PHYS 1442 – Section 001
Lecture #5
Wednesday, June 17, 2009
Dr. Jaehoon Yu

• Chapter 18
  – The Electric Battery
  – Ohm’s Law: Resistors
  – Resistivity
  – Electric Power
  – Alternating Current
  – Power Delivered by AC

Today’s homework is #3, due 9pm, Thursday, June 24!!
Announcements

- 1st term exam Monday, June 29
  - 6:00 – 7:30pm
  - SH103
  - Covers Appendix A + CH16 – What we cover next
  Wednesday, June 24
  - Mixture of Multiple choice and free response problems
  - Please do not miss the exam!
Reminder: Special Project – Magnitude of Forces

• What is the magnitude of the Coulomb force one proton exerts to another 1m away? (10 points)
• What is the magnitude of the gravitational force one proton exerts to another 1m away? (10 points)
• Which one of the two forces is larger and by how many times? (10 points)
• Due at the beginning of the class Monday, June 22.
Electric Current and Resistance

• So far we have been studying static electricity
  – What is the static electricity?
    • The charges so far has not been moving but staying put at the location they are placed.

• Now we will learn dynamics of electricity

• What is the electric current?
  – A flow of electric charge
  – A few examples of the things that use electric current in everyday lives?

• In an electrostatic situation, there is no electric field inside a conductor but when there is current, there is field inside a conductor
  – Electric field is needed to keep charges moving
The Electric Battery

• **What is a battery?**
  – A device that produces electrical energy from the stored chemical energy and produces electricity.

• **Electric battery was invented by Volta in 1790s in Italy**
  – It was made of disks of zinc and silver based on his research that certain combinations of materials produce a greater electromotive force (emf), or potential, than others.

• **Simplest batteries contain two plates made of dissimilar metals, electrodes**
  – Electrodes are immersed in a solution, electrolyte
  – This unit is called a cell and many of these form a battery.

• **Zinc and Iron in the figure are called terminals**
How does a battery work?

- One of the electrodes in the figure is zinc and the other carbon.
- The acid electrolyte reacts with the zinc electrode and dissolves it.
- Each zinc atom leaves two electrons in the electrode and enters into the solution as a positive ion \( \rightarrow \) zinc electrode acquires negative charge and the electrolyte becomes positively charged.
- The carbon electrode picks up the positive charge.
- Since the two terminals are oppositely charged, there is potential difference between them.
How does a battery work?

- When the terminals are not connected, only the necessary amount of zinc is dissolved into the solution.

- How is a particular potential maintained?
  - As too many of zinc ion gets produced, if the terminals are not connected,
    - zinc electrode gets increasingly charged up negative
    - zinc ions get recombined with the electrons in zinc electrode

- Why does battery go dead?
  - When the terminals are connected, the negative charges will flow away from the zinc electrode
  - More zinc atoms dissolve into the electrolyte to produce more charge
  - One or more electrode get used up not producing any more charge.
Electric Current

- When a circuit is powered by a battery (or a source of emf), the charge can flow through the circuit.

- Electric Current: Any flow of charge
  - Current can flow whenever there is potential difference between the ends of a conductor (or when the two ends have opposite charges)
    - The current can flow even through the empty space
  - Electric current in a wire can be defined as the net amount of charge that passes through the wire’s full cross section at any point per unit time (just like the flow of water through a conduit…)
  - Average current is defined as: $\overline{I} = \Delta Q / \Delta t$
  - The instantaneous current is: $I = dQ / dt$
  - What kind of a quantity is the current? Scalar

Unit of the current? C/s 1A=1C/s

In a single circuit, conservation of electric charge guarantees that the current at one point of the circuit is the same as any other points on the circuit.
Example 18 – 1

Current is the flow of charge: A steady current of 2.5A flows in a wire for 4.0min. (a) How much charge passed by any point in the circuit? (b) How many electrons would this be?

