PHYS 1442 – Section 001
Lecture #4

Monday, June 15, 2009

Dr. Jaehoon Yu

- Chapter 17
  - Electric Potential and Electric Field
  - Equi-potential Lines
  - The Electron Volt, a Unit of Energy
  - Capacitor and Capacitance
  - Di-electrics
  - Storage of Electric Energy

Announcements

- E-mail distribution list
  - 11 of you have subscribed to the list
  - Your three extra credit points for e-mail subscription is till midnight this Wednesday, June 17! Please take a full advantage of the opportunity.
- Quiz Results
  - Class Average: 21/38
    - Equivalent to 55/100!!
  - Top score: 36/38
  - Quiz is 10% of the total
- Quiz next Monday, June 22
  - Covers CH16 and CH17
- 1st term exam Monday, June 29
  - Covers Appendix A + CH16 – What we cover next Wednesday, June 24

Reminder: Special Project – Magnitude of Forces

- What is the magnitude of the Coulomb force one proton exerts to another 1m away? (10 points)
- What is the magnitude of the gravitational force one proton exerts to another 1m away? (10 points)
- Which one of the two forces is larger and by how many times? (10 points)
- Due at the beginning of the class Monday, June 22.

Electric Potential and Electric Field

- The effect of a charge distribution can be described in terms of electric field or electric potential.
  - What kind of quantities are the electric field and the electric potential?
    - Electric Field: Vector
    - Electric Potential: Scalar
  - Since electric potential is a scalar quantity, it is often easier to handle.
- Well other than the above, what are the connections between these two quantities?

Electric Potential and Electric Field

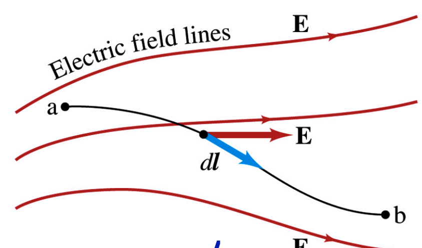
- The potential energy is expressed in terms of a conservative force

$$U_b - U_a = -\vec{F} \cdot \vec{D}$$

- For the electrical case, we are more interested in the potential difference:

$$V_{ba} = V_b - V_a = \frac{U_b - U_a}{q} = -\frac{\vec{F}}{q} \cdot \vec{D} = -\vec{E} \cdot \vec{D} = -ED\cos\theta$$

  – This formula can be used to determine V_{ba} when the electric field is given.



- When the field is uniform

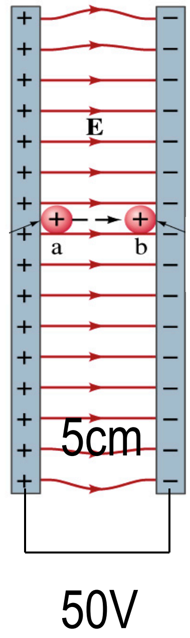
$$V_b - V_a = -\vec{E} \cdot \vec{D} = -ED\cos\theta = -Ed \quad \text{so} \quad E = -V_{ba}/d$$

What does "-"sign mean? The direction of E is along that of decreasing potential.

Unit of the electric field in terms of potential? V/m Can you derive this from N/C?

Example 17 – 3

Uniform electric field obtained from voltage: Two parallel plates are charged to a voltage of 50V. If the separation between the plates is 5.0cm, calculate the magnitude of the electric field between them, ignoring any fringe effect.



What is the relationship between electric field and the potential for a uniform field?

$$V = Ed$$

Solving for E

$$E = \frac{V}{d} = \frac{50V}{5.0cm} = \frac{50V}{5\times10^{-2}m} = 1000V/m$$

Which direction is the field? Direction of decreasing potential!

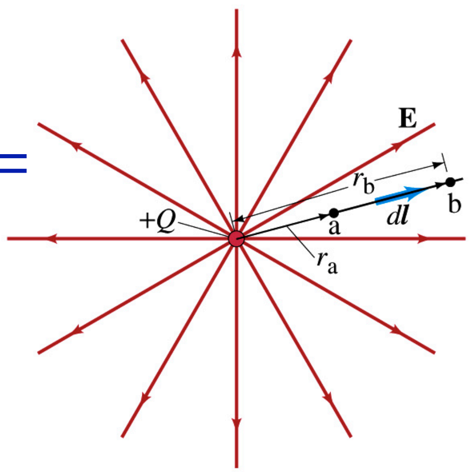
Electric Potential due to Point Charges

- What is the electric field by a single point charge Q at a distance r?

