PHYS 1441 – Section 001
Lecture # 14

Tuesday, June 24, 2008
Dr. Jaehoon Yu

- Rolling Motion
- Rotational Dynamics
  - Torque
  - Equilibrium
  - Moment of Inertia
  - Torque and Angular Acceleration
- Moment of Inertia
- Work, Power and Energy in Rotation
- Rotational Kinetic Energy

Announcements

- Quiz Results
  - Class Average: 3.2/6
    - Equivalent to 53.3/100
    - Previous quizzes: 61/100 and 69/100
- Top score: 6/6
- Quiz Thursday, June 26
  - Beginning of the class
  - Covers CH8.1 – what we complete tomorrow, Wednesday, June 25.
- Final exam
  - 8 – 10am, Monday, June 30, in SH103
  - Comprehensive exam: Covers CH 1 – What we finish this Thursday, June 26 + Appendix

Rolling Motion of a Rigid Body

What is a rolling motion?

A more generalized case of a motion where the rotational axis moves together with an object

A rotational motion about a moving axis

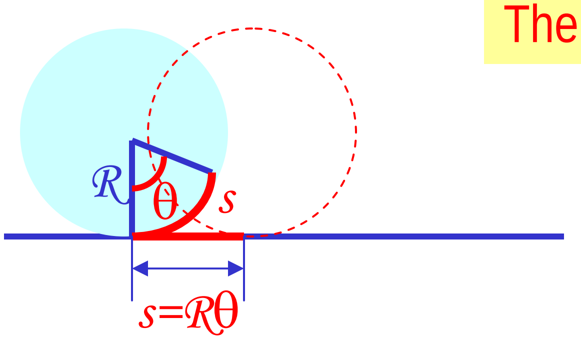
To simplify the discussion, let's make a few assumptions

1. Limit our discussion on very symmetric objects, such as cylinders, spheres, etc
2. The object rolls on a flat surface

Let's consider a cylinder rolling on a flat surface, without slipping.

Under what condition does this "Pure Rolling" happen?

The total linear distance the CM of the cylinder moved is $s = R\theta$



Thus the linear speed of the CM is

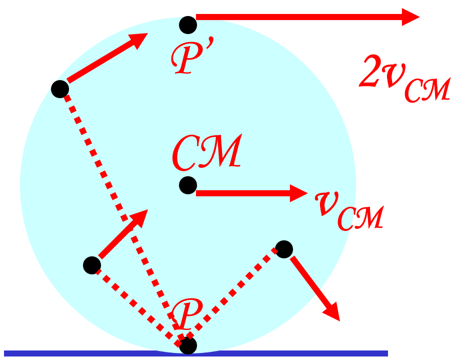
$$\bar{v}_{CM} = \frac{\Delta s}{\Delta t} = R\frac{\Delta\theta}{\Delta t} = R\omega$$

The condition for a "Pure Rolling motion"

More Rolling Motion of a Rigid Body

The magnitude of the linear acceleration of the CM is

$$a_{CM} = \frac{\Delta v_{CM}}{\Delta t} = R\frac{\Delta \omega}{\Delta t} = R\alpha$$



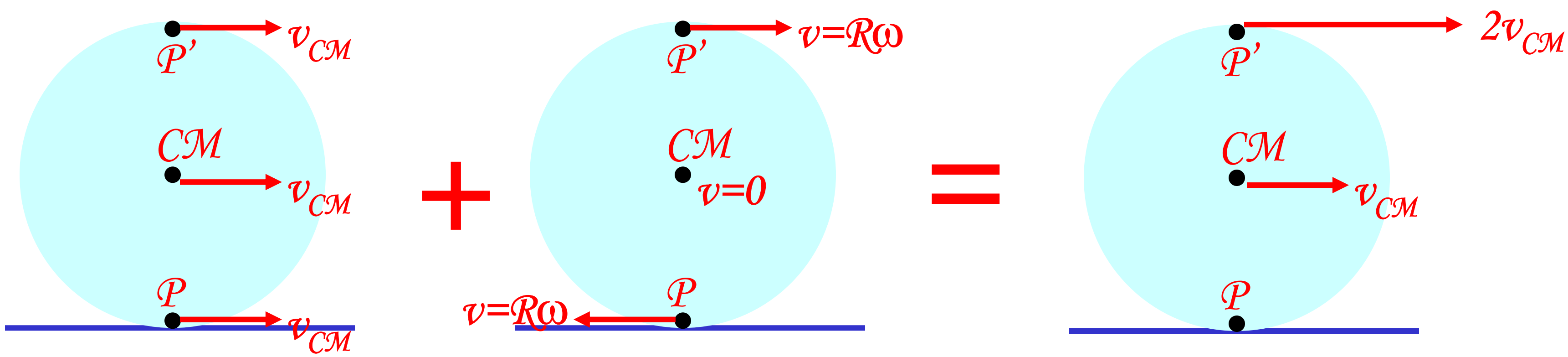
As we learned in rotational motion, all points in a rigid body moves at the same angular speed but at different linear speeds.