Current is total amount charge flow through a circuit in a given time. So from \( \Delta Q = I \Delta t \) we obtain

\[
\Delta Q = I \Delta t = 2.5 \times 4.0 \times 60 = 600C
\]

The total number of electrons passed through the circuit is

\[
N_e = \frac{\Delta Q}{e} = \frac{600C}{1.6 \times 10^{-19} C} = 3.8 \times 10^{21} \text{ electrons}
\]
Direction of the Electric Current

• What do conductors have in abundance?
  – Free electrons

• What happens if a continuous loop of conducting wire is connected to the terminals of a battery?
  – Electrons start flowing through the wire continuously as soon as both the terminals are connected to the wire. How?
    • The potential difference between the battery terminals sets up an electric field inside the wire and in the direction parallel to it
    • Free electrons in the conducting wire get attracted to the positive terminal
    • The electrons leaving negative terminal flow through the wire and arrive at the positive terminal
      – Electrons flow from negative to positive terminal
  – Due to historical convention, the direction of the current is opposite to the direction of flow of electrons ➔ Conventional Current
Ohm’s Law: Resistance and Resistors

• What do we need to produce electric current?
  – Potential difference

• Georg S. Ohm experimentally established that the current is proportional to the potential difference ($I \propto V$)
  – If we connect a wire to a 12V battery, the current flowing through the wire is twice that of 6V, three times that of 4V and four times that of 3V battery.
  – What happens if we reverse the sign of the voltage?
    • It changes the direction of the current flow
    • Does not change the magnitude of the current
  – Just as in water flow case, if the height difference is large the flow rate is large $\Rightarrow$ If the potential difference is large, the current is large.
Ohm’s Law: Resistance

• The exact amount of current flow in a wire depends on
  – The voltage
  – The resistance of the wire to the flow of electrons
    • Just like the gunk in water pipe slows down water flow
    • Electrons are slowed down due to interactions with the atoms of the wire

• The higher the resistance the less the current for the given potential difference V
  – So how would you define resistance?
    • So that current is inversely proportional to the resistance
  – Often it is rewritten as
  – What does this mean?
    • The metal conductor’s resistance R is a constant independent of V.
  – This linear relationship is not valid for some materials like diodes, vacuum tubes, transistors etc. ➔ These are called non-ohmic
Example 18 – 3

Flashlight bulb resistance: A small flashlight bulb draws 300mA from its 1.5V battery. (a) What is the resistance of the bulb? (b) If the voltage drops to 1.2V, how would the current change?

From Ohm’s law, we obtain

\[ R = \frac{V}{I} = \frac{1.5V}{300mA} = \frac{1.5V}{0.3A} = 5.0\, \Omega \]

Would the current increase or decrease, if the voltage reduces to 1.2V?

If the resistance did not change, the current is

\[ I = \frac{V}{R} = \frac{1.2V}{5.0\, \Omega} = 0.24\, A = 240\, mA \]
Ohm’s Law: Resistors

• All electric devices offer resistance to the flow of current.
  – Filaments of light bulbs or heaters are wires with high resistance to cause electrons to lose their energy in the wire
  – In general connecting wires have low resistance compared to other devices on the circuit

• In circuits, resistors are used to control the amount of current
  – Resistors offer resistance of less than one ohm to millions of ohms
  – Main types are
    • “wire-wound” resistors which consists of a coil of fine wire
    • “composition” resistors which are usually made of semiconductor carbon
    • thin metal films

• When drawn in the circuit, the symbol for a resistor is:  

• Wires are drawn simply as straight lines
Ohm’s Law: Resistor Values

- Resistors have its resistance color-coded on its body
- The color-coding follows the convention below:

<table>
<thead>
<tr>
<th>Color</th>
<th>Number</th>
<th>Multiplier</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>0</td>
<td>$1 \times 10^0$</td>
<td>0%</td>
</tr>
<tr>
<td>Brown</td>
<td>1</td>
<td>$10^1$</td>
<td>1%</td>
</tr>
<tr>
<td>Red</td>
<td>2</td>
<td>$10^2$</td>
<td>2%</td>
</tr>
<tr>
<td>Orange</td>
<td>3</td>
<td>$10^3$</td>
<td>3%</td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
<td>$10^4$</td>
<td>4%</td>
</tr>
<tr>
<td>Green</td>
<td>5</td>
<td>$10^5$</td>
<td>5%</td>
</tr>
<tr>
<td>Blue</td>
<td>6</td>
<td>$10^6$</td>
<td>6%</td>
</tr>
<tr>
<td>Violet</td>
<td>7</td>
<td>$10^7$</td>
<td>7%</td>
</tr>
<tr>
<td>Gray</td>
<td>8</td>
<td>$10^8$</td>
<td>8%</td>
</tr>
<tr>
<td>White</td>
<td>9</td>
<td>$10^9$</td>
<td>9%</td>
</tr>
<tr>
<td>Gold</td>
<td>10^-1</td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td>Silver</td>
<td>10^-2</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>None</td>
<td>20%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What is the resistance of the resistor in this figure?

