PHYS 1441 – Section 001
Lecture # 12

Thursday, June 19, 2008
Dr. Jaehoon Yu

- Center of Mass

- Fundamentals of Rotational Motion

- Equations of Rotational Kinematics

- Relationship Between Linear and Rotational Quantities

- Rolling Motion

- Rotational Dynamics

Today's homework is homework #7, due 9pm, Monday, June 23!!

Announcements

- Please redownload homework #7 if you downloaded before 4:10pm yesterday.
- I gave all of you 100% for problem #1 homework #5 since its computer answers was incorrect
  - In the process of doing this, I made a mistake of setting problem #1 of homework #6 incorrectly. So I removed the problem from the list. Sorry....
- Quiz next Monday, June 23
  - Beginning of the class
  - Covers CH 6.7 – what we cover today
- Don't forget the Mid-term grade discussions
  - Next Monday, June 23, bottom half of the class
- Final exam
  - 8 – 10am, Monday, June 30, in SH103
  - Comprehensive exam: Covers CH 1 – What we finish Thursday, June 26

Reminder: Extra-Credit Special Project

- Derive the formula for the final velocities of the two objects which underwent an elastic collision as a function of known quantities m_1, m_2, v_{01} and v_{02} in page 19 of yesterday's lecture note. Must be done in far greater detail than what is covered in the lecture note.

    - 20 points extra credit

- Describe in detail what happens to the final velocities if $m_1 = m_2$.

    - 5 point extra credit

- Due: Start of the class next Wednesday, June 25

PHYS 1441-001, Summer 2008
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Center of Mass

We've been solving physical problems treating objects as sizeless points with masses, but in realistic situations objects have shapes with masses distributed throughout the body.

Center of mass of a system is the average position of the system's mass and represents the motion of the system as if all the mass is on that point.

What does above statement tell you concerning the forces being exerted on the system?

The total external force exerted on the system of total mass M causes the center of mass to move at an acceleration given by $\vec{a} = \sum \vec{F} / M$ *as if all the mass of the system is concentrated on the center of mass.*
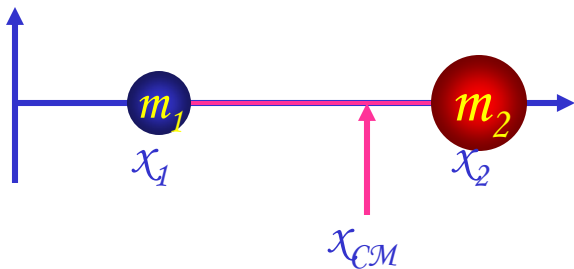
Consider a massless rod with two balls attached at either end.

The position of the center of mass of this system is the mass averaged position of the system

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

CM is closer to the heavier object

Example for Center of Mass

Thee people of roughly equivalent mass M on a lightweight (air-filled) banana boat sit along the x axis at positions $x_1=1.0m$, $x_2=5.0m$, and $x_3=6.0m$. Find the position of CM.
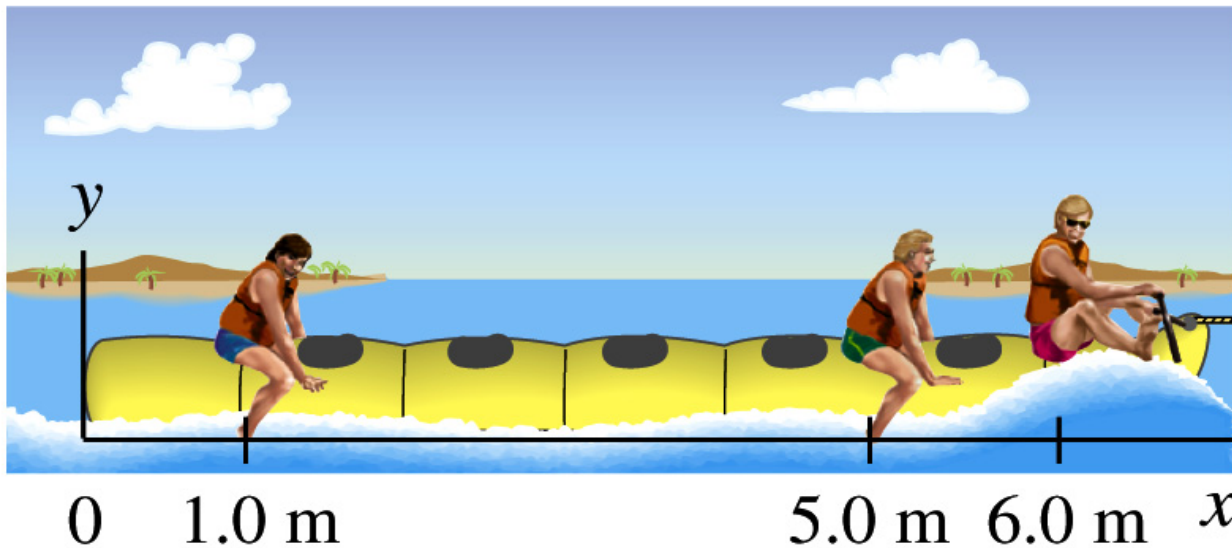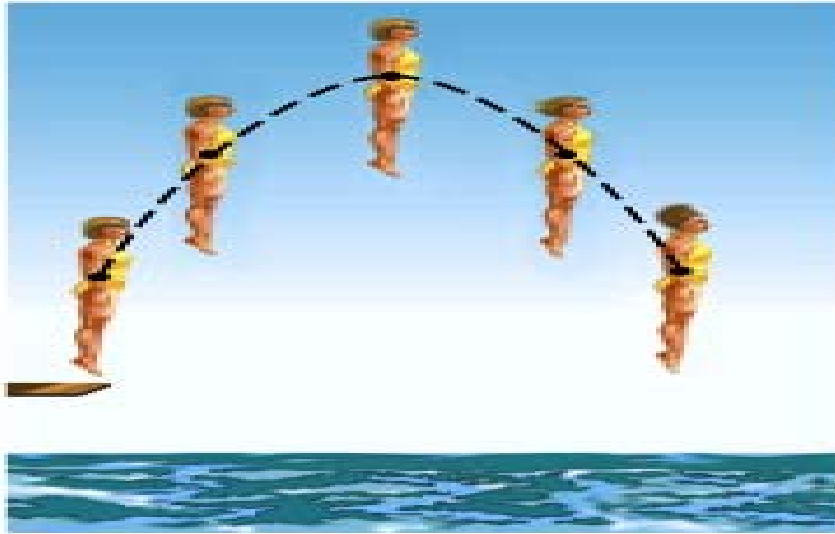