CM is moving at the same speed at all times.

At any given time, the point that comes to P has 0 linear speed while the point at P' has twice the speed of CM

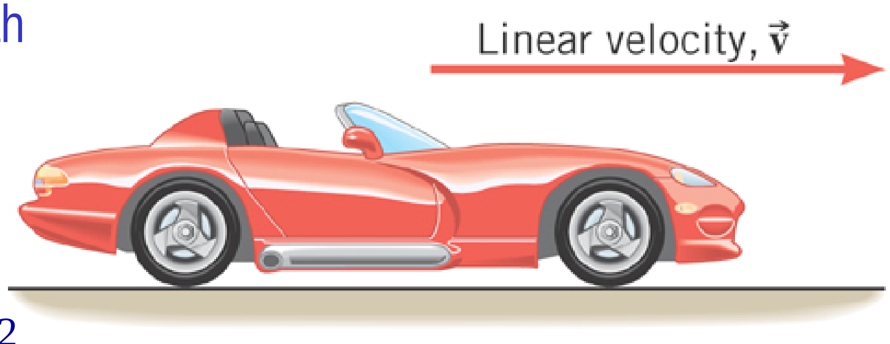
Why??

A rolling motion can be interpreted as the sum of Translation and Rotation



Ex. 8 An Accelerating Car

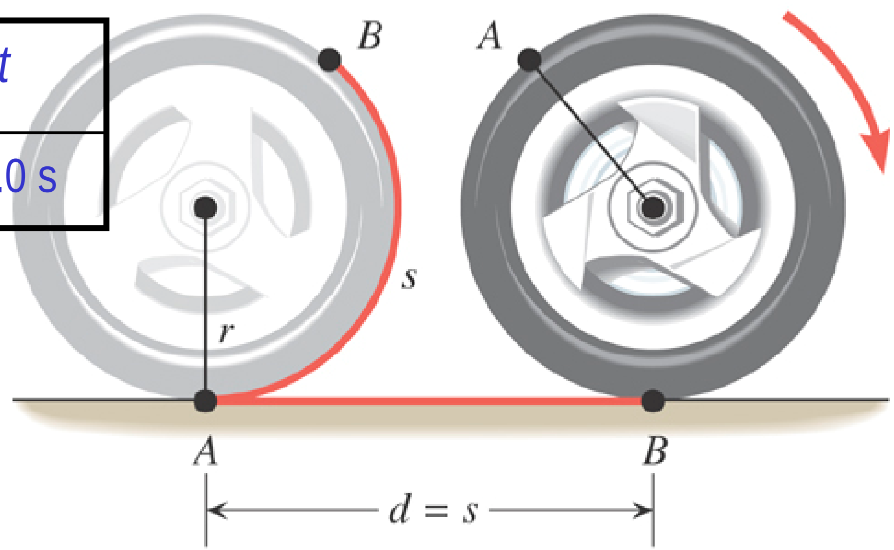
Starting from rest, a car accelerates for 20.0 s with a constant linear acceleration of 0.800 m/s^2. The radius of the tires is 0.330 m. What is the angle through which each wheel has rotated?



