PHYS 1442 – Section 001
Lecture #2

Monday, June 8, 2009

Dr. Jaehoon Yu

- Chapter 16
  - Coulomb's Law
  - Vector Operations Recap
  - The Electric Field & Field Lines
  - Electric Fields and Conductors
  - Motion of a Charged Particle in an Electric Field

Announcements

- Your five extra credit points for e-mail subscription extended till midnight Wednesday, June 10.   Please take a full advantage of the opportunity.
  - Five of you have subscribed so far. Thank you!!!
- 11 of you have registered in the homework system.
  - Fantastic job!!
  - You need my enrollment approval… So move quickly…
  - Remember, the due is 9pm Thursday.
- Reading assignment: CH16 – 10, 16 – 11, 16 – 12
- Quiz beginning of the class this Wednesday, June 10
  - Appendix A1 – A8 and CH16

Special Project – Angels & Demons

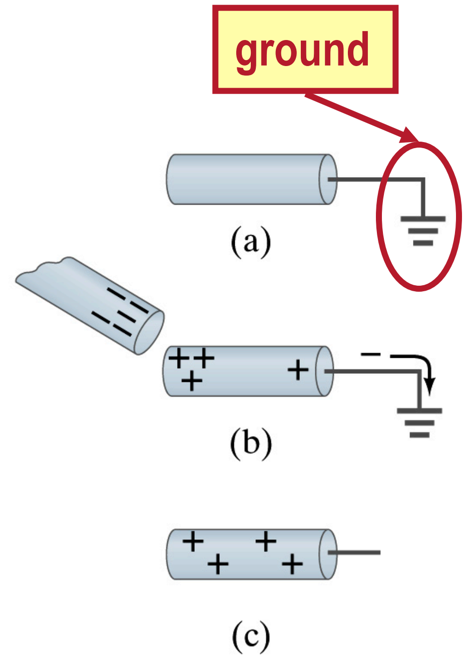
- Compute the total possible energy released from an annihilation of x-grams of anti-matter and the same quantity of matter, where x is the last two digits of your SS#. (20 points)
  - Use the famous Einstein's formula for mass-energy equivalence
- Compute the power output of this annihilation when the energy is released in x ns, where x is again the last two digits of your SS#. (10 points)
- Compute how many cups of gasoline (8MJ) this energy corresponds to. (5 points)
- Compute how many months of electricity usage it corresponds to (3.6GJ). (5 points)
- Due by the beginning of the class Monday, June 15.

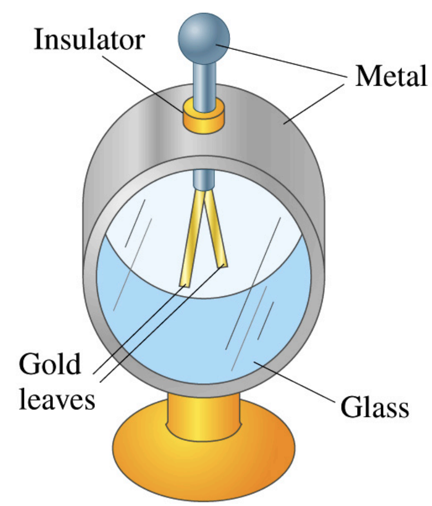
Special Project – Magnitude of Forces

- What is the magnitude of the Coulomb force one proton exerts to another 1m away? (10 points)
- What is the magnitude of the gravitational force one proton exerts to another 1m away? (10 points)
- Which one of the two forces is larger and by how many times? (10 points)
- Due at the beginning of the class Monday, June 22.

Induced Charge

- We can induce a net charge on a metal object by connecting a wire to the ground.
  - The object is "grounded" or "earthed".
- Since it is so large and conducts, the Earth can give or accept charge.
  - The Earth acts as a reservoir for charge.
- If the negative charge is brought close to a neutral metal
  - The positive charges will be induced toward the negatively charged metal.
  - The negative charges in the neutral metal will be gathered on the opposite side, transferring through the wire to the Earth.
  - If the wire is cut, the metal bar has net positive charge.
- An **electroscope** is a device that can be used for detecting charge and signs.
  - How would this work?





