PHYS 1442 – Section 001
Lecture #8

Monday, July 6, 2009

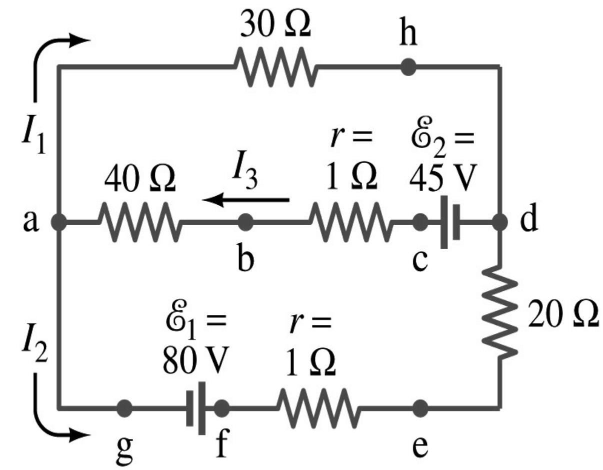
Dr. Jaehoon Yu

- Chapter 19
  - EMFs in Series and Parallel
  - Capacitors in Series and Parallel
  - RC Circuits
  - Electric Hazards

Announcements

- Quiz #3
  - At the beginning of the class this Wednesday, July 8
  - Covers CH19+ What we finish today
- Exam results
  - Class Average: 56/96
    - Equivalent to 59/100
  - Top score: 93/96
- Second non-comprehensive exam
  - Date will be moved to Monday, July 27
  - Covers from CH19 – what we finish Monday, July 20
  - There will be a help session Wednesday, July 22, in class

Special Project



1. Calculate the currents I_1, I_2 and I_3 in each of the branches of the circuit in the figure. Choose two different junctions for the second rule. (15 points)

2. Do the same as above but this time change the direction of one of the currents I_1 or I_2 and pick yet two other junctions than the ones used above and explain the result. (15 points)

3. You must show your own detailed work. Do not copy from the book or your friend's work! You will get 0 upon any indication of copying.

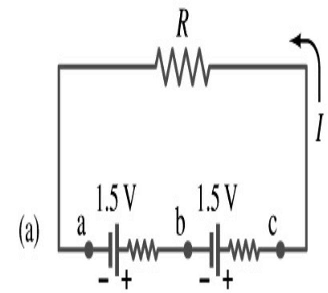
4. Due for this project is this Wednesday, July 8.

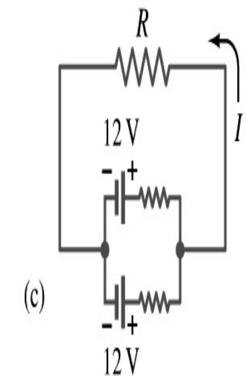
EMFs in Series and Parallel: Charging a Battery

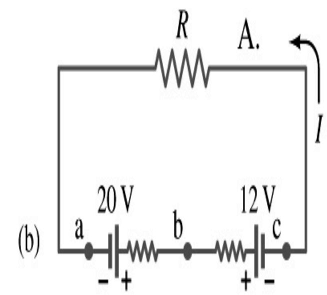
- When two or more sources of emfs, such as batteries, are connected in series
  - The total voltage is the algebraic sum of their voltages, if their direction is the same
    - V_{ac}=1.5 + 1.5=3.0V in figure (a).
  - If the batteries are arranged in an opposite direction, the total voltage is the difference between them
    - V_{ac}=20 – 12=8.0V in figure (b)

This is the way we jump start a car

    - Connecting batteries in opposite direction is wasteful.
    - This, however, is the way a battery charger works.
    - Since the 20V battery is at a higher voltage, it forces charges into 12V battery
    - Some battery are rechargeable since their chemical reactions are reversible but most the batteries can not reverse their chemical reactions