(a)

$$\alpha = \frac{a}{r} = \frac{0.800\,\mathrm{m/s^2}}{0.330\ \mathrm{m}} = 2.42\,\mathrm{rad/s^2}$$

θ	α	ω	ω_o	t
?	-2.42 rad/s^2		0 rad/s	20.0 s



(b)

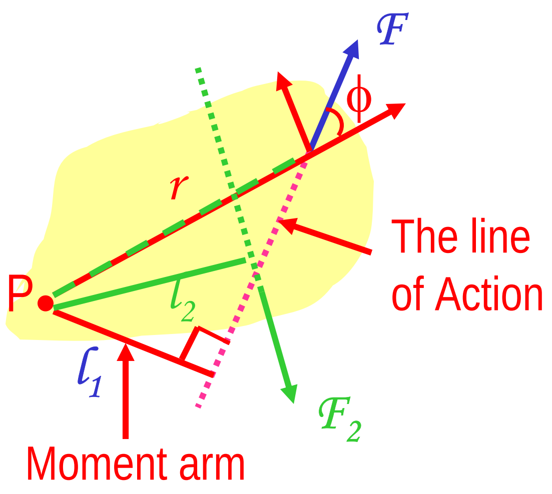
$$\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$

$$= \tfrac{1}{2}\left(-2.42\,\mathrm{rad/s^2}\right)^2 (20.0\ \mathrm{s})^2$$

$$= -484\ \mathrm{rad}$$

Torque

Torque is the tendency of a force to rotate an object about an axis. Torque, τ, is a vector quantity.



Consider an object pivoting about the point P by the force *F* being exerted at a distance r from P.

The line that extends out of the tail of the force vector is called the line of action.

The perpendicular distance from the pivoting point P to the line of action is called the moment arm.

$$\left|\vec{\tau}\right| \equiv (\text{Magnitude of the Force}) \times (\text{Lever Arm}) = (F)(r\sin\phi) = Fl$$

Magnitude of torque is defined as the product of the force exerted on the object to rotate it and the moment arm.

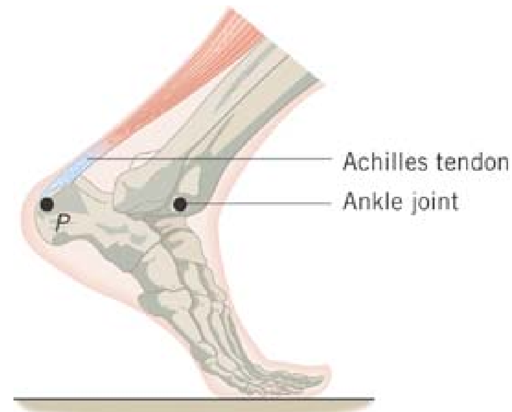
When there are more than one force being exerted on certain points of the object, one can sum up the torque generated by each force vectorially. The convention for sign of the torque is <u>positive if rotation is in counter-clockwise</u> and <u>negative if clockwise</u>.

$$\sum \tau = \tau_1 + \tau_2 = F_1 l_1 - F_2 l_2$$

Unit? $N \cdot m$

Ex. 2 The Achilles Tendon

The tendon exerts a force of magnitude 790 N on the point P. Determine the torque (magnitude and direction) of this force about the ankle joint which is located $3.6\text{x}10^{-2}$m away from point P.



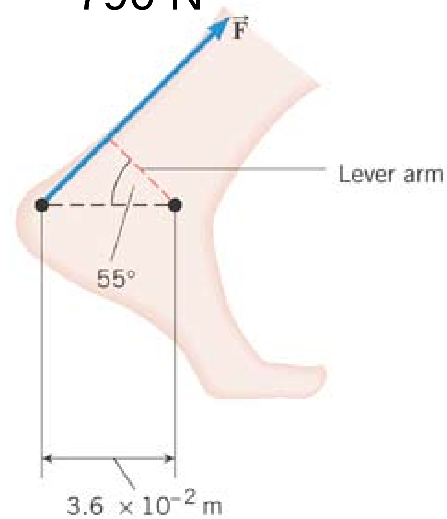
(a)

First, let's find the lever arm length

$$\cos 55^{\circ} = \frac{\ell}{3.6\times10^{-2}\,\mathrm{m}}$$

$$\ell = 3.6\times10^{-2}\cos 55^{\circ} =$$

$$= 3.6\times10^{-2}\sin\left(90^{\circ}-55^{\circ}\right) = 2.1\times10^{-2}\,(m)$$



(b)

So the torque is

$$\tau = F\ell$$

$$= (790\ \mathrm{N})\left(3.6\times10^{-2}\,\mathrm{m}\right)\cos 55^{\circ}$$

$$= (790\ \mathrm{N})\left(3.6\times10^{-2}\,\mathrm{m}\right)\sin 35^{\circ} = 15\ \mathrm{N}\cdot\mathrm{m}$$

Since the rotation is in clock-wise $\tau = -15N\cdot m$

Conditions for Equilibrium

What do you think the term "An object is at its equilibrium" means?