Using the formula for CM

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$= \frac{M \cdot 1.0 + M \cdot 5.0 + M \cdot 6.0}{M + M + M} = \frac{12.0M}{3M} = 4.0(m)$$

Motion of a Diver and the Center of Mass

(a)

Diver performs a simple dive. The motion of the center of mass follows a parabola since it is a projectile motion.
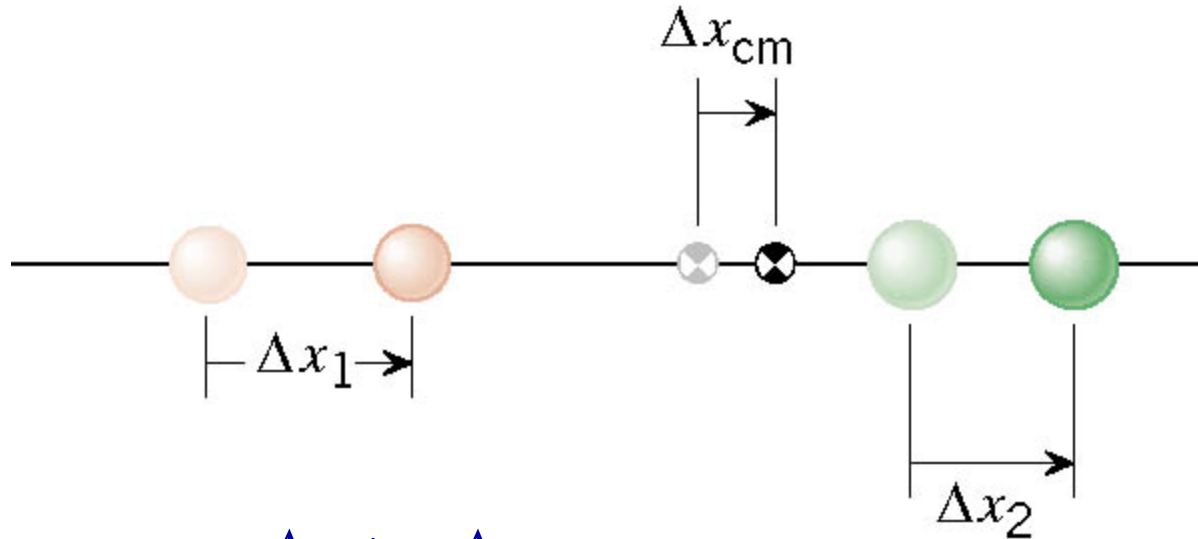

(b)

Diver performs a complicated dive. The motion of the center of mass still follows the same parabola since it still is a projectile motion.

The motion of the center of mass of the diver is always the same.

Velocity of Center of Mass

$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

$$\Rightarrow \quad v_{cm} = \frac{\Delta x_{cm}}{\Delta t} = \frac{m_1 \left(\Delta x_1 / \Delta t\right) + m_2 \left(\Delta x_2 / \Delta t\right)}{m_1 + m_2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

Another Look at the Ice Skater Problem

Starting from rest, two skaters push off against each other on ice where friction is negligible. One is a 54-kg woman and one is a 88-kg man. The woman moves away with a speed of +2.5 m/s. What is the velocity of the center of mass of the two?