$25 \times 10^3 \pm 10\%$
Resistivity

• It is experimentally found that the resistance \( R \) of a metal wire is directly proportional to its length \( l \) and inversely proportional to its cross-sectional area \( A \)

\[
R = \rho \frac{l}{A}
\]

– How would you formulate this?

– The proportionality constant \( \rho \) is called the resistivity and depends on the material used. What is the unit of this constant?
  
  • ohm-m or \( \text{W} \cdot \text{m} \)
  • The values depends on purity, heat treatment, temperature, etc

– How would you interpret the resistivity?
  
  • The higher the resistivity the higher the resistance
  • The lower the resistivity the lower the resistance and the higher the conductivity ➔ Silver has the lowest resistivity.
    
    – So the silver is the best conductor

– The reciprocal of the resistivity is called the conductivity, \( \sigma \),

\[
\sigma = \frac{1}{\rho}
\]
Example 18 – 5

**Speaker wires:** Suppose you want to connect your stereo to remote speakers. (a) If each wire must be 20m long, what diameter copper wire should you use to keep the resistance less than 0.1-Ω per wire? (b) If the current on each speaker is 4.0A, what is the voltage drop across each wire?

The resistivity of a copper is \( \rho_{Cu} = 1.68 \times 10^{-8} \, \Omega \cdot m \) (Table 25.1)

From the formula for resistance, we can obtain the formula for area

\[
R = \rho \frac{l}{A} \quad \Rightarrow \quad A = \rho \frac{l}{R} = \pi r^2
\]

Solve for \( A \)

\[
d = 2r = 2\sqrt{\frac{\rho l}{\pi R}} = 2\sqrt{\frac{1.68 \times 10^{-8} \, \Omega \cdot m \cdot 20m}{\pi \cdot 0.1\Omega}} = 2.1 \times 10^{-3} \, m = 2.1 \, mm
\]

From Ohm’s law, \( V=IR \), we obtain

\[
V = IR = 4.0 \, A \cdot 0.1\Omega = 0.4V
\]
Example 18 – 6

**Stretching changes resistance:** A wire of resistance R is stretched uniformly until it is twice its original length. What happens to its resistance?

What is the constant quantity in this problem? The volume!

What is the volume of a cylinder of length L and radius r? \[ V = AL = \pi r^2 L \]

What happens to A if L increases factor two, L' = 2L?

The cross-sectional area, A, halves. A' = A/2

The original resistance is \[ R = \rho \frac{l}{A} \]

The new resistance is \[ R' = \rho \frac{L'}{A'} = \rho \frac{2L}{A/2} = 4 \rho \frac{L}{A} = 4R \]

The resistance of the wire increases by a factor of four if the length increases twice.
Temperature Dependence of Resistivity

• Do you think the resistivity depends on temperature?
  – Yes

• Would it increase or decrease with the temperature?
  – Increase
  – Why?
    – Since the atoms are vibrating more rapidly as temperature increases and are arranged in a less orderly fashion. So?
      • They might interfere more with the flow of electrons.

• If the temperature change is not too large, the resistivity of metals usually increase nearly linearly w/ temperature
  \[ \rho_T = \rho_0 \left[ 1 + \alpha (T - T_0) \right] \]
  – \( \alpha \) is the temperature coefficient of resistivity
  – \( \alpha \) of some semiconductors can be negative due to increased number of freed electrons.
Electric Power

• Why is the electric energy useful?
  – It can transform into different forms of energy easily.
    • Motors, pumps, etc, transform electric energy to mechanical energy
    • Heaters, dryers, cook-tops, etc, transforms electricity to thermal energy
    • Light bulb filament transforms electric energy to light energy
      – Only about 10% of the energy turns to light and the 90% lost via heat
      – Typical household light bulb and heating elements have resistance of order few ohms to few hundred of ohms

