conservative and non-conservative forces act simultaneously on an object, so the work done by the net external force can be written as

$$W = W_c + W_{nc}$$

$$W = \mathrm{KE_f} - \mathrm{KE_o} = \Delta\mathrm{KE}$$

$$W_c = W_{\mathrm{gravity}} = mgh_o - mgh_f = \mathrm{PE_o} - \mathrm{PE_f} =$$

$$= -\left(\mathrm{PE_f} - \mathrm{PE_o}\right) = -\Delta\mathrm{PE}$$

$$\Delta\mathrm{KE} = -\Delta\mathrm{PE} + W_{nc}$$

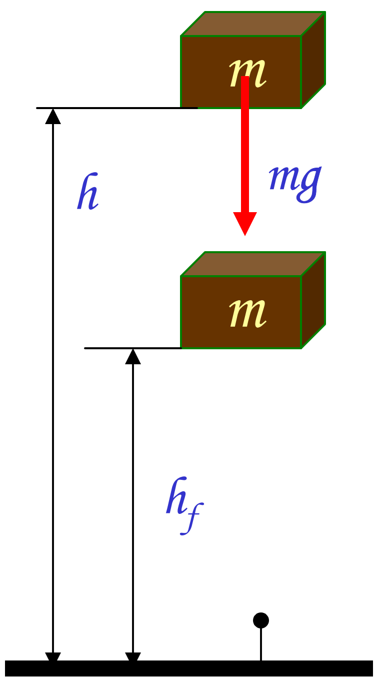
$$W_{nc} = \Delta\mathrm{KE} + \Delta\mathrm{PE}$$ THE WORK-ENERGY THEOREM

Work done by a non-conservative force causes changes in kinetic energy as well as the potential energy (or the total mechanical energy) of an object in motion.

Conservation of Mechanical Energy

Total mechanical energy is the sum of kinetic and potential energies

$$E \equiv KE + PE$$



Let's consider a brick of mass *m* at the height *h* from the ground

What is the brick's potential energy?

$$PE_g = mgh$$

What happens to the energy as the brick falls to the ground?

$$\Delta PE = PE_f - PE_0$$

The brick gains speed

By how much?

$$v = gt$$

So what?

The brick's kinetic energy increased

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(gt)^2$$

And?

The lost potential energy is converted to kinetic energy!!

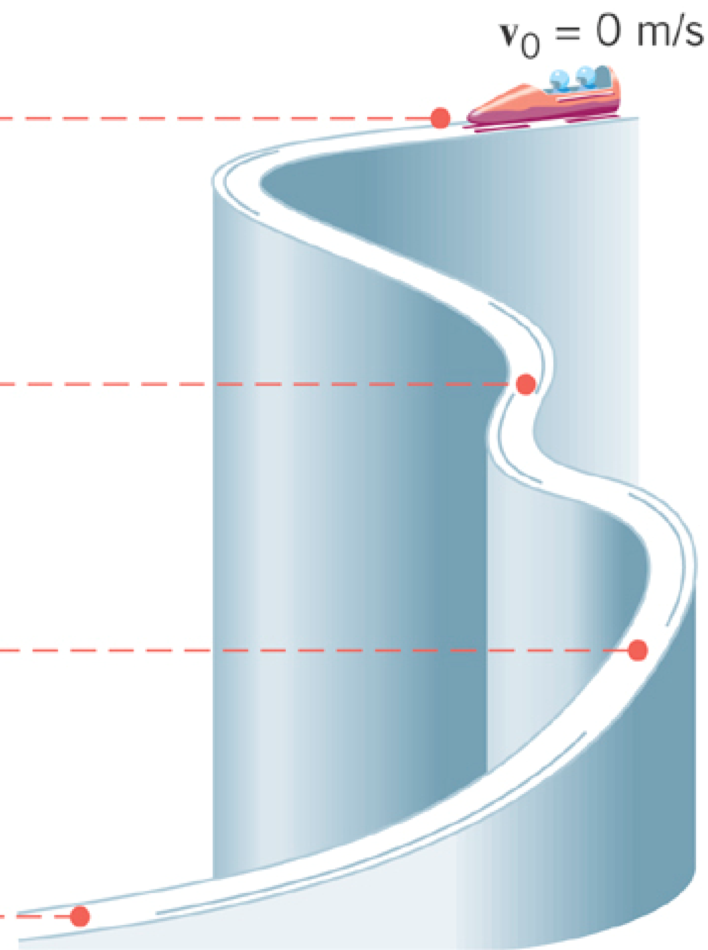
What does this mean?