$$v_{10} = 0 \, m/s \qquad v_{20} = 0 \, m/s$$
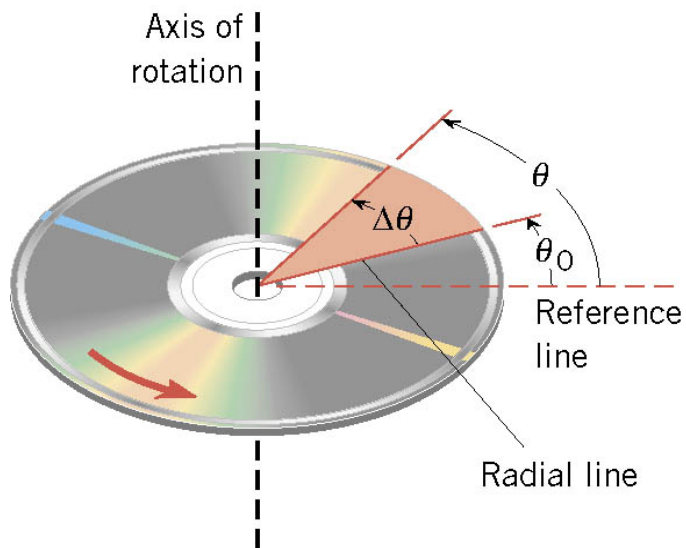
$$v_{cm0} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$

$$v_{1f} = +2.5 \, m/s \qquad v_{2f} = -1.5 \, m/s$$

$$v_{cmf} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2}$$

$$= \frac{54 \cdot (+2.5) + 88 \cdot (-1.5)}{54 + 88} = \frac{3}{142} = 0.02 \approx 0 \, m/s$$

m_2 m_1

(a) Before

v_{f2} v_{f1}

(b) After
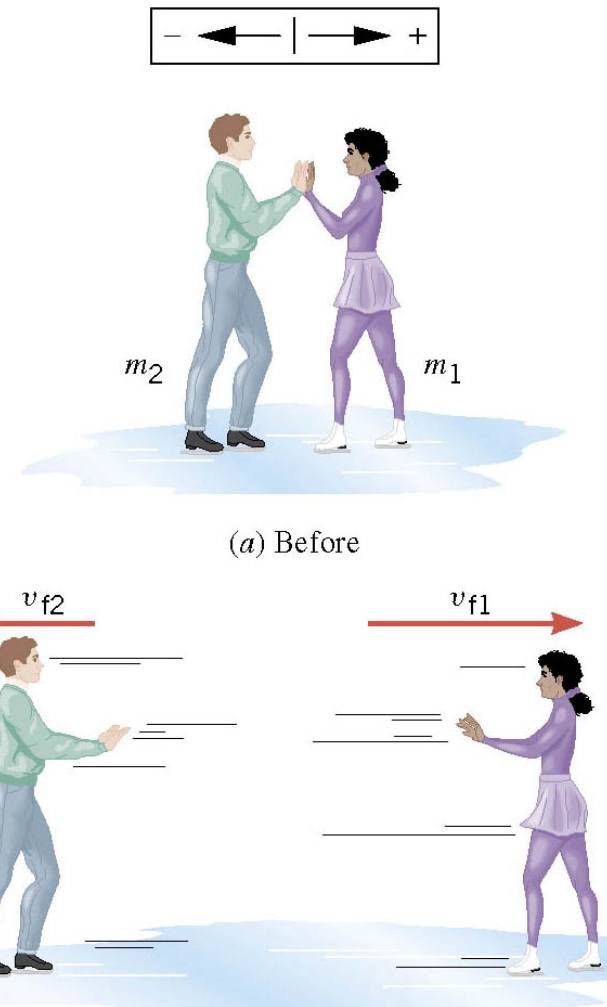
Rotational Motion and Angular Displacement

In the simplest kind of rotation, points on a rigid object move on circular paths around an *axis of rotation.*

Axis of rotation

The angle swept out by a line passing through any point on the body and intersecting the axis of rotation perpendicularly is called the *angular displacement.*
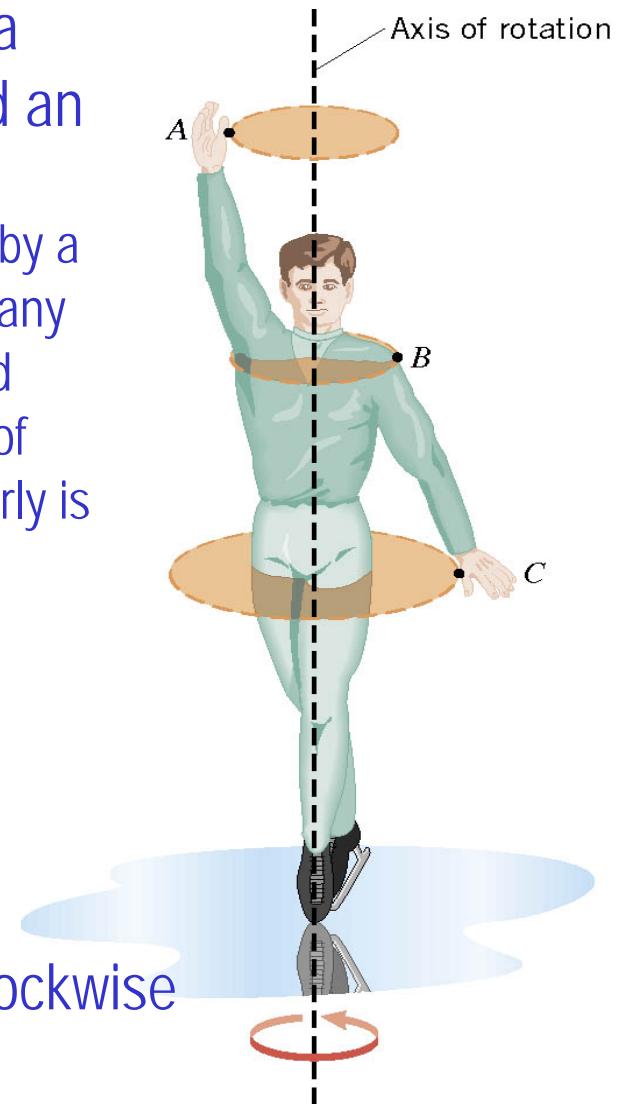
Axis of rotation