• How does electric energy transforms to thermal energy?
  – Flowing electrons collide with the vibrating atoms of the wire.
  – In each collision, part of electron’s kinetic energy is transferred to the atom it collides with.
  – The kinetic energy of wire’s atoms increases, and thus the temperature of the wire increases.
  – The increased thermal energy can be transferred as heat through conduction and convection to the air in a heater or to food on a pan, through radiation to bread in a toaster or radiated as light.
Electric Power

• How do we find out the power transformed by an electric device?
  – What is definition of the power?
    • The rate at which work is done or the energy is transformed
  – What is the energy transformed when an infinitesimal charge dq moves through a potential difference V?
    – dU=Vdq
    – If dt is the time required for an amount of charge dq to move through the potential difference V, the power P is
      – \( P = \frac{dU}{dt} = V \frac{dq}{dt} \)
      – Thus, we obtain \( P = VI \). In terms of resistance
      \[ P = I^2 R = \frac{V^2}{R} \]
    – What is the unit?
      – Watts = J/s
    – What kind of quantity is the electrical power?
      • Scalar
    – P=IV can apply to any devices while the formula with resistance can only apply to resistors.
Headlights: Calculate the resistance of a 40-W automobile headlight designed for 12V.

Since the power is 40W and the voltage is 12V, we use the formula with V and R.

\[ P = \frac{V^2}{R} \]

Solve for R

\[ R = \frac{V^2}{P} = \frac{(12V)^2}{40W} = 3.6\Omega \]
Power in Household Circuits

- Household devices usually have small resistance
  - But since they draw current, if they become large enough, wires can heat up (overloaded)
    - Why is using thicker wires safer?
      - Thicker wires has less resistance, lower heat
    - Overloaded wire can set off a fire at home
- How do we prevent this?
  - Put in a switch that would disconnect the circuit when overloaded
    - Fuse or circuit breakers
    - They open up the circuit when the current is over certain value
Example 18 – 11

Will a fuse blow?: Calculate

Determine the total current drawn by all the devices in the circuit in the figure.

The total current is the sum of current drawn by individual device.

\[ P = IV \quad \text{Solve for } I = \frac{P}{V} \]

Bulb \( I_B = \frac{100W}{120V} = 0.8A \)

Heater \( I_H = \frac{1800W}{120V} = 15.0A \)

Stereo \( I_S = \frac{135W}{120V} = 2.9A \)

Dryer \( I_D = \frac{1200W}{120V} = 10.0A \)

Total current

\[ I_T = I_B + I_H + I_S + I_D = 0.8A + 15.0A + 2.9A + 10.0A = 28.7A \]

What is the total power?

\( P_T = (100W + 1800W + 350W + 1200W) = 3450W \)

Jaehoon Yu
Alternating Current

• Does the direction of the flow of current change when a battery is connected to a circuit?
  – No. Why?
    • Because its source of potential difference stays put.
  – This kind of current is called the Direct Current (DC), and it does not change its direction of flow.
    • How would DC look as a function of time?
      – A straight line

• Electric generators at electric power plant produce alternating current (AC)
  – AC reverses direction many times a second
  – AC is sinusoidal as a function of time

• Most the currents supplied to homes and business are AC.
Alternating Current

• The voltage produced by an AC electric generator is sinusoidal
  – This is why the current is sinusoidal
• Voltage produced can be written as
  \[ V = V_0 \sin 2\pi f = V_0 \sin \omega t \]
• What are the maximum and minimum voltages?
  – \( V_0 \) and \(-V_0\)
  – The potential oscillates between \( +V_0 \) and \(-V_0 \), the peak voltages or amplitude
  – What is \( f \)?
    • The frequency, the number of complete oscillations made per second. What is the unit of \( f \)? What is the normal size of \( f \) in the US?
      – \( f=60\text{Hz} \) in the US and Canada.
      – Many European countries have \( f=50\text{Hz} \).
    – \( w=2\pi f \)
Alternating Current