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = k\frac{Q}{r^2}$$

- Electric potential due to the field E for moving from point r_a to r_b in radial direction away from the charge Q is, using calculus,

$$V_b - V_a = -\int_{r_a}^{r_b} \vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{\hat{r}}{r^2}\cdot\hat{r}dr =$$

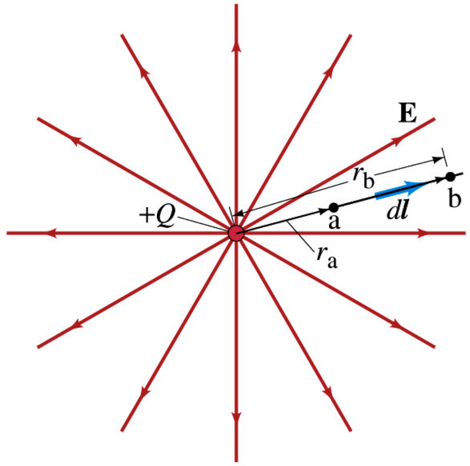


$$= -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{1}{r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Electric Potential due to Point Charges

- Since only the differences in potential have physical meaning, we can choose $V_b = 0$ at $r_b = \infty$.
- The electrical potential V at a distance r from a single point charge is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$



- So the absolute potential by a single point charge can be thought of as **the potential difference by a single point charge between r and infinity**

Properties of the Electric Potential

- What are the differences between the electric potential and the electric field?
  - Electric potential
    - Electric potential energy per unit charge
    - Inversely proportional to the distance
    - Simply add the potential by each of the source charges to obtain the total potential from multiple charges, since potential is a scalar quantity

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$$

  - Electric field
    - Electric force per unit charge
    - Inversely proportional to the square of the distance
    - Need vector sums to obtain the total field from multiple source charges

$$\boxed{\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

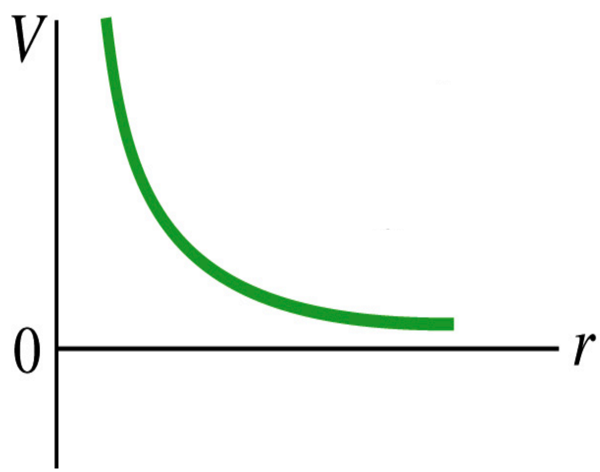
- Potential for the positive charge is large positive near the charge and decreases towards 0 at the large distance.
- Potential for the negative charge is large negative near the charge and increases towards 0 at a large distance.

Shape of the Electric Potential

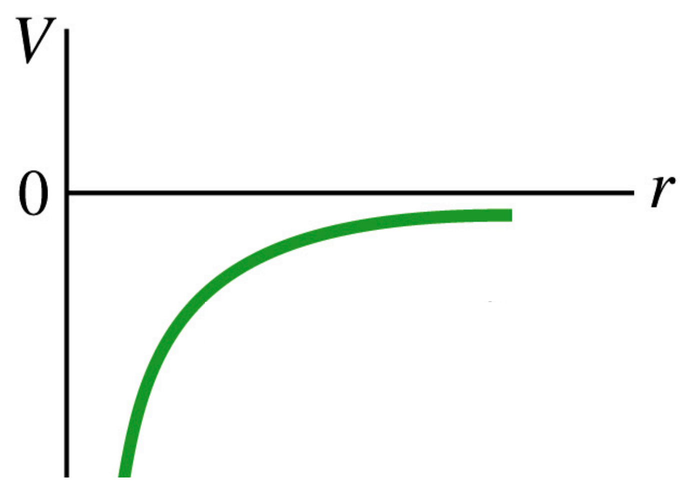
- So, how does the electric potential look like as a function of distance from the source charge?
  - What is the formula for the potential by a single charge?

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Positive Charge



Negative Charge



Uniformly charged sphere would have the potential the same as a single point charge.

What does this mean? **Uniformly charged sphere behaves like all the charge is on the single point in the center.**

Example 17 – 4

Potential due to a positive or negative charge: Determine the potential at a point 0.50m (a) from a +20μC point charge and (b) from a -20μC point charge.

The formula for absolute potential at a point r away from the charge Q is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

(a) For +20μC charge: $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = 9.0\times10^{9}\cdot\frac{\left(+20\times10^{-6}\right)}{0.50} = 3.6\times10^{5}V$

(b) For -20μC charge: $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = 9.0\times10^{9}\cdot\frac{\left(-20\times10^{-6}\right)}{0.50} = -3.6\times10^{5}V$

It is important to express electric potential with the proper sign!!