θ

$\Delta\theta$

θ_0

Reference line

Radial line

A

B

C

$$\Delta\theta = \theta - \theta_o$$

It's a vector!! So there must be directions…
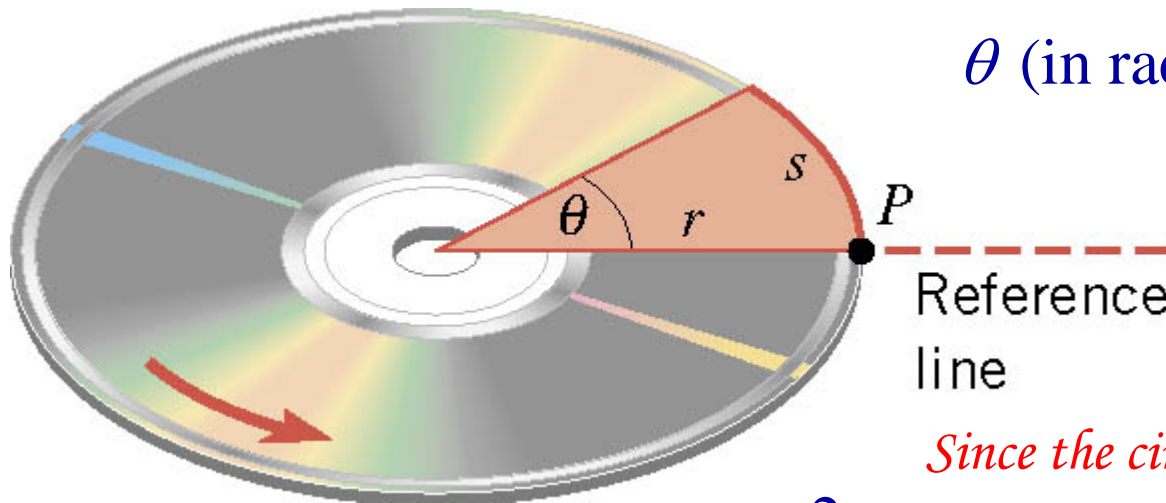
How do we define directions?   +:if counter-clockwise
                                 -:if clockwise

The direction vector points gets determined based on the right-hand rule.   These are just conventions!!

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SI Unit of the Angular Displacement

$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$

Dimension? None

For one full revolution:

Since the circumference of a circle is $2\pi r$

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ rad} \quad \Longrightarrow \quad 2\pi \text{ rad} = 360°$$

How many degrees is in one radian?

1 radian is $\quad 1 \text{ rad} = \frac{360°}{2\pi} \cdot 1rad = \frac{180°}{\pi} \cdot 1rad \cong \frac{180°}{3.14} \cdot 1rad \cong 57.3°$

How many radians is in one degree?

And one degrees is $\quad 1° = \frac{2\pi}{360°} \cdot 1° = \frac{\pi}{180°} \cdot 1° \cong \frac{3.14}{180°} \cdot 1° \cong 0.0175rad$

How many radians are in 10.5 revolutions? $10.5rev = 10.5rev \cdot 2\pi \frac{rad}{rev} = 21\pi \left(rad\right)$

Very important: In solving angular problems, all units, degrees or revolutions, must be converted to radians.