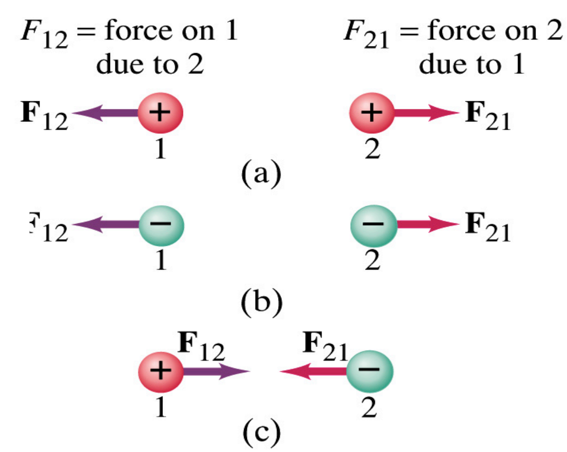
Coulomb's Law

- Charges exert force to each other. What factors affect the magnitude of this force?
  - Any guesses?
- Charles Coulomb figured this out in 1780's.
- Coulomb found that the electrical force is
  - Proportional to the multiplication of the two charges
    - If one of the charges doubles, the force doubles.
    - If both the charges double, the force quadruples.
  - Inversely proportional to the square of the distances between them.
  - Electric charge is a fundamental property of matter, just like mass.
- How would you put the above into a formula?

Coulomb's Law – The Formula

$$F \propto \frac{Q_1 \times Q_2}{r^2} \quad \xrightarrow{\text{Formula}} \quad F = k\frac{Q_1 Q_2}{r^2}$$

- Is Coulomb force a scalar quantity or a vector quantity? Unit?
  - A vector quantity. Newtons
- Direction of electric (Coulomb) force is always along the line joining the two objects.
  - If the two charges are the same sign: forces are directed away from each other.
  - If the two charges are opposite sign: forces are directed toward each other.
- Coulomb force is precise to 1 part in 10^{16}.
- Unit of charge is called Coulomb, C, in SI.
- The value of the proportionality constant, k, in SI unit is $k = 8.988 \times 10^9 \ \mathrm{N \cdot m^2} / C^2$
- Thus, 1C is the charge that gives **F~9x10⁹N** of force when placed 1m apart from each other.



Electric Force and Gravitational Force

$$F = k\frac{Q_1 Q_2}{r^2} \quad \Longleftrightarrow \quad F = G\frac{M_1 M_2}{r^2}$$

Extremely Similar

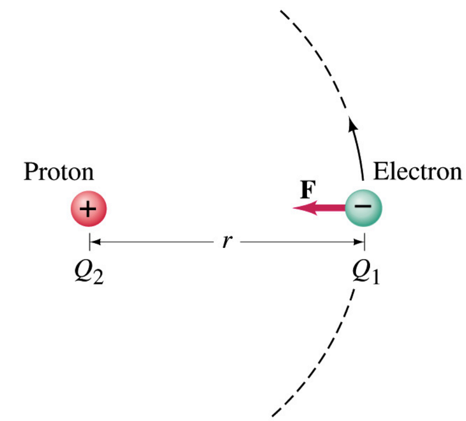
- Does the electric force look similar to another force? What is it?
  - **Gravitational Force**
- What are the sources of the forces?
  - Electric Force: Charges, fundamental properties of matter
  - Gravitational Force: Masses, fundamental properties of matter
- What else is similar?
  - Inversely proportional to the square of the distance between the sources of the force ➔ What is this kind law called?
    - Inverse Square Law
- What is the difference?
  - Gravitational force is always attractive.
  - Electric force depends on the signs of the two charges.

The Elementary Charge and Permittivity

- Elementary charge, the smallest unit charge, is that of an electron: $e = 1.602 \times 10^{-19} C$
  - Since electron is a negatively charged particle, its charge is $-e$.
- An object cannot gain or lose fraction of an electron.
  - Electric charge is quantized.
    - It changes always in integer multiples of e.
- The proportionality constant k is often written in terms of another constant, ε_0, the permittivity of free space. They are related $k = 1/4\pi\varepsilon_0$ and $\varepsilon_0 = 1/4\pi k = 8.85 \times 10^{-12}\, C^2/N \cdot m^2$.
- Thus the electric force can be written: $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$
- Note that this force is for "point" charges at rest.

Example 16 – 1

- **Electric force on electron by proton**. Determine the magnitude of the electric force on the electron of a hydrogen atom exerted by the single proton (Q_2=+e) that is its nucleus. Assume the electron "orbits" the proton at its average distance of r=0.53x10^{-10}m.