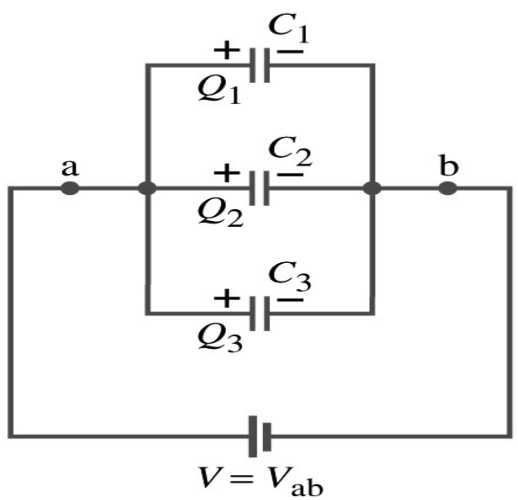


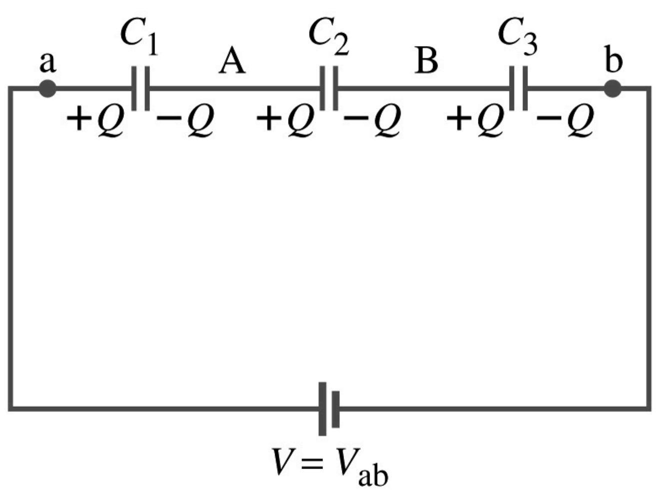


Parallel arrangements (c) are used only to increase currents.

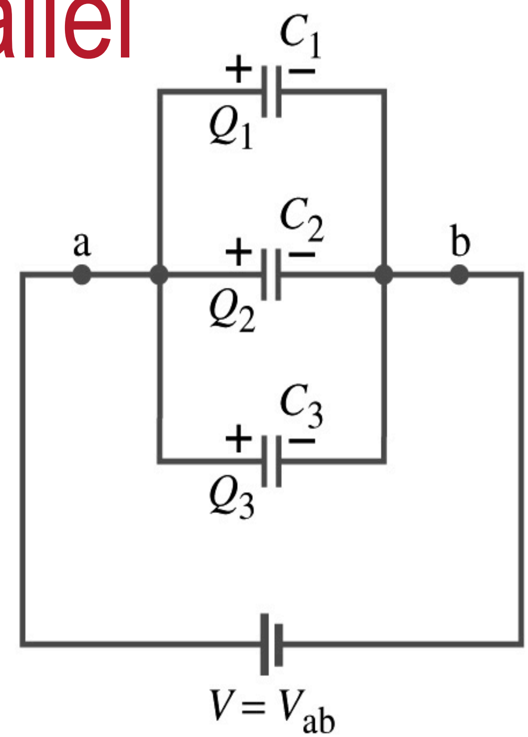
Capacitors in Series or Parallel

- Capacitors are also used in many electric circuits.
- So what is an electric circuit again?
  - A closed path of conductors, usually wires, connecting capacitors, resisters and other electrical devices, in which
    - charges can flow
    - And includes a voltage source such as a battery
- Capacitors can be connected in various ways.
  - In parallel and in Series or in combination





Capacitors in Parallel

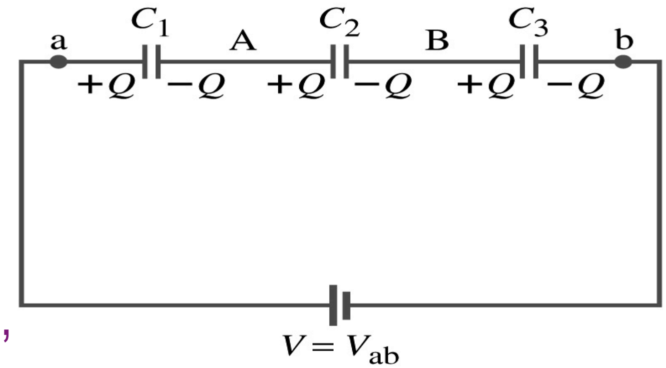


- Parallel arrangement provides the **same voltage** across all the capacitors.
  - Left hand plates are at V_a and right hand plates are at V_b
  - So each capacitor plate acquires charges given by the formula
    - $Q_1=C_1V$, $Q_2=C_2V$, and $Q_3=C_3V$
- The total charge Q that must leave the battery is then
  - $Q=Q_1+Q_2+Q_3=V(C_1+C_2+C_3)$
- Consider that the three capacitors behave like an equivalent one
  - $Q=C_{eq}V= V(C_1+C_2+C_3)$
- Thus the equivalent capacitance in parallel is $\boxed{C_{eq} = C_1 + C_2 + C_3}$

What is the net effect?