Ex.1 Adjacent Synchronous Satellites

Synchronous satellites are put into an orbit whose radius is 4.23×10^7 m. If the angular separation of the two satellites is 2.00 degrees, find the arc length that separates them.
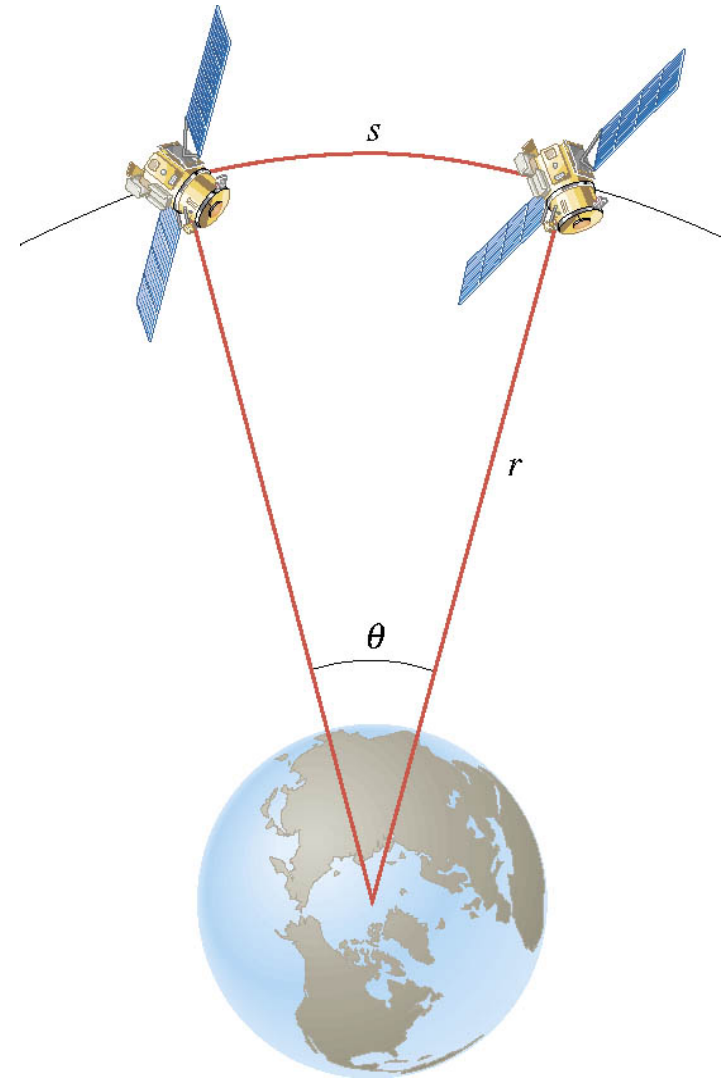
What do we need to find out? The Arc length!!!

$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$

Convert degrees to radians

$$2.00 \deg \left(\frac{2\pi \text{ rad}}{360 \deg} \right) = 0.0349 \text{ rad}$$

$$s = r\theta = \left(4.23 \times 10^7 \text{ m} \right) \left(0.0349 \text{ rad} \right)$$

$$= 1.48 \times 10^6 \text{ m} \quad (920 \text{ miles})$$

Ex. 2 A Total Eclipse of the Sun

The diameter of the sun is about 400 times greater than that of the moon. By coincidence, the sun is also about 400 times farther from the earth than is the moon. For an observer on the earth, compare the angle subtended by the moon to the angle subtended by the sun and explain why this result leads to a total solar eclipse.

$$\theta \text{ (in radians)} =$$

$$\frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$

(a)

(b)
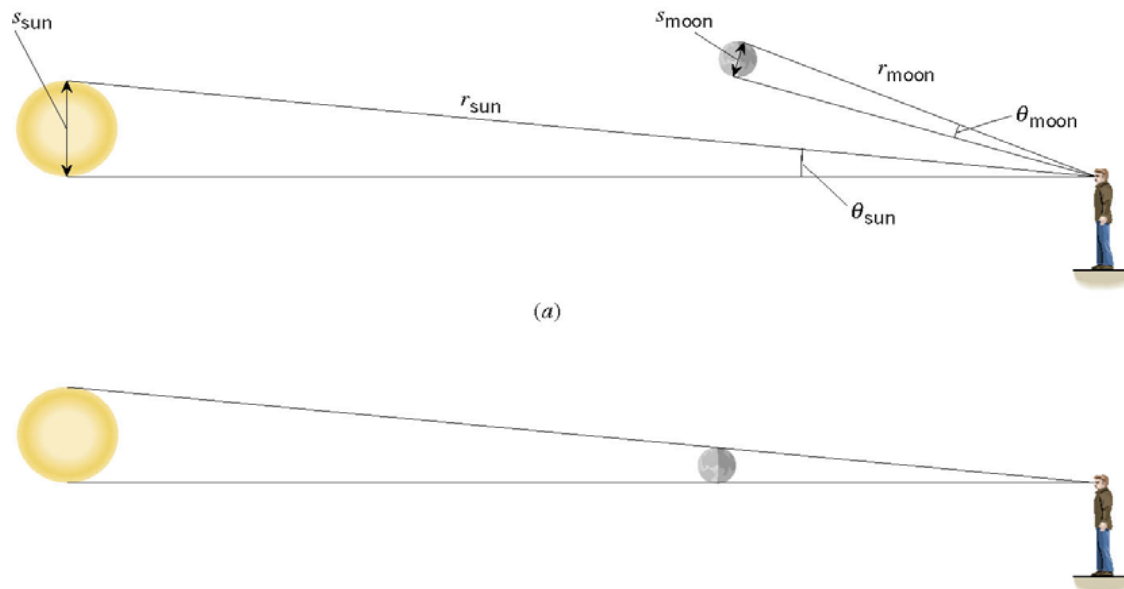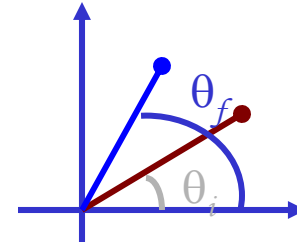
I can even cover the entire sun with my thumb!! Why?