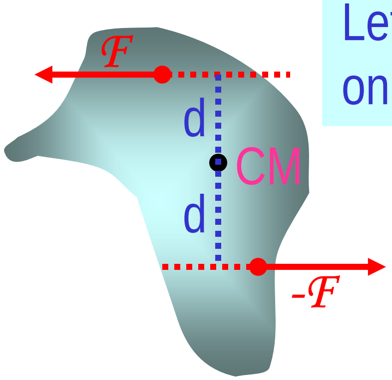
The object is either at rest (Static Equilibrium) or its center of mass is moving at a constant velocity (Dynamic Equilibrium).

When do you think an object is at its equilibrium?

Translational Equilibrium: Equilibrium in linear motion $\sum \vec{F} = 0$

Is this it?

The above condition is sufficient for a point-like object to be at its translational equilibrium. However for an object with size this is not sufficient. One more condition is needed. What is it?



Let's consider two forces equal in magnitude but in opposite direction acting on a rigid object as shown in the figure. What do you think will happen?

The object will rotate about the CM. The net torque acting on the object about any axis must be 0. $\sum \vec{\tau} = 0$

For an object to be at its *static equilibrium*, the object should not have linear or angular speed. $v_{CM} = 0 \quad \omega = 0$

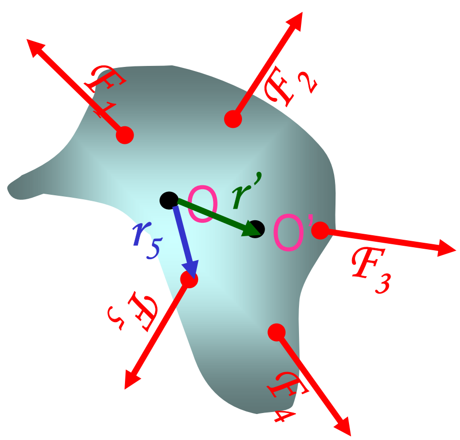
More on Conditions for Equilibrium

To simplify the problem, we will only deal with forces acting on x-y plane, giving torque only along z-axis. What do you think the conditions for equilibrium be in this case?

The six possible equations from the two vector equations turns to three equations.

$$\sum \vec{F} = 0 \Rightarrow \sum F_x = 0, \quad \sum F_y = 0 \quad \textbf{AND} \quad \sum \vec{\tau} = 0 \Rightarrow \sum \tau_z = 0$$

What happens if there are many forces exerting on an object?



If an object is at its translational static equilibrium, and if the net torque acting on the object is 0 about one axis, the net torque must be 0 about any arbitrary axis.

Why is this true?

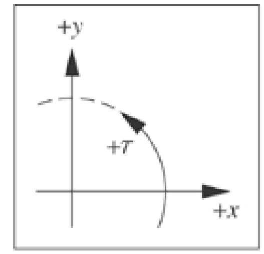
Because the object is ***not moving***, no matter what the rotational axis is, there should not be any motion. It is simply a matter of mathematical manipulation.

How do we solve equilibrium problems?

1. Select the object to which the equations for equilibrium are to be applied.
2. Identify all the forces and their directions and locations.
3. Draw a free-body diagram with forces indicated on it with their directions and locations properly noted.
4. Choose a convenient set of x and y axes and write down force equation for each x and y component with proper signs.
5. Apply the equations that specify the balance of forces at equilibrium. Set the net force in the x and y directions equal to 0.
6. Select a rotational axis for torque calculations ➔ Selecting the axis such that the torque of one of the unknown forces become 0 makes the problem easier to solve.
7. Write down the torque equation with proper signs.
8. Solve the equations for the desired unknown quantities.