*The total mechanical energy of a system remains constant in any isolated system of objects that interacts only through <u>conservative forces</u>: <u>**Principle of mechanical energy conservation**</u>*

$$E_i = E_f$$

$$K_i + \sum PE_0 = K_f + \sum PE_f$$

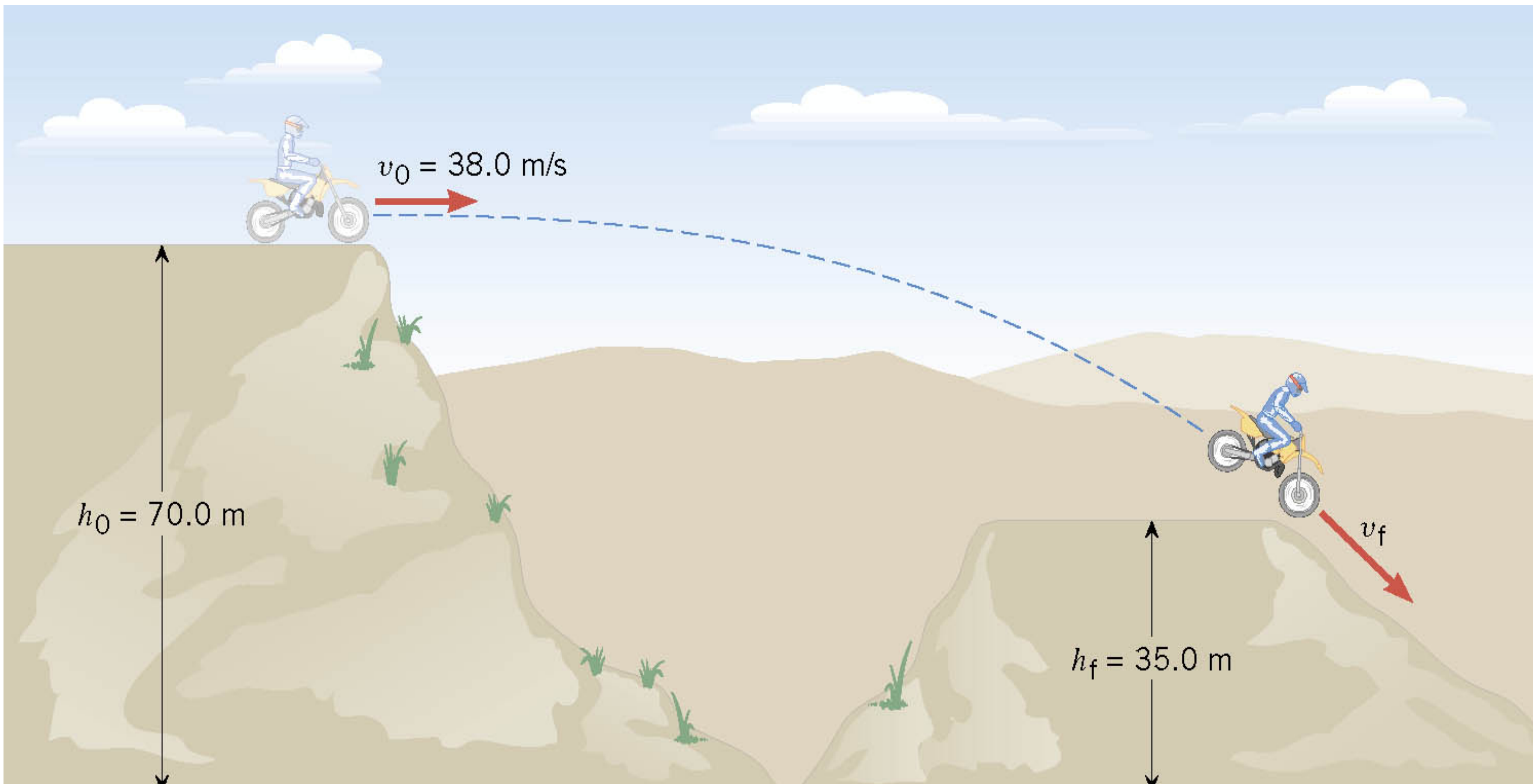
KE	PE	E = KE + PE
0 J	600 000 J	600 000 J
200 000 J	400 000 J	600 000 J
400 000 J	200 000 J	600 000 J
600 000 J	0 J	600 000 J