Example 17 – 5

Work to bring two positive charges close together: What minimum work is required by an external force to bring a charge q=3.00μC from a great distance away (r=infinity) to a point 0.500m from a charge Q=20.0 μC?

What is the work done by the electric field in terms of potential energy and potential?

$$W = -qV_{ba} = -\frac{q}{4\pi\varepsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right)$$

Since $r_b = 0.500m, r_a = \infty$ we obtain

$$W = -\frac{q}{4\pi\varepsilon_0}\left(\frac{Q}{r_b} - 0\right) = -\frac{q}{4\pi\varepsilon_0}\frac{Q}{r_b} = -\frac{\left(8.99\times10^9 N\cdot m^2/C^2\right)\cdot\left(3.00\times10^{-6}C\right)\left(20.00\times10^{-6}C\right)}{0.500m} = -1.08J$$

Electric force does negative work. In other words, the external force must work +1.08J to bring the charge 3.00μC from infinity to 0.500m to the charge 20.0μC.

Electric Potential by Charge Distributions

- Let's consider that there are n individual point charges in a given space and V=0 at r=infinity.
- Then the potential due to the charge Q_i at a point a, distance r_{ia} from Q_i is

$$V_{ia} = \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}}$$

- Thus the total potential V_a by all n point charges is

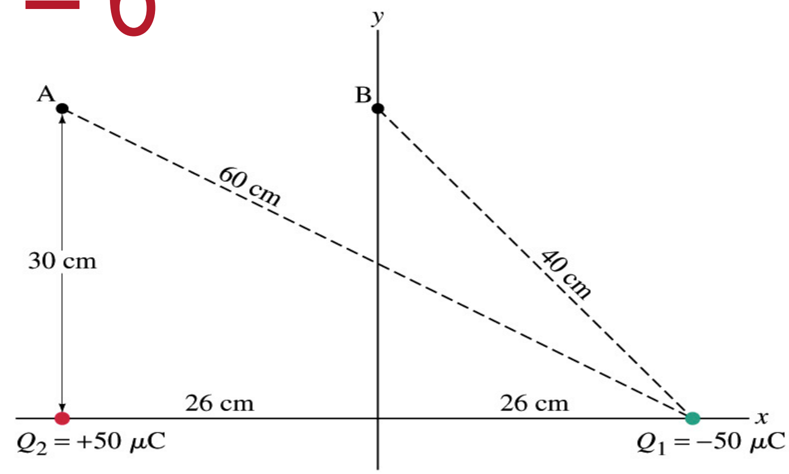
$$V_a = \sum_{i=1}^{n} V_{ia} = \sum_{i=1}^{n} \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}}$$

- For a continuous charge distribution, we obtain

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

Example 17 – 6

- **Potential due to two charges**: Calculate the electric potential (a) at point A in the figure due to the two charges shown, and (b) at point B.



- Potential is a scalar quantity, so one adds the potential by each of the source charge, as if they are numbers.

(a) potential at A is $V_A = V_{1A} + V_{2A} = \sum \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{iA}} =$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_{1A}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_{2A}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{r_{1A}} + \frac{Q_2}{r_{2A}}\right)$$

$$= 9.0\times10^9\left(\frac{-50\times10^{-6}}{0.60} + \frac{50\times10^{-6}}{0.30}\right) = 7.5\times10^5 V$$

Electric field at A?

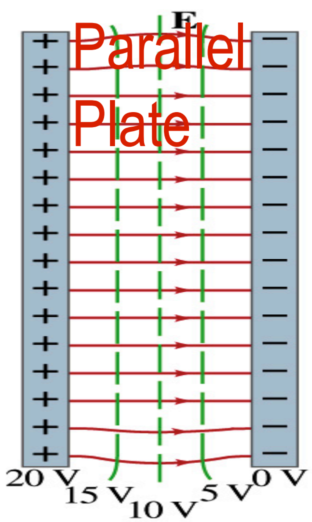
(b) How about potential at B?