Because the distance (r) from my eyes to my thumb is far shorter than that to the sun.

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Angular Displacement, Velocity, and Acceleration

Angular displacement is defined as

$$\Delta\theta = \theta_f - \theta_i$$

How about the average angular velocity, the rate of change of angular displacement?

$$\overline{\omega} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Unit?   rad/s   Dimension? $[T^{-1}]$

By the same token, the average angular acceleration, rate of change of the angular velocity, is defined as…

$$\overline{\alpha} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Unit?   rad/s^2 Dimension? $[T^{-2}]$

When rotating about a fixed axis, every particle on a rigid object rotates through the same angle and has the same angular speed and angular acceleration.

Ex. 3 Gymnast on a High Bar

A gymnast on a high bar swings through two revolutions in a time of 1.90 s. Find the average angular velocity of the gymnast.

What is the angular displacement?

$$\Delta\theta = -2.00 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$$

$$= -12.6 \text{ rad}$$

Why negative? Because he is rotating clockwise!!

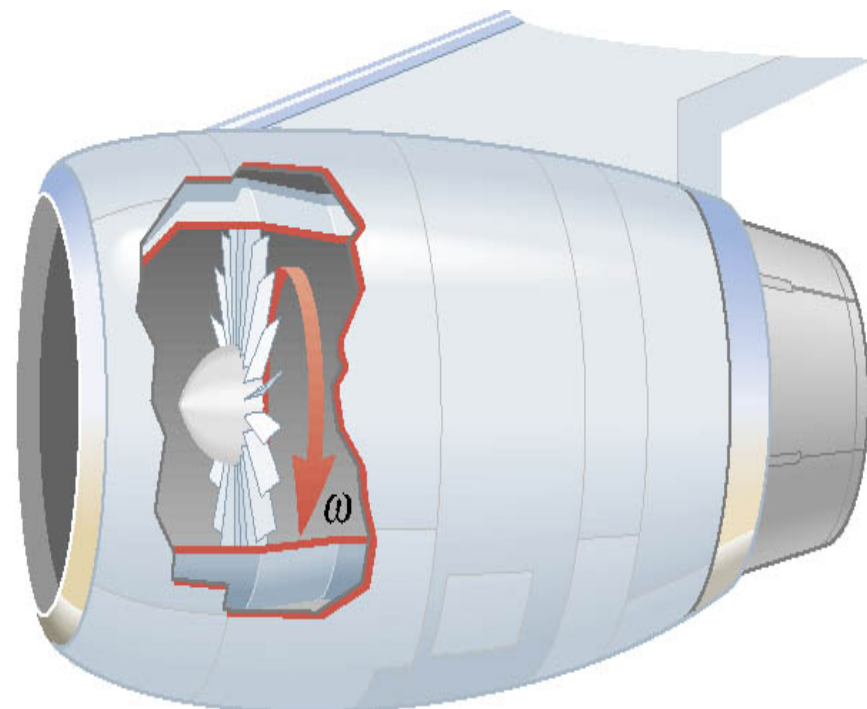$$\overline{\omega} = \frac{-12.6 \text{ rad}}{1.90 \text{ s}} = -6.63\,\text{rad/s}$$

PHYS 1441-001, S

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Ex. 4 A Jet Revving Its Engines

As seen from the front of the engine, the fan blades are rotating with an angular speed of -110 rad/s. As the plane takes off, the angular velocity of the blades reaches -330 rad/s in a time of 14 s. Find the angular acceleration, assuming it to be constant.

$$\overline{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

$$= \frac{\left(-330\,\mathrm{rad/s}\right) - \left(-110\,\mathrm{rad/s}\right)}{14\ \mathrm{s}} = -16\,\mathrm{rad/s}^2$$

Rotational Kinematics

The first type of motion we have learned in linear kinematics was under a constant acceleration. We will learn about the rotational motion under constant angular acceleration, because these are the simplest motions in both cases.

Just like the case in linear motion, one can obtain

Angular velocity under constant angular acceleration:

Linear kinematics $v = v_o + at$

$$\omega_f = \omega_0 + \alpha t$$

Angular displacement under constant angular acceleration:

Linear kinematics $x_f = x_0 + v_o t + \frac{1}{2} a t^2$

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

One can also obtain

Linear kinematics $v_f^2 = v_o^2 + 2a\left(x_f - x_i\right)$

$$\omega_f^2 = \omega_0^2 + 2\alpha\left(\theta_f - \theta_0\right)$$

Problem Solving Strategy

- Visualize the problem by drawing a picture
- Decide which directions are to be called positive (+) and negative (-).
- Write down the values that are given for any of the five kinematic variables and convert them to SI units.
- Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.
- When the motion is divided into segments, remember that the final angular velocity of one segment is the initial velocity for the next.
- Keep in mind that there may be two possible answers to a kinematics problem.