Using Coulomb's law $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2} = k\frac{Q_1Q_2}{r^2}$

Each charge is $Q_1 = -e = -1.602\times10^{-19}C$ and $Q_2 = +e = 1.602\times10^{-19}C$

So the magnitude of the force is

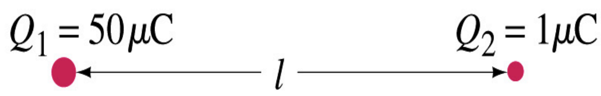
$$F = \left|k\frac{Q_1Q_2}{r^2}\right| = 9.0\times10^9\ N\cdot m^2/C^2\ \frac{\left(1.6\times10^{-19}C\right)\left(1.6\times10^{-19}C\right)}{\left(0.53\times10^{-10}m\right)^2}$$

$$= 8.2\times10^{-8}\ N$$

Which direction? Toward each other…

Example 16 – 2

- **Which charge exerts greater force?** Two positive point charges, Q_1=50μC and Q_2=1μC, are separated by a distance L. Which is larger in magnitude, the force that Q_1 exerts on Q_2 or the force that Q_2 exerts on Q_1?



What is the force that Q_1 exerts on Q_2?

$$F_{12} = k\frac{Q_1 Q_2}{L^2}$$

What is the force that Q_2 exerts on Q_1?

$$F_{21} = k\frac{Q_2 Q_1}{L^2}$$

Therefore the magnitudes of the two forces are identical!!

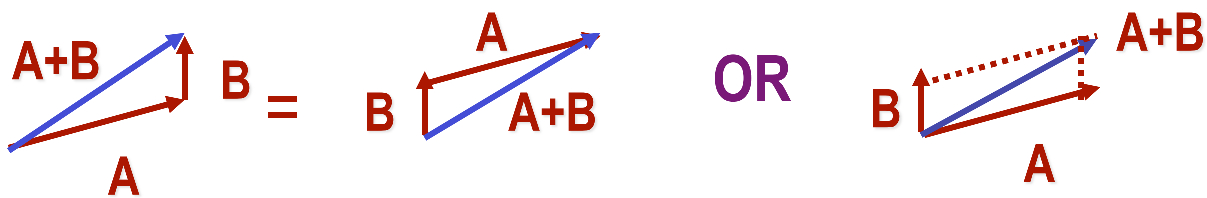
Well then what is different? The direction.

Which direction? Opposite to each other!

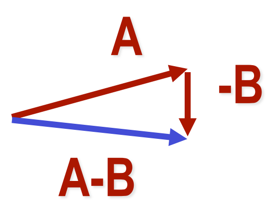
What is this law? Newton's third law, the law of action and reaction!!

Vector Additions and Subtractions

- Addition:
  - Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  - Parallelogram method: Connect the tails of the two vectors and extend
  - Addition is commutative: Changing order of operation does not affect the results **A+B=B+A**, **A+B+C+D+E=E+C+A+B+D**



- Subtraction:
  - The same as adding a negative vector:**A** - **B** = **A** + (-**B**)



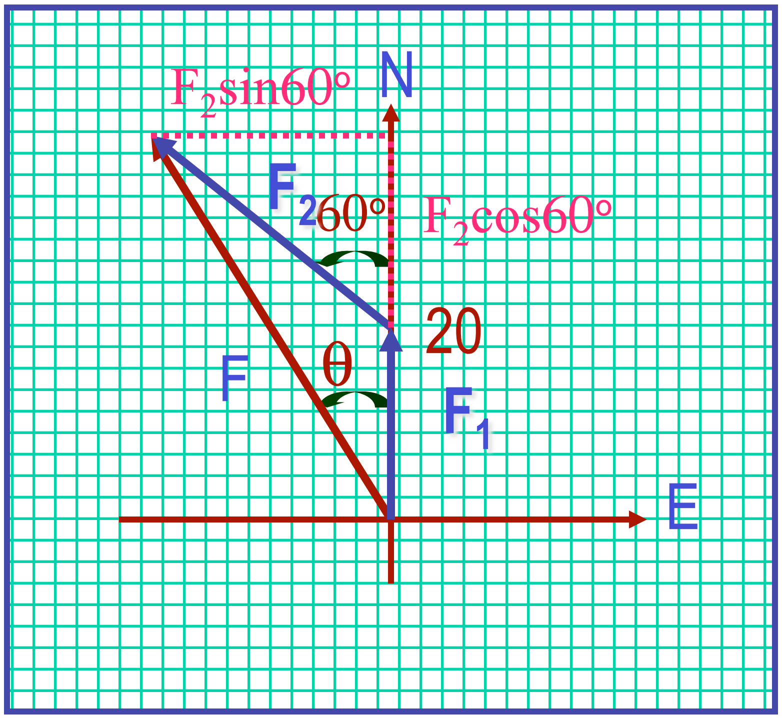
Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude **A**, **B**=2**A**