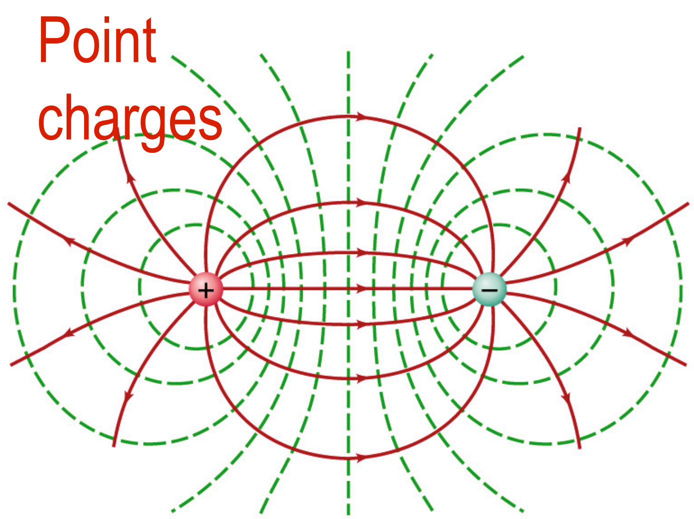
Equi-potential Surfaces

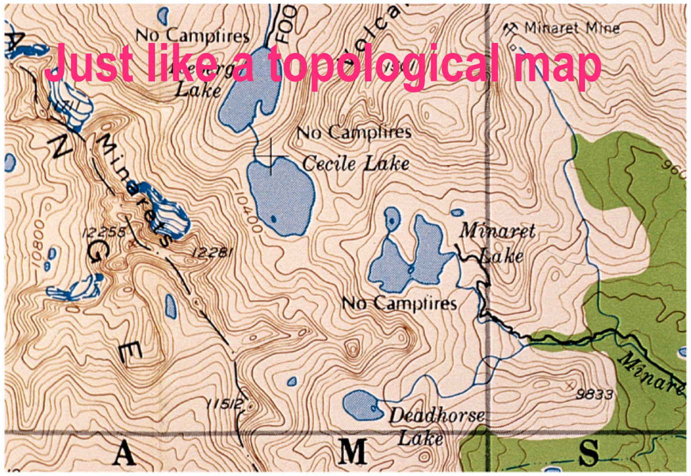
- Electric potential can be visualized using equipotential lines in 2-D or equipotential surfaces in 3-D
- Any two points on equipotential surfaces (lines) are on the same potential
- What does this mean in terms of the potential difference?
  - The potential difference between the two points on an equipotential surface is 0.
- How about the potential energy difference?
  - Also 0.
- What does this mean in terms of the work to move a charge along the surface between these two points?
  - No work is necessary to move a charge between these two points.

Equi-potential Surfaces

- An equipotential surface (line) must be perpendicular to the electric field. Why?
  - If there are any parallel components to the electric field, it would require work to move a charge along the surface.
- Since the equipotential surface (line) is perpendicular to the electric field, we can draw these surfaces or lines easily.
- There can be no electric field inside a conductor in static case, thus the entire volume of a conductor must be at the same potential.
- So the electric field must be perpendicular to the conductor surface.







Electrostatic Potential Energy

- Consider a point charge q is moved between points a and b where the electrostatic potentials due to other charges are V_a and V_b
- The change in electrostatic potential energy of q in the field by other charges is

$$\Delta U = U_b - U_a = q\left(V_b - V_a\right) = qV_{ba}$$

- Now what is the electrostatic potential energy of a system of charges?
  - Let's choose V=0 at r=infinity
  - If there are no other charges around, single point charge Q_1 in isolation has no potential energy and is exerted on with no electric force

Electrostatic Potential Energy; Two charges

- If a second point charge Q_2 is brought close to Q_1 at the distance r_{12}, the potential due to Q_1 at the position of Q_2 is

$$V = \frac{Q_1}{4\pi\varepsilon_0}\frac{1}{r_{12}}$$

- The potential energy of the two charges relative to V=0 at r=infinity is $U = Q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{12}}$
  - This is the work that needs to be done by an external force to bring Q_2 from infinity to a distance r_{12} from Q_1.
  - It is also a negative of the work needed to separate them to infinity.

Electrostatic Potential Energy; Three Charges

- So what do we do for three charges?
- Work is needed to bring all three charges together
  - Work needed to bring Q_1 to a certain place without the presence of any charge is 0.
  - Work needed to bring Q_2 to a distance to Q_1 is $U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{12}}$
  - Work need to bring Q_3 to a distance to Q_1 and Q_2 is

$$U_3 = U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2Q_3}{r_{23}}$$

- So the total electrostatic potential of the three charge system is

$$U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_2Q_3}{r_{23}}\right) \quad [V = 0 \text{ at } r = \infty]$$

  - What about a four charge system?