The capacitance increases!!!

Capacitors in Series



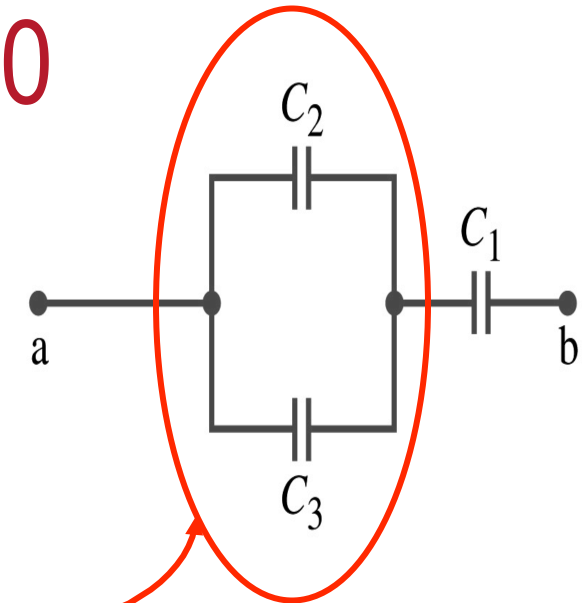
- Series arrangement is more interesting
  - When a battery is connected, +Q flows to the left plate of C_1 and –Q flows to the right plate of C_3 inducing opposite sign charges on the other plates.
  - Since the capacitor in the middle was originally neutral, charges get induced to neutralize the induced charges
  - So the charge on each capacitor is the same value, Q. (**Same charge**)
- Consider that the three capacitors behave like an equivalent one
  - $Q=C_{eq}V \Rightarrow V=Q/C_{eq}$
- The total voltage V across the three capacitors in series must be equal to the sum of the voltages across each capacitor.
  - $V=V_1+V_2+V_3=(Q/C_1+Q/C_2+Q/C_3)$
- Putting all these together, we obtain:
- $V=Q/C_{eq}=Q(1/C_1+1/C_2+1/C_3)$
- Thus the equivalent capacitance is

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

What is the net effect?

The capacitance smaller than the smallest C!!!

Example 19 – 10



Equivalent Capacitor: Determine the capacitance of a single capacitor that will have the same effect as the combination shown in the figure. Take $C_1=C_2=C_3=C$.

$\boxed{\text{We should do these first!!}}$

How? These are in parallel so the equivalent capacitance is:

$$C_{eq1} = C_1 + C_2 = 2C$$

Now the equivalent capacitor is in series with C_1.

$$\frac{1}{C_{eq}} = \frac{1}{C_{eq1}} + \frac{1}{C_2} = \frac{1}{2C} + \frac{1}{C} = \frac{3}{2C}$$

Solve for C_{eq}

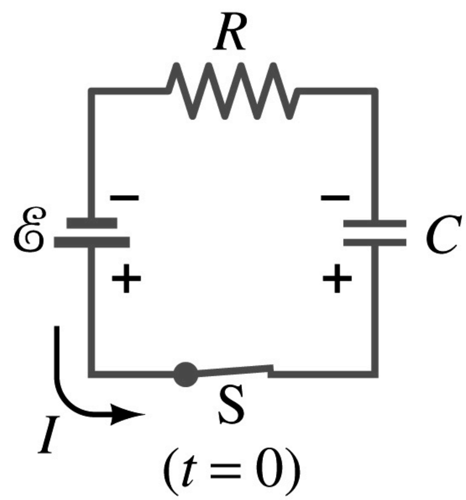
$$C_{eq} = \frac{2C}{3}$$

Resister and Capacitor Arrangements

- Parallel Capacitor arrangements $$C_{eq} = \sum_i C_i$$
- Parallel Resister arrangements $$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$
- Series Capacitor arrangements $$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$
- Series Resister arrangements $$R_{eq} = \sum_i R_i$$