$$|\mathcal{B}| = 2|\mathcal{A}|$$

Example for Vector Addition

A force of 20.0N applies to north while another force of 35.0N applies in the direction 60.0° west of north. Find the magnitude and direction of resultant force.



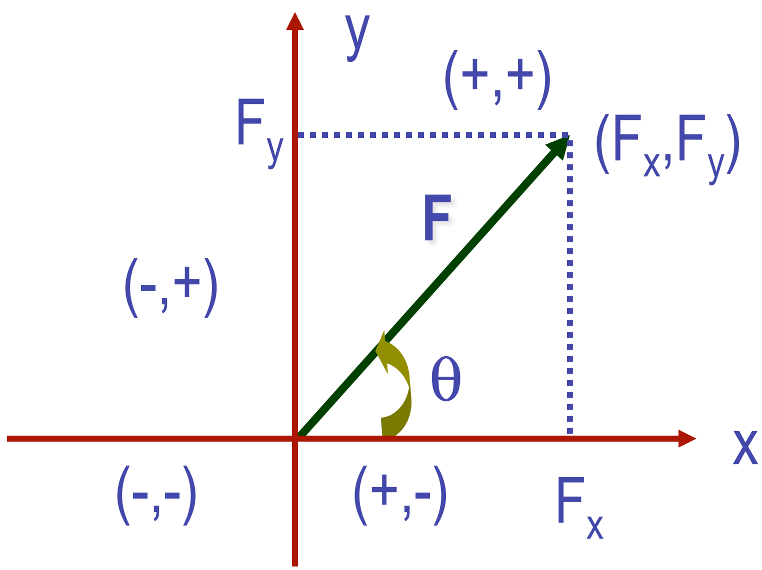
$$F = \sqrt{\left(F_1 + F_2\cos 60^\circ\right)^2 + \left(F_2\sin 60^\circ\right)^2}$$
$$= \sqrt{F_1^2 + F_2^2\left(\cos^2 60^\circ + \sin^2 60^\circ\right) + 2F_1F_2\cos 60^\circ}$$
$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60^\circ}$$
$$= \sqrt{(20.0)^2 + (35.0)^2 + 2\times 20.0\times 35.0\cos 60}$$
$$= \sqrt{2325} = 48.2(N)$$

$$\theta = \tan^{-1}\frac{\left|\vec{F}_2\right|\sin 60^\circ}{\left|\vec{F}_1\right| + \left|\vec{F}_2\right|\cos 60^\circ}$$
$$= \tan^{-1}\frac{35.0\sin 60}{20.0 + 35.0\cos 60}$$
$$= \tan^{-1}\frac{30.3}{37.5} = 38.9^\circ \text{ to W wrt N}$$

Find other ways to solve this problem…

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$F_x = \left|\vec{F}\right| \cos\theta$$

$$F_y = \left|\vec{F}\right| \sin\theta$$

} Components

$$\left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2}$$

} Magnitude

$$\left|\vec{F}\right| = \sqrt{\left(\left|\vec{F}\right|\cos\theta\right)^2 + \left(\left|\vec{F}\right|\sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{F}\right|^2\left(\cos^2\theta + \sin^2\theta\right)} \quad = \left|\vec{F}\right|$$

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
- **Dimensionless**
- **Magnitudes are exactly 1**
- Unit vectors are usually expressed in **i, j, k** or

$$\vec{i},\ \vec{\mathrm{j}},\ \vec{\mathrm{k}}$$

So the vector **F** can be re-written as

$$\vec{F} = F_x \vec{i} + F_y \vec{j} = \left|\vec{F}\right| \cos\theta \vec{i} + \left|\vec{F}\right| \sin\theta \vec{j}$$

Examples of Vector Operations

Find the resultant force which is the sum of **F1**=(2.0**i**+2.0**j**)N and **F2** =(2.0**i**-4.0**j**)N.