Electrostatic Potential Energy: electron Volt

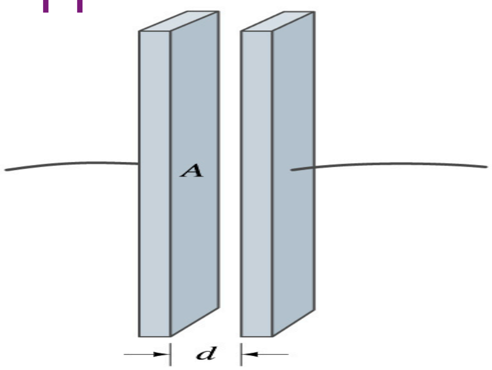
- What is the unit of electrostatic potential energy?
  - Joules
- Joules is a very large unit in dealing with electrons, atoms or molecules in atomic scale problems
- For convenience a new unit, electron volt (eV), is defined
  - 1 eV is defined as the energy acquired by a particle carrying the charge equal to that of an electron (q=e) when it moves across a potential difference of 1V.
  - How many Joules is 1 eV then? $1eV = 1.6\times10^{-19}C\cdot1V = 1.6\times10^{-19}J$
- eV however is **<u>not a standard SI unit</u>**. You must convert the energy to Joules for computations.
- What is the speed of an electron with kinetic energy 5000eV?

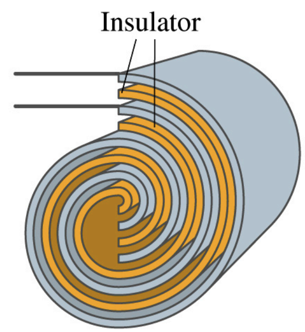
Capacitors (or Condensers)

- What is a capacitor?
  - A device that can store electric charge
  - But does not let them flow through
- What does it consist of?
  - Usually consists of two conducting objects (plates or sheets) placed near each other without touching
  - Why can't they touch each other?
    - The charge will neutralize…
- Can you give some examples?
  - Camera flash, UPS, Surge protectors, binary circuits, etc…
- How is a capacitor different than a battery?
  - Battery provides potential difference by storing energy (usually chemical energy) while the capacitor stores charges but very little energy.

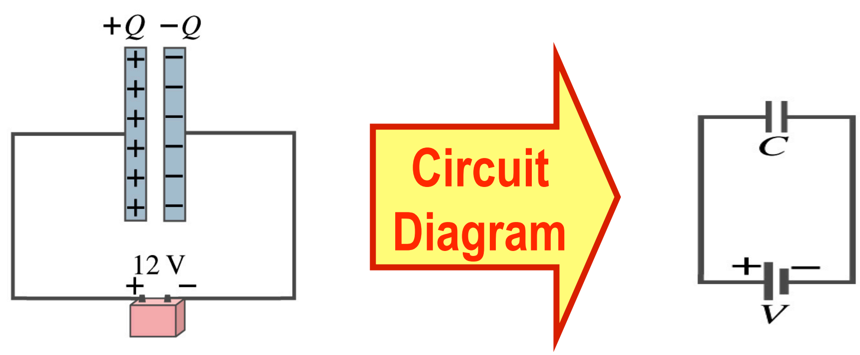
Capacitors

- A simple capacitor consists of a pair of parallel plates of area $\mathcal{A}$ separated by a distance d.
  - A cylindrical capacitors are essentially parallel plates wrapped around as a cylinder.



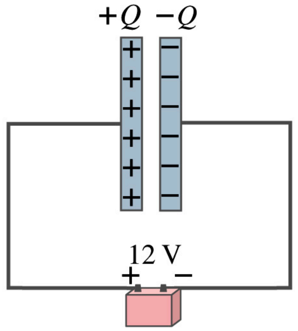


- How would you draw symbols for a capacitor and a battery?
  - Capacitor -||-
  - Battery (+) -|i- (-)



Capacitors

- What do you think will happen if a battery is connected (or the voltage is applied) to a capacitor?
  - The capacitor gets charged quickly, one plate positive and the other negative in equal amount.



- Each battery terminal, the wires and the plates are conductors. What does this mean?
  - All conductors are at the same potential. And?
  - So the full battery voltage is applied across the capacitor plates.
- So for a given capacitor, the amount of charge stored in the capacitor is proportional to the potential difference V_{ba} between the plates. How would you write this formula?

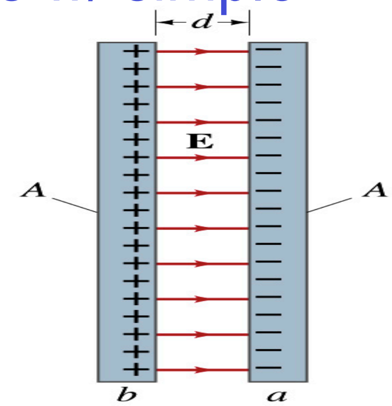
$$\boxed{Q = CV_{ba}}$$

C is a property of a capacitor so does not depend on Q or V.

  - C is a proportionality constant, called capacitance of the device.
  - What is the unit? C/V or Farad (F) Normally use mF or pF.