• Since $V=IR$, if a voltage $V$ exists across a resistance $R$, the current $I$ is

$$I = \frac{V}{R} = \frac{V_0}{R} \sin 2\pi f t = I_0 \sin \omega t$$

• What are the maximum and minimum currents?
  – $I_0$ and $-I_0$
  – The current oscillates between $+I_0$ and $-I_0$, the peak currents or amplitude. The current is positive when electron flows to one direction and negative when they flow opposite.
  – AC is as many times positive as negative. What’s the average current?
    • Zero. So there is no power and no heat is produced in a heater?
      – Yes there is! The electrons actually flow back and forth, so power is delivered.
Power Delivered by Alternating Current

• AC power delivered to a resistance is:

\[ P = I^2R = I_0^2R\sin^2\omega t \]

- Since the current is squared, the power is always positive

• The average power delivered is

\[ \bar{P} = \frac{1}{2}I_0^2R \]

• Since the power is also \( P = \frac{V^2}{R} \), we can obtain

\[ P = \left(\frac{V_0^2}{R}\right)\sin^2\omega t \]

Average power

• The average of the square of current and voltage are important in calculating power:

\[ \bar{I}^2 = \frac{1}{2}I_0^2 \]

\[ \bar{V}^2 = \frac{1}{2}V_0^2 \]
Power Delivered by Alternating Current

- The square root of each of these are called root-mean-square, or rms:
  \[ I_{rms} = \sqrt{I^2} = \frac{I_0}{\sqrt{2}} = 0.707I_0 \]
  \[ V_{rms} = \sqrt{V^2} = \frac{V_0}{\sqrt{2}} = 0.707V_0 \]

- rms values are sometimes called effective values
  - These are useful quantities since they can substitute current and voltage directly in power, as if they are in DC
    \[ P = \frac{1}{2} I_0^2 R = I_{rms}^2 R \]
    \[ P = \frac{1}{2} \frac{V_0^2}{R} = \frac{V_{rms}^2}{R} \]
    \[ P = I_{rms} V_{rms} \]
  - In other words, an AC of peak voltage \( V_0 \) or peak current \( I_0 \) produces as much power as DC voltage of \( V_{rms} \) or DC current \( I_{rms} \).
  - So normally, rms values in AC are specified or measured.
    - US uses 115V rms voltage. What is the peak voltage?
      \[ V_0 = \sqrt{2} V_{rms} = \sqrt{2} \cdot 115V = 162.6V \]
    - Europe uses 240V
      \[ V_0 = \sqrt{2} V_{rms} = \sqrt{2} \cdot 240V = 340V \]
Example 18 – 12

Hair Dryer. (a) Calculate the resistance and the peak current in a 1000-W hair dryer connected to a 120-V AC line. (b) What happens if it is connected to a 240-V line in Britain?

The rms current is: 
\[ I_{rms} = \frac{\overline{P}}{V_{rms}} = \frac{1000W}{120V} = 8.33A \]

The peak current is: 
\[ I_0 = \sqrt{2}I_{rms} = \sqrt{2} \cdot 8.33A = 11.8A \]

Thus the resistance is: 
\[ R = \frac{\overline{P}}{I_{rms}^2} = \frac{1000W}{(8.33A)^2} = 14.4\Omega \]

(b) If connected to 240V in Britain …

The average power provide by the AC in UK is 
\[ \overline{P} = \frac{V_{rms}^2}{R} = \frac{(240V)^2}{14.4\Omega} = 4000W \]

So? The heating coils in the dryer will melt!
Microscopic View of Electric Current

• When a potential difference is applied to the two ends of a wire with uniform cross-section, the direction of electric field is parallel to the walls of the wire, this is possible since the charges are moving, electrodynamics.

• Let’s define a microscopic vector quantity, the current density, \( j \), the electric current per unit cross-sectional area
  – \( j = I/A \) or \( I = jA \) if the current density is uniform
  – If not uniform \( I = \int j \cdot dA \)

  – The direction of \( j \) is the direction the positive charge would move when placed at that position, generally the same as \( E \)

• The current density exists on any point in space while the current \( I \) refers to a conductor as a whole so macroscopic.
Microscopic View of Electric Current

• The direction of \( j \) is the direction of a positive charge. So in a conductor, since negatively charged electrons move, the direction is \(-j\).