$$\vec{F}_3 = \vec{F}_1 + \vec{F}_2 = \left(2.0\vec{i} + 2.0\vec{j}\right) + \left(2.0\vec{i} - 4.0\vec{j}\right)$$

$$= (2.0 + 2.0)\vec{i} + (2.0 - 4.0)\vec{j} = 4.0\vec{i} - 2.0\vec{j}\,(N)$$

$$\left|\vec{F}_3\right| = \sqrt{(4.0)^2 + (-2.0)^2}$$

$$= \sqrt{16 + 4.0} = \sqrt{20} = 4.5(N)$$

$$\theta = \tan^{-1}\frac{F_{3y}}{F_{3x}} = \tan^{-1}\frac{-2.0}{4.0} = -27^\circ$$

Find the resultant force of the sum of three forces: $\mathbf{F_1}$=(15**i**+30**j** +12**k**)N, $\mathbf{F_2}$=(23**i**+14**j** -5.0**k**)N, and $\mathbf{F_3}$=(-13**i**+15**j**)N.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \left(15\vec{i} + 30\vec{j} + 12\vec{k}\right) + \left(23\vec{i} + 14\vec{j} - 5.0\vec{k}\right) + \left(-13\vec{i} + 15\vec{j}\right)$$

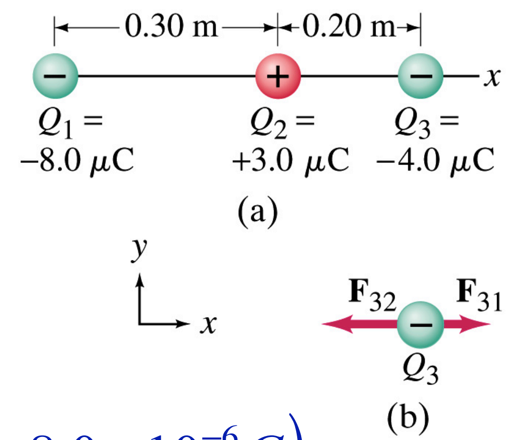
$$= (15 + 23 - 13)\vec{i} + (30 + 14 + 15)\vec{j} + (12 - 5.0)\vec{k} = 25\vec{i} + 59\vec{j} + 7.0\vec{k}\,(N)$$

Magnitude

$$\left|\vec{D}\right| = \sqrt{(25)^2 + (59)^2 + (7.0)^2} = 65(N)$$

Example 16 – 3

- **Three charges in a line.** Three charged particles are arranged in a line as shown in the figure. Calculate the net electrostatic force on particle 3 (the -4μC on the right) due to other two charges.



What is the force that Q_1 exerts on Q_3?

$$F_{13x} = k\frac{Q_1Q_3}{L^2} = \frac{\left(9.0\times10^9\ N\cdot m^2/C^2\right)\left(-4.0\times10^{-6}C\right)\left(-8.0\times10^{-6}C\right)}{\left(0.5m\right)^2} = 1.2N$$

What is the force that Q_2 exerts on Q_3?

$$F_{23x} = k\frac{Q_2Q_3}{L^2} = \frac{\left(9.0\times10^9\ N\cdot m^2/C^2\right)\left(-4.0\times10^{-6}C\right)\left(3.0\times10^{-6}C\right)}{\left(0.2m\right)^2} = -2.7N$$