Ex. 5 Blending with a Blender

The blades are whirling with an angular velocity of +375 rad/s when the "puree" button is pushed in. When the "blend" button is pushed, the blades accelerate and reach a greater angular velocity after the blades have rotated through an angular displacement of +44.0 rad. The angular acceleration has a constant value of +1740 rad/s². Find the final angular velocity of the blades.

θ	α	ω	ω_o	t
+44.0rad	+1740rad/s²	?	+375rad/s	

Which kinematics eq? $\omega^2 = \omega_o^2 + 2\alpha\theta$

$$\omega = \pm\sqrt{\omega_o^2 + 2\alpha\theta}$$

$$= \pm\sqrt{(375\,\text{rad/s})^2 + 2(1740\,\text{rad/s}^2)(44.0\,\text{rad})} = \pm542\,\text{rad/s}$$
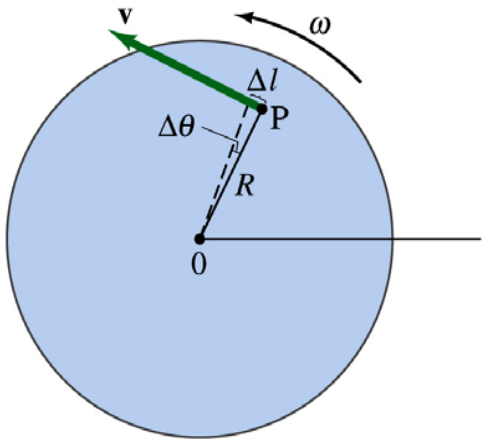
Which sign? $\omega = +542\,\text{rad/s}$ *Why?* *Because it is accelerating in counter-clockwise!*

Relationship Between Angular and Linear Quantities

What do we know about a rigid object that rotates about a fixed axis of rotation?

Every particle (or masslet) in the object moves in a circle centered at the same axis of rotation.

When a point rotates, it has both the linear and angular components in its motion.

What is the linear component of the motion you see?

Linear velocity along the tangential direction.

How do we related this linear component of the motion with angular component?

The direction of ω follows a right-hand rule.

The arc-length is $\boxed{l = r\theta}$ So the tangential speed v is $v = \dfrac{\Delta l}{\Delta t} = \dfrac{\Delta(r\theta)}{\Delta t} = r\dfrac{\Delta\theta}{\Delta t} = r\omega$

What does this relationship tell you about the tangential speed of the points in the object and their angular speed?:

Although every particle in the object has the same angular speed, its tangential speed differs and is proportional to its distance from the axis of rotation.

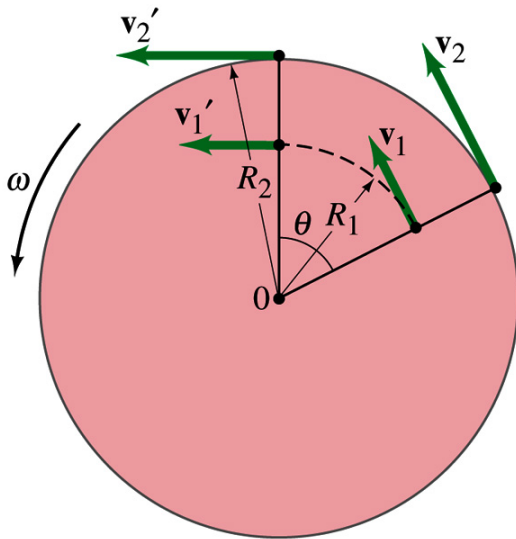The farther away the particle is from the center of rotation, the higher the tangential speed.

PHYS 144

Dr

Is the lion faster than the horse?

A rotating carousel has one child sitting on a horse near the outer edge and another child on a lion halfway out from the center. (a) Which child has the greater linear speed? (b) Which child has the greater angular speed?

(a) Linear speed is the distance traveled divided by the time interval. So the child sitting at the outer edge travels more distance within the given time than the child sitting closer to the center. Thus, the horse is faster than the lion.

(b) Angular speed is the angle traveled divided by the time interval. The angle both the children travel in the given time interval is the same. Thus, both the horse and the lion have the same angular speed.

How about the acceleration?
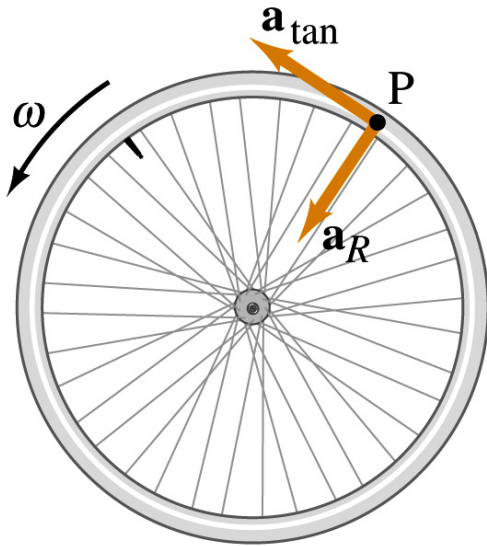
How many different linear acceleration components do you see in a circular motion and what are they? **Two**

Tangential, a_t, and the radial acceleration, a_r

Since the tangential speed v is $\quad v_T = r\omega$

The magnitude of tangential acceleration a_t is

$$a_t = \frac{v_{Tf} - v_{T0}}{\Delta t} = \frac{r\omega_f - r\omega_0}{\Delta t} = r\frac{\omega_f - \omega_0}{\Delta t} = r\alpha$$

What does this relationship tell you?