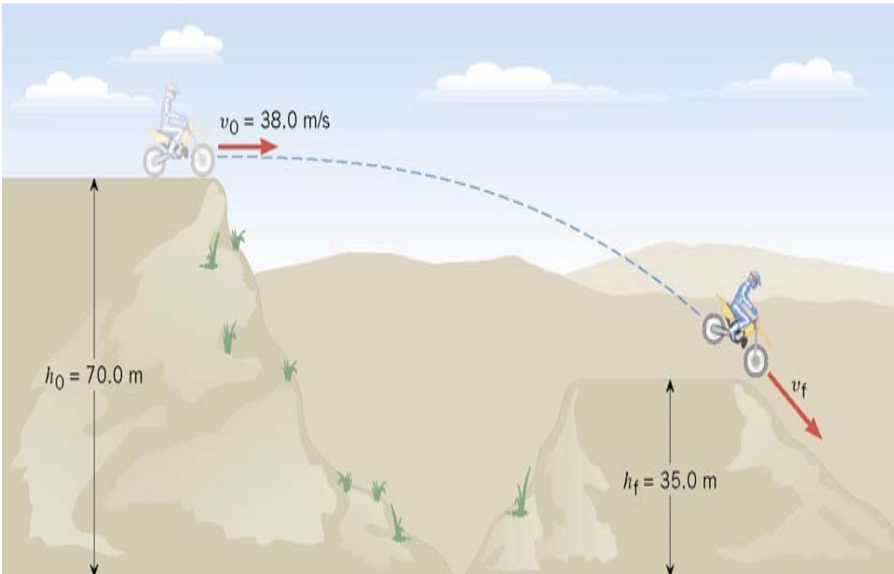
As the car drives down the frictionless hill, the total mechanical energy, which is the sum of KE and PE, stays the same. The form of the energy changes throughout the motion.

Ex.8: Daredevil Motorcyclist

A motorcyclist is trying to leap across the canyon by driving horizontally off a cliff 38.0 m/s. Ignoring air resistance, find the speed with which the cycle strikes the ground on the other side.



Using Mechanical Energy Conservation



	t=0	t=t
PE	mgh_o	mgh_f
+		
KE	$\frac{1}{2}mv_o^2$	$\frac{1}{2}mv_f^2$
ME	$mgh_o + \frac{1}{2}mv_o^2$	$mgh_f + \frac{1}{2}mv_f^2$

$$E_o = E_f = mgh_o + \frac{1}{2}mv_o^2 = mgh_f + \frac{1}{2}mv_f^2$$

$$\cancel{m}gh_o + \frac{1}{2}\cancel{m}v_o^2 = \cancel{m}gh_f + \frac{1}{2}\cancel{m}v_f^2$$

$$gh_o + \frac{1}{2}v_o^2 = gh_f + \frac{1}{2}v_f^2$$

$$v_f^2 = 2g\left(h_o - h_f\right) + v_0^2$$

Solve for v_f

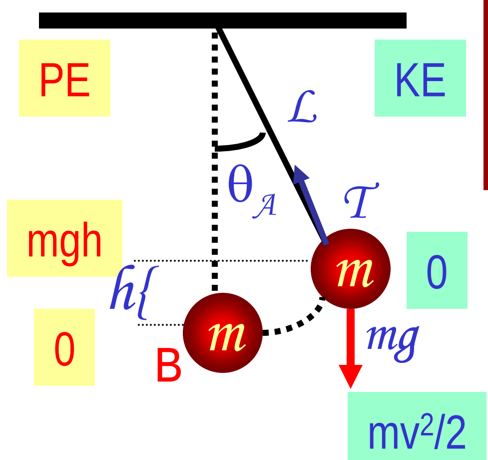
$$v_f = \sqrt{2g\left(h_o - h_f\right) + v_o^2}$$