• Let’s think about the current in a microscopic view again:
  – When voltage is applied to the end of a wire
  – Electric field is generated by the potential difference
  – Electrons feel force and get accelerated
  – Electrons soon reach to a steady average speed due to collisions with atoms in the wire, called drift velocity, \( v_d \)
  – The drift velocity is normally much smaller than electrons’ average random speed.
Microscopic View of Electric Current

• How do we relate $v_d$ with the macroscopic current $I$?
  – In time interval $\Delta t$, the electrons travel $I = v_d \Delta t$ on average.
  – If wire’s x-sectional area is $A$, in time $\Delta t$ electrons in a volume $V = IA = Av_d \Delta t$ will pass through the area $A$.
  – If there are $n$ free electrons (of charge $-e$) per unit volume, the total charge $\Delta Q$ that pass through $A$ in time $\Delta t$ is
    \[ \Delta Q = \text{(total number of charge, } N) \times \text{(charge per particle)} = (nV)(-e) = -(nAv_d \Delta te) \]
  – The current $I$ in the wire is
    \[ I = \frac{\Delta Q}{\Delta t} = -neAv_d \]
  – The density in vector form is
    \[ \vec{j} = \frac{I}{A} = -ne\vec{v}_d \]
  – For any types of charge:
    \[ I = \sum_i n_i q_i \vec{v}_{di} A \quad \text{and} \quad \vec{j} = \sum_i n_i q_i \vec{v}_{di} \]
Microscopic View of Electric Current

- The drift velocity of electrons in a wire is only about 0.05mm/s. How could we get light turned on immediately then?
  - While the electrons in a wire travels slow, the electric field travels essentially at the speed of light. Then what is all the talk about electrons flowing through?
    - It is just like water. When you turn on the facet, water flows right off the facet despite the fact that the water travels slow.
    - Electricity is the same. Electrons fill the conductor wire and when the switch is flipped on or a potential difference is applied, the electrons closed to the positive terminal flows into the bulb.
    - Interesting, isn’t it? Why is the field travel at the speed of light then?
Ohm’s Law in Microscopic View

• Ohm’s law can be written in microscopic quantities.
  – Resistance in terms of resistivity is \( R = \rho \frac{l}{A} \)
  – We can rewrite \( V \) and \( I \) as: \( I = JA \), \( V = E\ell \)
  – If electric field is uniform, from \( V = IR \), we obtain
  
  \[ V = IR \]
  \[ EL = (JA) \left( \rho \frac{l}{A} \right) = j\rho l \]
  So
  \[ j = \frac{E}{\rho} = \sigma E \]
  – In a metal conductor, \( r \) or \( s \) does not depend on \( V \), thus, the current density \( j \) is proportional to the electric field \( E \)
  
  Microscopic statement of Ohm’s Law
  – In vector form, the density can be written as
  \[ \vec{j} = \frac{\vec{E}}{\rho} = \sigma \vec{E} \]
Superconductivity

- At the temperature near absolute 0K, resistivity of certain material becomes 0.
  - This state is called the “superconducting” state.
  - Observed in 1911 by H. K. Onnes when he cooled mercury to 4.2K (-269°C).
    - Resistance of mercury suddenly dropped to 0.
  - In general superconducting materials become superconducting below a transition temperature.
  - The highest temperature superconductivity seen is 160K
    - First observation above the boiling temperature of liquid nitrogen is in 1987 at 90k observed from a compound of yttrium, barium, copper and oxygen.

- Since much smaller amount of material can carry just as much current more efficiently, superconductivity can make electric cars more practical, computers faster, and capacitors store higher energy.
Electric Hazards: Leakage Currents

• How does one feel shock by electricity?
  – Electric current stimulates nerves and muscles, and we feel a shock
  – The severity of the shock depends on the amount of current, how long it acts and through what part of the body it passes
  – Electric current heats tissues and can cause burns

• Currents above 70mA on a torso for a second or more is fatal, causing heart to function irregularly, “ventricular fibrillation”

• A dry human body between two points on opposite side of the body is about $10^4$ to $10^6$ W.

• When wet, it could be $10^3$W.

• A person in good contact with the ground who touches 120V DC line with wet hands can get the current: $I = \frac{V}{R} = \frac{120V}{1000\Omega} = 120mA$
  – Could be lethal

Wednesday, June 17, 2009