Determination of Capacitance

- C can be determined analytically for capacitors w/ simple geometry and air in between.
- Let's consider a parallel plate capacitor.
  - Plates have area A each and separated by d.
    - d is smaller than the length, and so E is uniform.
  - E for parallel plates is $E=\sigma/\varepsilon_0$, $\sigma=Q/A$ is the surface charge density.



- E and V are related
- $$V_{ba} = Ed = \frac{Q}{A}d$$
- Since Q=CV, we obtain:

$$\boxed{C = \frac{Q}{V_{ba}} = \frac{Q}{Qd/\varepsilon_0 A} = \frac{\varepsilon_0 A}{d}}$$

C only depends on the area and the distance of the plates and the permittivity of the medium between them.

Example 17 – 8

Capacitor calculations: (a) Calculate the capacitance of a capacitor whose plates are 20cmx3.0cm and are separated by a 1.0mm air gap. (b) What is the charge on each plate if the capacitor is connected to a 12-V battery? (c) What is the electric field between the plates? (d) Estimate the area of the plates needed to achieve a capacitance of 1F, given the same air gap.

(a) Using the formula for a parallel plate capacitor, we obtain

$$C = \frac{\varepsilon_0 A}{d} =$$

$$= \left(8.85\times 10^{-12}\, C^2/N\cdot m^2\right)\frac{0.2\times 0.03 m^2}{1\times 10^{-3}\, m} = 53\times 10^{-12}\, C^2/N\cdot m = 53\,pF$$

(b) From Q=CV, the charge on each plate is

$$Q = CV = \left(53\times 10^{-12}\, C^2/N\cdot m\right)\left(12V\right) = 6.4\times 10^{-10}\, C = 640\,pC$$

Example 17 – 8

(C) Using the formula for the electric field in two parallel plates

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{6.4\times10^{-10}\,C}{6.0\times10^{-3}\,m^2 \times 8.85\times10^{-12}\,C^2/N\cdot m^2} = 1.2\times10^4\,N/C = 1.2\times10^4\,V/m$$

Or, since $V = Ed$ we can obtain $E = \frac{V}{d} = \frac{12V}{1.0\times10^{-3}\,m} = 1.2\times10^4\,V/m$

(d) Solving the capacitance formula for A, we obtain

$$C = \frac{\varepsilon_0 A}{d}$$

Solve for A

$$A = \frac{Cd}{\varepsilon_0} = \frac{1F\cdot1\times10^{-3}\,m}{\left(9\times10^{-12}\,C^2/N\cdot m^2\right)} \approx 10^8\,m^2 \approx 100km^2$$

About 40% the area of Arlington ($256km^2$).

Capacitor Made of a Single Conductor

- A single isolated conductor can be said to have a capacitance, C.
- C can still be defined as the ratio of the charge to absolute potential V on the conductor.
  - So Q=CV.
- The potential of a single conducting sphere of radius r_b can be obtained as

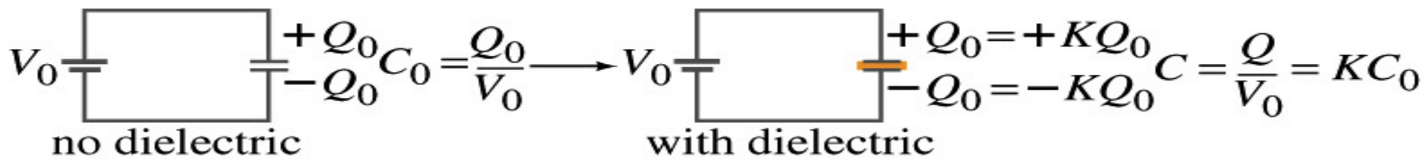
$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = \frac{Q}{4\pi\varepsilon_0 r_b} \quad \text{where} \quad r_a \to \infty$$

- So its capacitance is $\boxed{C = \frac{Q}{V} = 4\pi\varepsilon_0 r_b}$
- Single conductor alone is not considered as a capacitor. There must be another object near by to form a capacitor.