$$v_f = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(35.0\,\text{m}\right) + \left(38.0\,\text{m/s}\right)^2} =$$

$$= 46.2\,\text{m/s}$$

Example

A ball of mass m is attached to a light cord of length L, making up a pendulum. The ball is released from rest when the cord makes an angle θ_A with the vertical, and the pivoting point P is frictionless. Find the speed of the ball when it is at the lowest point, B.



Compute the potential energy at the maximum height, h. Remember where the 0 is.

$$h = L - L\cos\theta_A = L(1-\cos\theta_A)$$

$$U_i = mgh = mgL(1-\cos\theta_A)$$

Using the principle of mechanical energy conservation

$$K_i + U_i = K_f + U_f$$

$$0 + mgh = mgL(1-\cos\theta_A) = \frac{1}{2}mv^2$$

$$v^2 = 2gL(1-\cos\theta_A)$$

$$\therefore v = \sqrt{2gL(1-\cos\theta_A)}$$

b) Determine tension T at the point B.

Using Newton's 2nd law of motion and recalling the centripetal acceleration of a circular motion

$$\sum F_r = T - mg = ma_r = m\frac{v^2}{r} = m\frac{v^2}{L}$$

$$T = mg + m\frac{v^2}{L} = m\left(g + \frac{v^2}{L}\right) = m\left(g + \frac{2gL(1-\cos\theta_A)}{L}\right)$$

$$= m\frac{gL + 2gL(1-\cos\theta_A)}{L}$$

$$\therefore T = mg(3 - 2\cos\theta_A)$$

Cross check the result in a simple situation. What happens when the initial angle θ_A is 0? $T = mg$