Although every particle in the object has the same angular acceleration, its tangential acceleration differs proportional to its distance from the axis of rotation.

The radial or centripetal acceleration a_r is

$$a_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

What does this tell you?

The father away the particle is from the rotation axis, the more radial acceleration it receives. In other words, it receives more centripetal force.

Total linear acceleration is
$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$$

Ex. 6 A Helicopter Blade

A helicopter blade has an angular speed of 6.50 rev/s and an angular acceleration of 1.30 rev/s². For point 1 on the blade, find the magnitude of (a) the tangential speed and (b) the tangential acceleration.
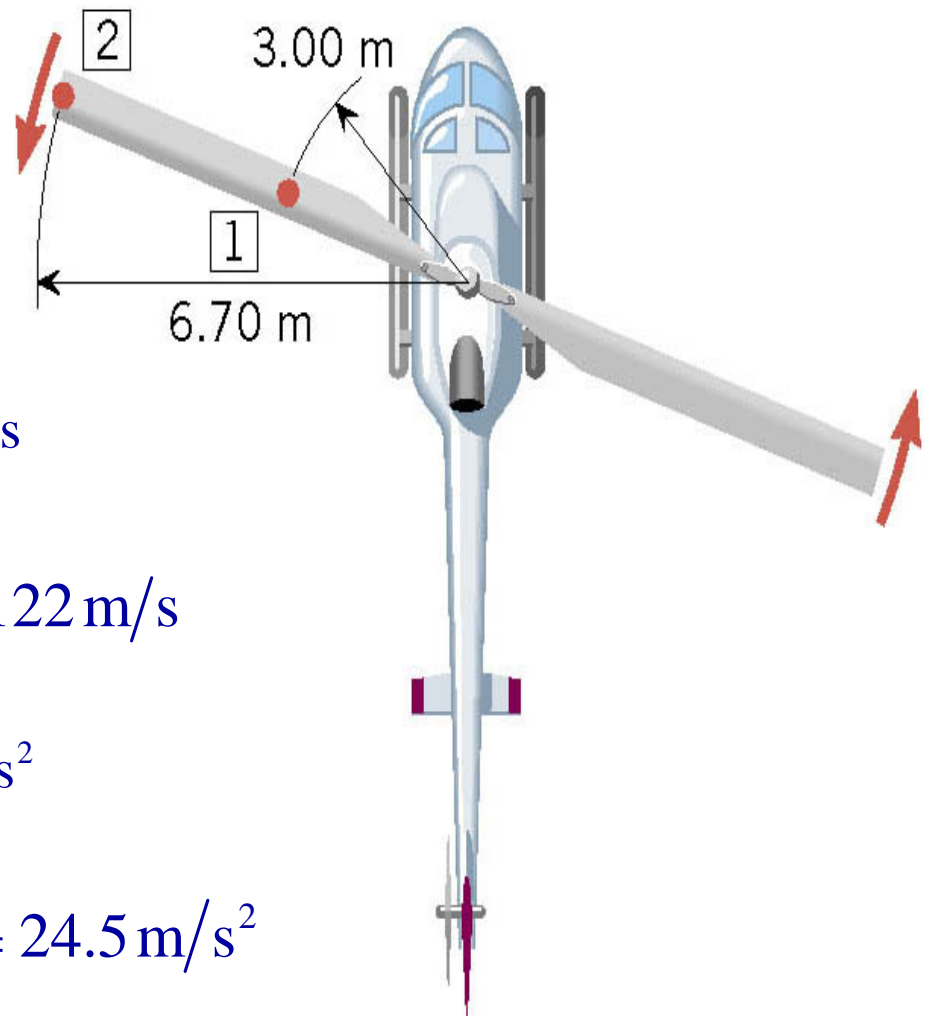
$$\omega = \left(6.50 \; \frac{\text{rev}}{\text{s}} \right) \left(\frac{2\pi \; \text{rad}}{1 \; \text{rev}} \right) = 40.8 \; \text{rad/s}$$

$$v_T = r\varpi = \left(3.00 \; \text{m} \right) \left(40.8 \, \text{rad/s} \right) = 122 \, \text{m/s}$$

$$\alpha = \left(1.30 \; \frac{\text{rev}}{\text{s}^2} \right) \left(\frac{2\pi \; \text{rad}}{1 \; \text{rev}} \right) = 8.17 \; \text{rad/s}^2$$

$$a_T = r\alpha = \left(3.00 \; \text{m} \right) \left(8.17 \, \text{rad/s}^2 \right) = 24.5 \, \text{m/s}^2$$