Ex. 3 A Diving Board

A woman whose weight is 530 N is poised at the right end of a diving board with length 3.90 m. The board has negligible weight and is supported by a fulcrum 1.40 m away from the left end. Find the forces that the bolt and the fulcrum exert on the board.



First the torque eq. $\sum \tau = F_2 l_2 - W l_W = 0$

How large is the torque by the bolt?

None Why?

Because the lever arm is 0.

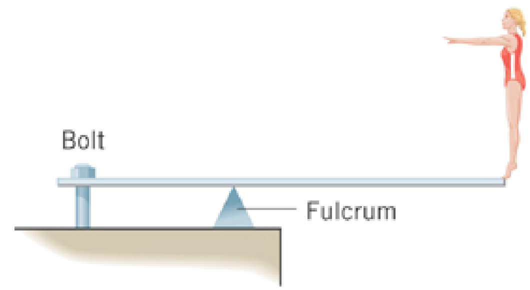
So the force by the fulcrum is $F_2 = \dfrac{W l_W}{l_2}$

$$F_2 = \frac{(530\ \mathrm{N})(3.90\ \mathrm{m})}{1.40\ \mathrm{m}} = 1480\ \mathrm{N}$$

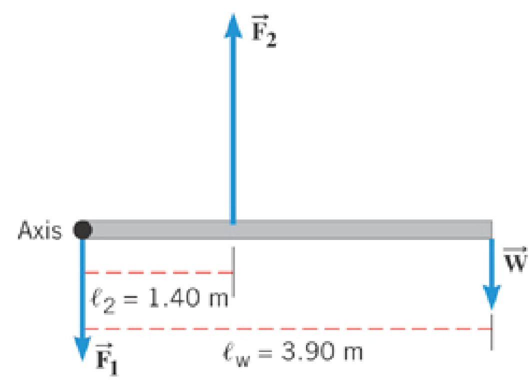
Now the force eq. $\sum F_y = -F_1 + F_2 - W = 0$

$$-F_1 + 1480\ \mathrm{N} - 530\ \mathrm{N} = 0$$

So the force by the bolt is $F_1 = 950\ \mathrm{N}$



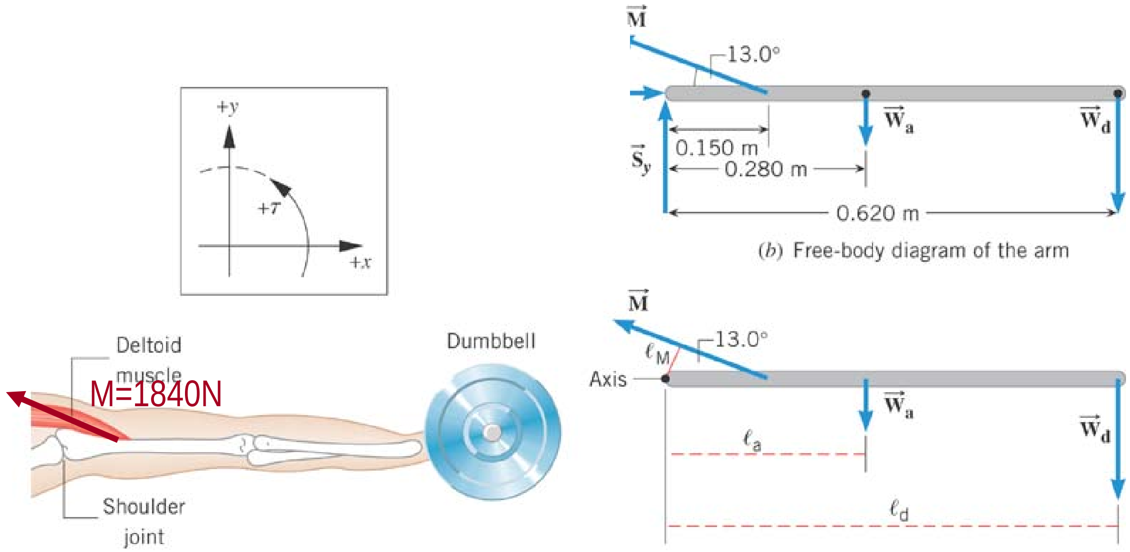
(a)



(b) Free-body diagram of the diving board

Ex. 5 Bodybuilding

The arm is horizontal and weighs 31.0 N. The deltoid muscle can supply 1840 N of force. What is the weight of the heaviest dumbell he can hold?