Effect of a Dielectric Material

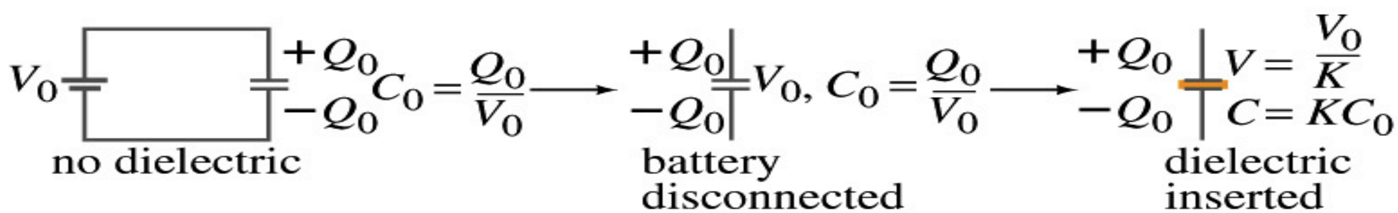
- Let's consider the two cases below:

Case #1 : constant V



(a) Voltage constant

Case #2 : constant Q

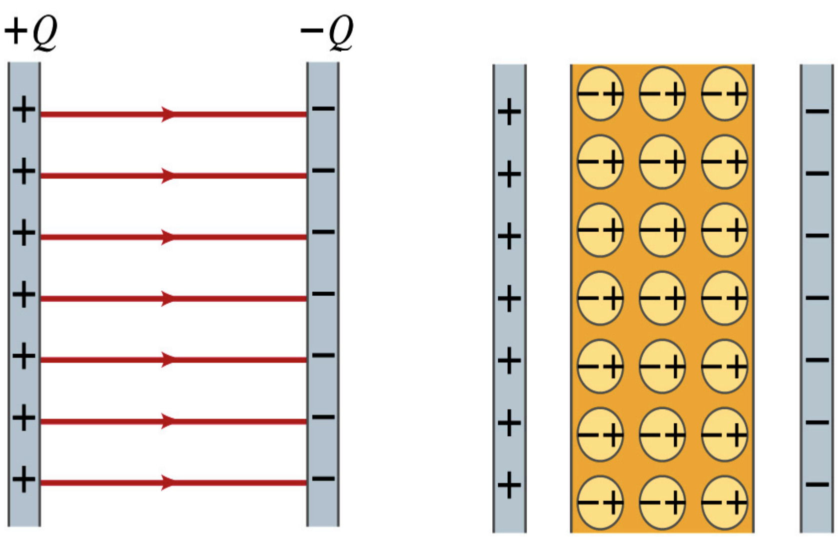


(b) Charge constant

- Constant voltage: Experimentally observed that the total charge on the each plate of the capacitor increases by K as the dielectric material is inserted between the gap ➔ $Q=KQ_0$
  - The capacitance increased to $C=Q/V_0=KQ_0/V_0=KC_0$
- Constant charge: Voltage found to drop by a factor K ➔ $V=V_0/K$
  - The capacitance increased to $C=Q_0/V=KQ_0/V_0=KC_0$

Molecular Description of Dielectric

- So what in the world makes dielectrics behave the way they do?
- We need to examine this in a microscopic scale.
- Let's consider a parallel plate capacitor that is charged up $+Q(=C_0V_0)$ and $-Q$ with air in between.
  - Assume there there is no way any charge can flow in or out

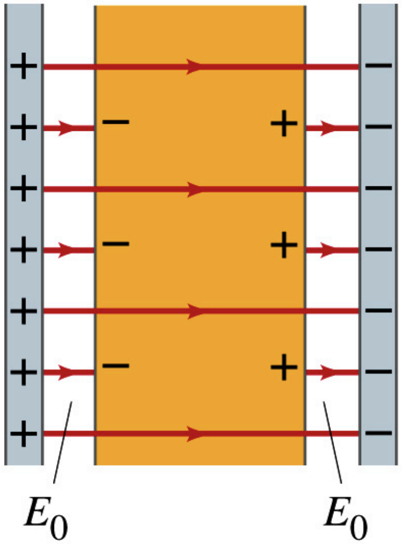


(a) (b)

- Now insert a dielectric
  - Dielectrics can be polar ➔ could have permanent dipole moment. What will happen?
- Due to the electric field molecules may be aligned.

Molecular Description of Dielectric

- OK. Then what happens?
- Then effectively, there will be some negative charges close to the surface of the positive plate and positive charges close to the negative plate
  - Some electric field do not pass through the whole dielectric but stops at the negative charge

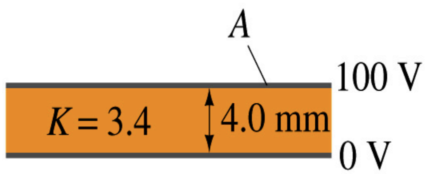


(c)

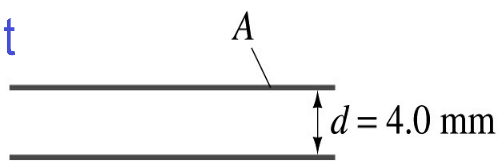
  - So the field inside dielectric is smaller than the air
- Since electric field is smaller, the force is smaller
  - The work need to move a test charge inside the dielectric is smaller
  - Thus the potential difference across the dielectric is smaller than across the air

Example

Dielectric Removal: A parallel-plate capacitor, filled with a dielectric with K=3.4, is connected to a 100-V battery. After the capacitor is fully charged, the battery is disconnected. The plates have area $A=4.0m^2$, and are separated by d=4.0mm. (a) Find the capacitance, the charge on the capacitor, the electric field strength, and the energy stored in the capacitor. (b) The dielectric is carefully removed, without changing the plate separation nor does any charge leave the capacitor. Find the new value of capacitance, electric field strength, voltage between the plates and the energy stored in the capacitor.