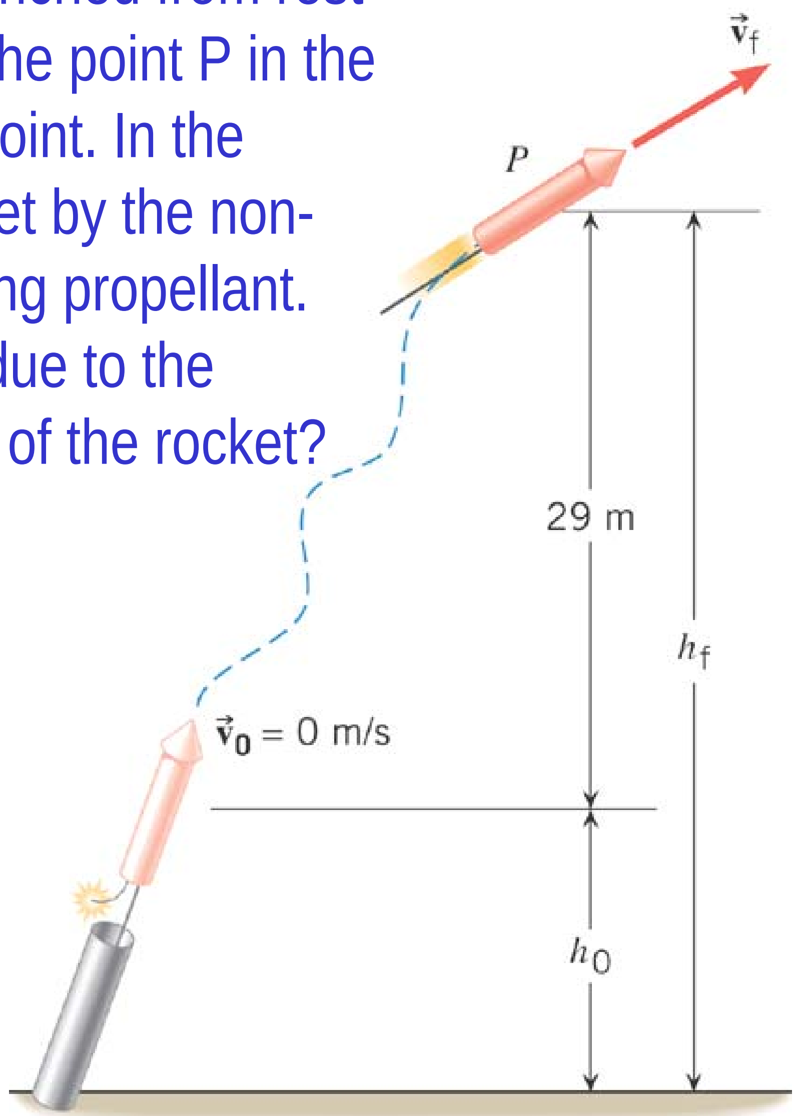
Ex. 12 Fireworks

A 0.2kg rocket in a fireworks display is launched from rest and follows an erratic flight path to reach the point P in the figure. Point P is 29m above the starting point. In the process, 425J of work is done on the rocket by the non-conservative force generated by the burning propellant. Ignoring air resistance and the mass lost due to the burning propellant, what is the final speed of the rocket?

$$W_{nc} = ME_f - ME_0$$

$$= \left(mgh_f + \tfrac{1}{2}mv_f^2\right)$$

$$-\left(mgh_o + \tfrac{1}{2}mv_o^2\right)$$



Now using Work-Energy Theorem

$$W_{nc} = \Delta PE + \Delta KE = mgh_f - mgh_o + \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