(a)

(b) Free-body diagram of the arm

(c)

First the torque eq.

$$\sum \tau = -W_a l_a - W_d l_d + M l_M = 0$$

the lever arm by the deltoid muscle is $l_M = (0.150 \text{ m}) \sin 13.0^{\circ}$

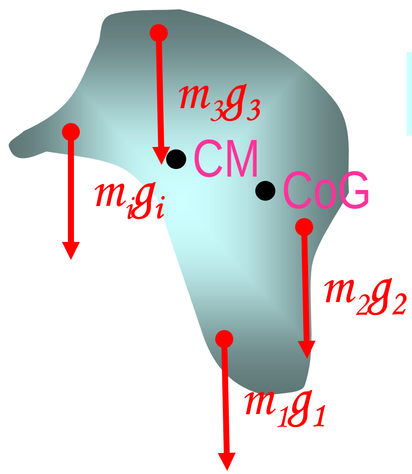
$$W_d = \frac{-W_a l_a + M l_M}{l_d}$$

$$= \frac{-(31.0 \text{ N})(0.280 \text{ m}) + (1840 \text{ N})(0.150 \text{ m}) \sin 13.0^{\circ}}{0.620 \text{ m}} = 86.1 \text{ N}$$

Center of Gravity

When is the center of gravity of a rigid body the same as the center of mass?

Under the uniform gravitational field throughout the body of the object.



Let's consider an arbitrary shaped object

The center of mass of this object is at

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\sum m_i x_i}{M}$$

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\sum m_i y_i}{M}$$

Let's now examine the case that the gravitational acceleration on each point is g_i

Since the CoG is the point as if all the gravitational force is exerted on, the torque due to this force becomes

$$(m_1 g_1 + m_2 g_2 + \cdots) x_{CoG} = m_1 g_1 x_1 + m_2 g_2 x_2 + \cdots$$

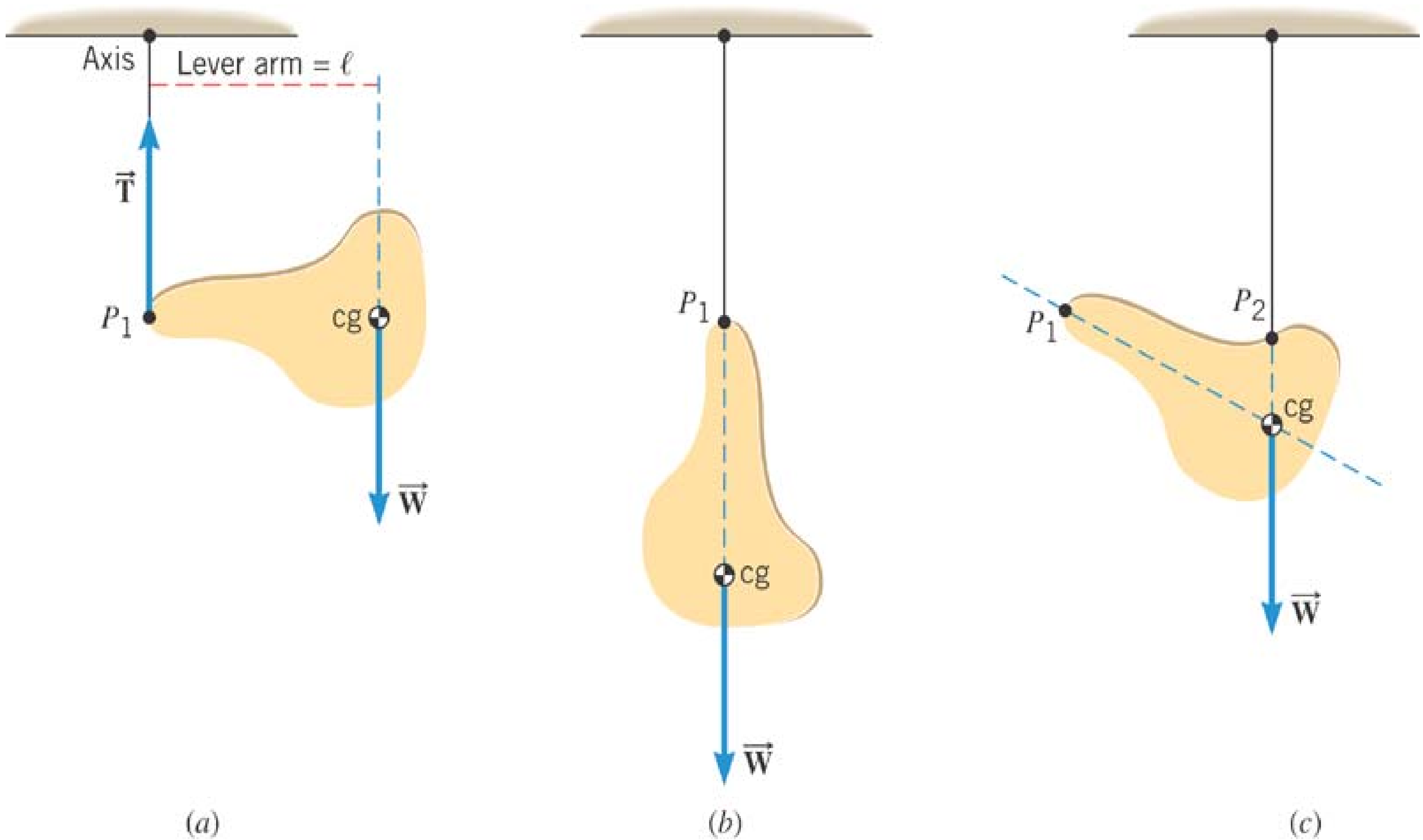
Generalized expression for different g throughout the body