(a)



(b)

(a) $$C=\frac{\varepsilon A}{d}=\frac{K\varepsilon_0 A}{d}=\left(3.4\times 8.85\times 10^{-12}\ C^2/N\cdot m^2\right)\frac{4.0m^2}{4.0\times 10^{-3}\ m}=3.0\times 10^{-8}\ F=30nF$$

$$Q=CV=\left(3.0\times 10^{-8}\ F\right)\times 100V=3.0\times 10^{-6}\ C=3.0\mu C$$

$$E=\frac{V}{d}=\frac{100V}{4.0\times 10^{-3}\ m}=2.5\times 10^{4}\ V/m$$

$$U=\frac{1}{2}CV^2=\frac{1}{2}\left(3.0\times 10^{-8}\ F\right)\left(100V\right)^2=1.5\times 10^{-4}\ J$$

Example cont'd

(b) Since the dielectric has been removed, the effect of dielectric constant must be removed as well.

$$C_0 = \frac{C}{K} = \left(8.85\times10^{-12}\,C^2/N\cdot m^2\right)\frac{4.0m^2}{4.0\times10^{-3}\,m} = 8.8\times10^{-9}\,F = 8.8nF$$

Since charge is the same ($Q_0 = Q$) before and after the removal of the dielectric, we obtain

$$V_0 = Q/C_0 = K\,Q/C = K\,V = 3.4\times100V = 340V$$

$$E_0 = \frac{V_0}{d} = \frac{340V}{4.0\times10^{-3}\,m} = 8.5\times10^4\,V/m = 84\,kV/m$$

$$U_0 = \frac{1}{2}C_0V_0^2 = \frac{1}{2}\frac{C}{K}(KV)^2 = \frac{1}{2}KCV^2 = KU = 3.4\times1.5\times10^{-4}\,J = 5.1\times10^{-4}\,J$$

Where did the extra energy come from?.

~~Th… ergy conservation law is violated in elec… ??~~

Wrong!
Wrong!
Wrong!

External force has done the work of 3.6×10^{-4}J on the system to remove dielectric!!

Electric Energy Storage

- A charged capacitor stores energy.
  - The stored energy is the work done to charge it.
- The net effect of charging a capacitor is removing one type of charge from a plate and put them on to the other.
  - Battery does this when it is connected to a capacitor.
- Capacitors do not charge immediately.
  - Initially when the capacitor is uncharged, no work is necessary to move the first bit of charge. Why?
    - Since there is no charge, there is no field that the external work needs to overcome.
  - When some charge is on each plate, it requires work to add more charge due to electric repulsion.

Electric Energy Storage

- The work needed to add a small amount of charge, Q, when a potential difference across the plate is V: $W=Q\langle V\rangle=QV_f/2$
- Since V=Q/C, the work needed to store total charge Q is

$$W = Q\frac{V_f}{2} = Q\frac{Q}{2C} = \frac{Q^2}{2C}$$

- Thus, the energy stored in a capacitor when the capacitor carries charges +Q and –Q is

$$\boxed{U = \frac{Q^2}{2C}}$$

- Since Q=CV, we can rewrite

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Example 17 – 11

Energy store in a capacitor: A camera flash unit stores energy in a 150mF capacitor at 200V. How much electric energy can be stored?

Use the formula for stored energy. Umm.. Which one?

What do we know from the problem? C and V

So we use the one with C and V: $U = \frac{1}{2}CV^2$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(150\times10^{-6}F\right)\left(200V\right)^2 = 3.0J$$

How do we get J from FV²? $FV^2 = \left(\frac{C}{V}\right)V^2 = CV = C\left(\frac{J}{C}\right) = J$

Electric Energy Density

- The energy stored in a capacitor can be considered as being stored in the electric field between the two plates
- For a uniform field E between two plates, V=Ed and $C=\varepsilon_0 A/d$
- Thus the stored energy is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2 Ad$$

- Since Ad is the gap volume V, we can obtain the energy density, stored energy per unit volume, as

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

Valid for any space that is vacuum

Electric energy stored per unit volume in any region of space is proportional to the square of E in that region.