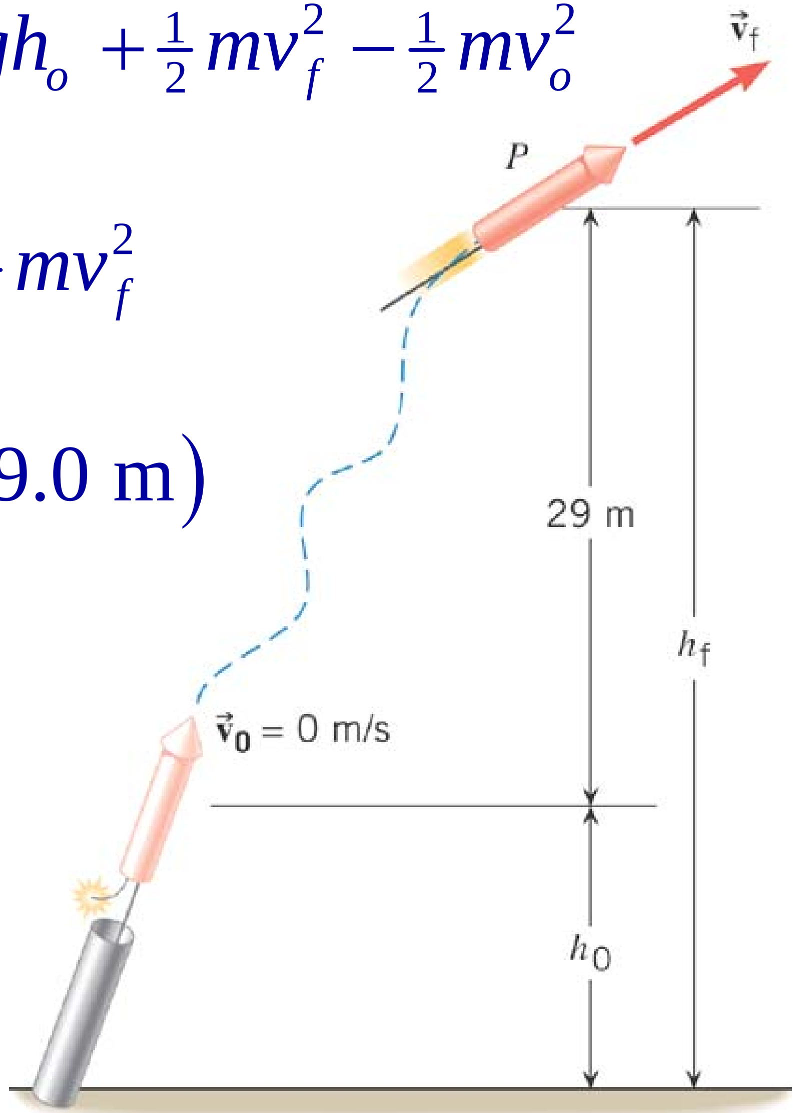
Since v_0=0

$$W_{nc} = mg\left(h_f - h_o\right) + \tfrac{1}{2}mv_f^2$$

$$425\ \mathrm{J} = (0.20\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(29.0\ \mathrm{m})$$

$$+\tfrac{1}{2}(0.20\ \mathrm{kg})v_f^2$$

$$v_f = 61\mathrm{m/s}$$



Power

- Rate at which the work is done or the energy is transferred
  - What is the difference for the same car with two different engines (4 cylinder and 8 cylinder) climbing the same hill?
  - ➔ The time… 8 cylinder car climbs up the hill faster!

Is the total amount of work done by the engines different? NO

Then what is different? The rate at which the same amount of work performed is higher for 8 cylinders than 4.

Average power
$$\overline{P} \equiv \frac{\Delta W}{\Delta t} = \frac{Fs}{\Delta t} = F\frac{s}{\Delta t} = F\,\overline{v}$$
Scalar quantity

Unit? $J/s = Watts$ $\quad 1HP \equiv 746Watts$

What do power companies sell? $1kWH = 1000Watts \times 3600s = 3.6 \times 10^{6}\,J$

Energy

Human Metabolic Rates

Activity	Rate (watts)
Running (15 km/h)	1340 W
Skiing	1050 W
Biking	530 W
Walking (5 km/h)	280 W
Sleeping	77 W

[a]For a young 70-kg male.

Energy Loss in Automobile

Automobile uses only 13% of its fuel to propel the vehicle.

Why?

67% in the engine:
- Incomplete burning
- Heat
- Sound

16% in friction in mechanical parts

4% in operating other crucial parts such as oil and fuel pumps, etc

13% used for balancing energy loss related to moving vehicle, like air resistance and road friction to tire, etc

Two frictional forces involved in moving vehicles

$$m_{car} = 1450kg \quad Weight = mg = 14200N$$

Coefficient of Rolling Friction; μ=0.016

$$\mu n = \mu mg = 227N$$

Air Drag

$$f_a = \frac{1}{2}D\rho A v^2 = \frac{1}{2}\times 0.5\times 1.293\times 2v^2 = 0.647v^2$$

Total Resistance

$$f_t = f_r + f_a$$

Total power to keep speed v=26.8m/s=60mi/h

$$P = f_t v = (691N)\cdot 26.8 = 18.5kW$$

Power to overcome each component of resistance

$$P_r = f_r v = (227)\cdot 26.8 = 6.08kW$$

$$P_a = f_a v = (464.7)\cdot 26.8 = 12.5kW$$

Ex. 13 The Power to Accelerate a Car

A 1.10×10^3kg car, starting from rest, accelerates for 5.00s. The magnitude of the acceleration is $a=4.60m/s^2$. Determine the average power generated by the net force that accelerates the vehicle.

What is the force that accelerates the car?

$$F = ma = \left(1.10 \times 10^3\right) \cdot \left(4.60\, m/s^2\right) = 5060\, N$$

Since the acceleration is constant, we obtain

$$\overline{v} = \frac{v_0 + v_f}{2} = \frac{0 + v_f}{2} = \frac{v_f}{2}$$

From the kinematic formula

$$v_f = v_0 + at = 0 + \left(4.60\, m/s^2\right) \cdot \left(5.00 s\right) = 23.0\, m/s$$

Thus, the average speed is

$$\frac{v_f}{2} = \frac{23.0}{2} = 11.5\, m/s$$

And, the average power is

$$\overline{P} = F\overline{v} = \left(5060\, N\right) \cdot \left(11.5\, m/s\right) = 5.82 \times 10^4 W$$
$$= 78.0 hp$$