If g is uniform throughout the body

$$(m_1 + m_2 + \cdots) g x_{CoG} = (m_1 x_1 + m_2 x_2 + \cdots) g$$

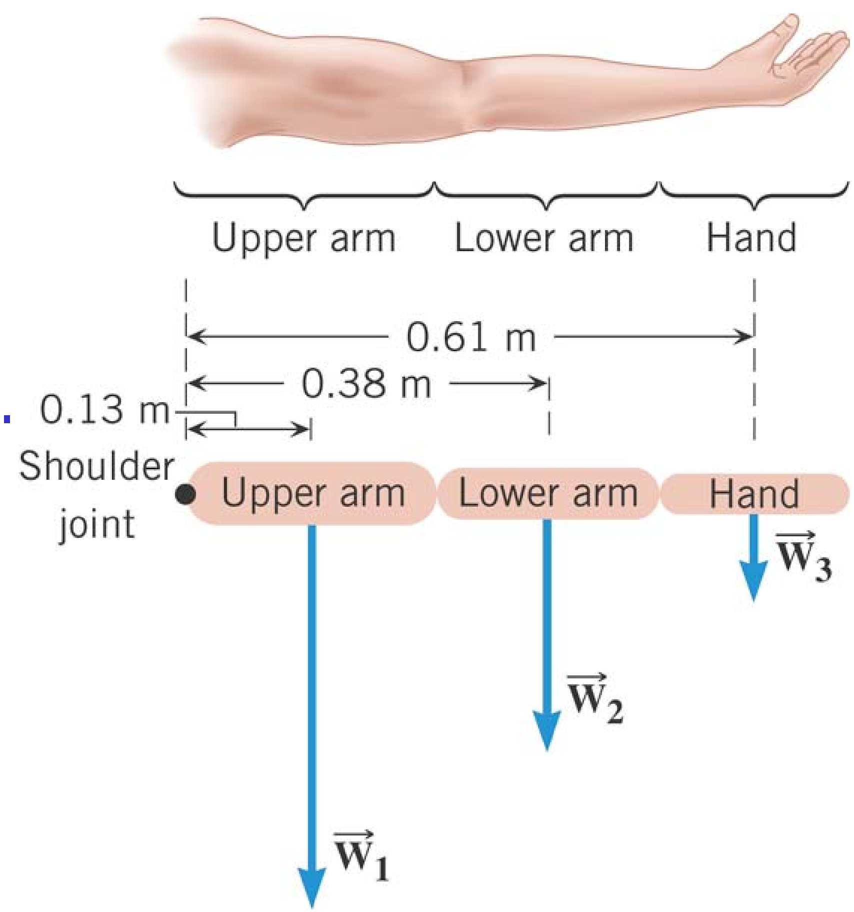
$$x_{CoG} = \frac{\sum m_i x_i}{\sum m_i} = x_{CM}$$

How can one find the COG?