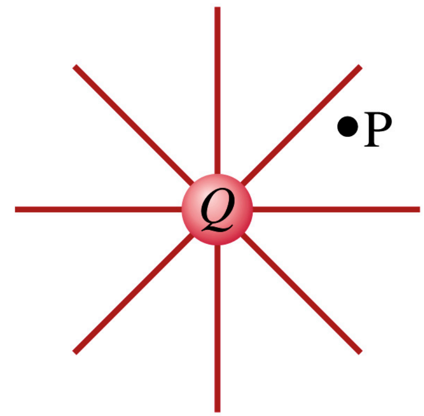
Using the vector sum of the two forces

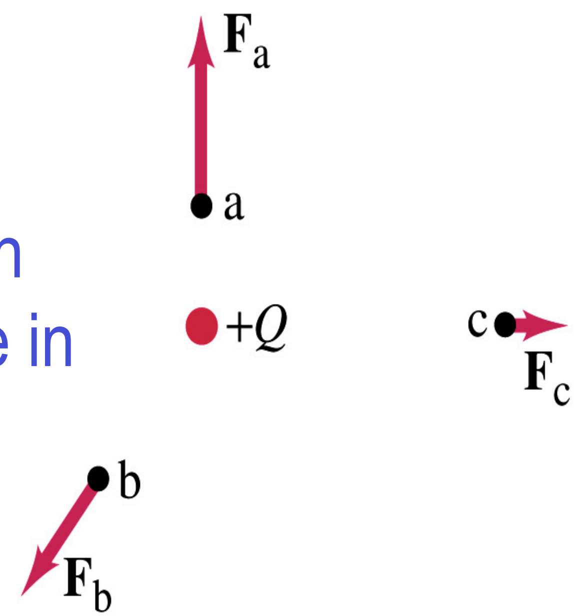
$$F_x = F_{13x} + F_{23x} = 1.2 + \left(-2.7\right) = -1.5\left(N\right) \qquad F_y = 0\left(N\right)$$

$$\vec{F} = -1.5\vec{i}\,(N)$$

The Electric Field

- Both gravitational and electric forces act over a distance without touching objects → What kind of forces are these?
  - Field forces
- Michael Faraday developed an idea of field.
  - Faraday argued that the electric field extends outward from every charge and permeates through all of space.
- Field by a charge or a group of charges can be inspected by placing a small test charge in the vicinity and measuring the force on it.





The Electric Field

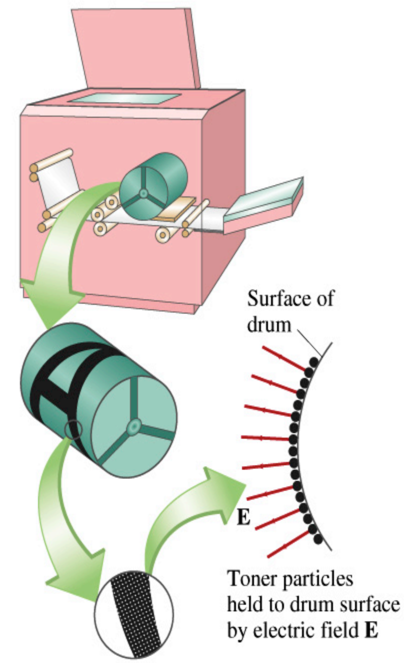
- The electric field at any point in space is defined as the force exerted on a tiny positive test charge divide by the magnitude of the test charge
  - Electric force per unit charge

$$\vec{E} = \frac{\vec{F}}{q} \Rightarrow \vec{F} = q\vec{E}$$

- What kind of quantity is the electric field?
  - Vector quantity.   Why?
- What is the unit of the electric field?
  - N/C
- What is the magnitude of the electric field at a distance r from a single point charge Q?

$$\left|\vec{E}\right| = \frac{\left|\vec{F}\right|}{q} = \frac{kQq/r^2}{q} = \frac{kQ}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Example 16 – 6



- **Electrostatic copier**. An electrostatic copier works by selectively arranging positive charges (in a pattern to be copied) on the surface of a nonconducting drum, then gently sprinkling negatively charged dry toner (ink) onto the drum. The toner particles temporarily stick to the pattern on the drum and are later transferred to paper and "melted" to produce the copy. Suppose each toner particle has a **mass of 9.0×10^{-16}kg** and carries the average of **20 extra electrons** to provide an electric charge. Assuming that the electric force on a toner particle must exceed **twice its weight** in order to ensure sufficient attraction, compute the required electric field strength near the surface of the drum.

The electric force must be the same as twice the gravitational force on the toner particle.

So we can write $F_e = qE = 2F_g = 2mg$

Thus, the magnitude of the electric field is

$$E = \frac{2mg}{q} = \frac{2\cdot\left(9.0\times10^{-16}\,kg\right)\cdot\left(9.8\,m/s^2\right)}{20\left(1.6\times10^{-19}\,C\right)} = 5.5\times10^{3}\,N/C.$$

Direction of the Electric Field

- If there are more than one charge, the individual field due to each charge is added vectorially to obtain the total field at any point.

$$\vec{E}_{Tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \ldots.$$

- This superposition principle of electric field has been verified by experiments.
- For a given electric field **E** at a given point in space, we can calculate the force **F** on any charge q, **F**=q**E**.
  - What happens to the direction of the force and the field depending on the sign of the charge q?
  - The two are in the same directions if q>0
  - The two are in opposite directions if q<0