Ex. 6 The Center of Gravity of an Arm

The horizontal arm is composed of three parts: the upper arm (17 N), the lower arm (11 N), and the hand (4.2 N). Find the center of gravity of the arm relative to the shoulder joint.



$$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + \cdots}{W_1 + W_2 + \cdots}$$

$$= \frac{(17\ \mathrm{N})(0.13\ \mathrm{m}) + (11\ \mathrm{N})(0.38\ \mathrm{m}) + (4.2\ \mathrm{N})(0.61\ \mathrm{m})}{17\ \mathrm{N} + 11\ \mathrm{N} + 4.2\ \mathrm{N}} = 0.28\ \mathrm{m}$$

Moment of Inertia

Rotational Inertia: Measure of resistance of an object to changes in its rotational motion. Equivalent to mass in linear motion.

For a group of objects $$I \equiv \sum_i m_i r_i^2$$

For a rigid body $$I \equiv \int r^2 dm$$

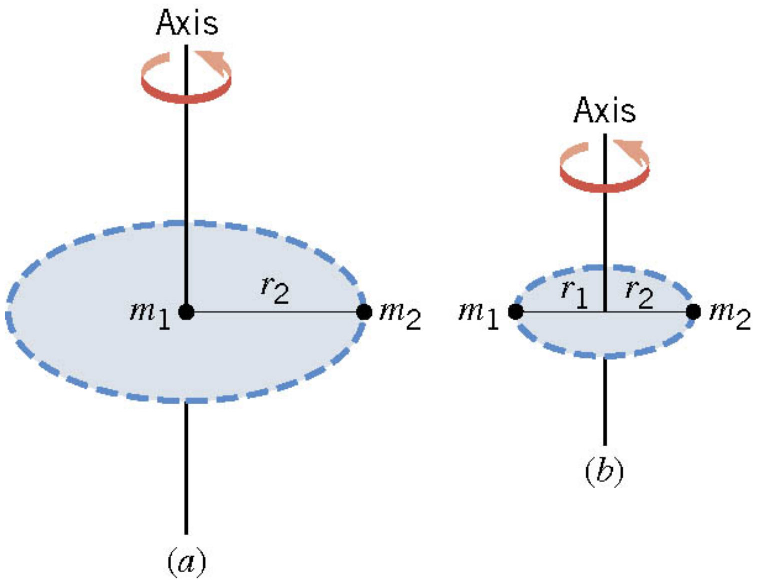
What are the dimension and unit of Moment of Inertia? $\left[ML^2\right]$ $kg \cdot m^2$

Determining Moment of Inertia is extremely important for computing equilibrium of a rigid body, such as a building.

Dependent on the axis of rotation!!!

Ex. 9 The Moment of Inertia Depends on Where the Axis Is.

Two particles each have mass and are fixed at the ends of a thin rigid rod. The length of the rod is *L*. Find the moment of inertia when this object rotates relative to an axis that is perpendicular to the rod at (a) one end and (b) the center.



(a) $I = \sum\left(mr^2\right) = m_1 r_1^2 + m_2 r_2^2$

$m_1 = m_2 = m \qquad r_1 = 0 \quad r_2 = L$

$$I = m(0)^2 + m(L)^2 = mL^2$$

(b) $I = \sum\left(mr^2\right) = m_1 r_1^2 + m_2 r_2^2$

$m_1 = m_2 = m \qquad r_1 = L/2 \quad r_2 = L/2$

$$I = m(L/2)^2 + m(L/2)^2 = \tfrac{1}{2} mL^2$$

Which case is easier to spin?

Case (b)

Why? Because the moment of inertia is smaller