RC Circuits

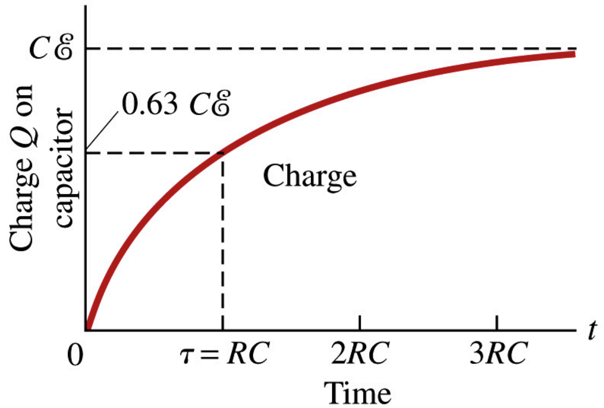
- Circuits containing both resisters and capacitors
  - RC circuits are used commonly in everyday life
    - Control windshield wiper
    - Timing of traffic light color change
    - Camera flashes and heart pacemakers
- How does an RC circuit look?
  - There should be a source of emf, capacitors and resisters
- What happens when the switch S is closed?
  - Current immediately starts flowing through the circuit.
  - Electrons flows out of negative terminal of the emf source, through the resister R and accumulates on the upper plate of the capacitor
  - The electrons from the bottom plate of the capacitor flow into the positive terminal of the battery, leaving only positive charge on the bottom plate
  - As the charge accumulates on the capacitor, the potential difference across it increases
  - The current reduces gradually to 0 till the voltage across the capacitor is the same as emf.
  - The charge on the capacitor increases till it reaches to its maximum, $C\mathcal{E}$.
  - What happens when the battery in the circuit is replaced with a wire?

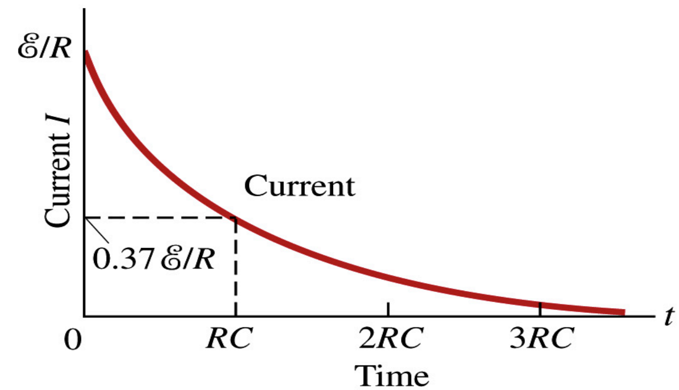


Capacitor discharges

RC Circuits

- How does all this look like in graphs?
  - Charge and the current on the capacitor as a function of time





  - From energy conservation (Kirchhoff's 2nd rule), the emf ε must be equal to the voltage drop across the capacitor and the resister
    - $\varepsilon = I\mathcal{R} + Q/C$
    - $\mathcal{R}$ includes all resistance in the circuit, including the internal resistance of the battery, I is the current in the circuit at any instant, and Q is the charge of the capacitor at that same instance.

Analysis of RC Circuits

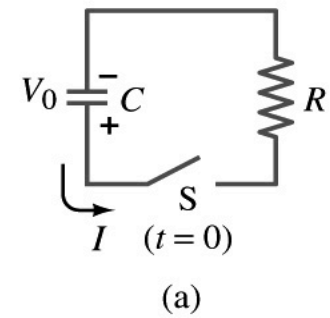
- From the energy conservation, we obtain $\varepsilon = I\mathcal{R} + Q/C$
- Which ones are constant in the above equation?
  - ε, $\mathcal{R}$ and C are constant
  - Q and I are functions of time
- How do we write the rate at which the charge is accumulated on the capacitor?
  - We can rewrite the above equation as $\varepsilon = R\dfrac{dQ}{dt} + \dfrac{1}{C}Q$
  - This equation can be solved by rearranging the terms as $\dfrac{dQ}{C\varepsilon - Q} = \dfrac{dt}{RC}$

Analysis of RC Circuits

- Charge $Q = C\varepsilon\left(1 - e^{-t/RC}\right)$ and voltage $V_C = \varepsilon\left(1 - e^{-t/RC}\right)$
- What can we see from the above equations?
  - Q and V_C increase from 0 at t=0 to maximum value $Q_{max}=C\varepsilon$ and $V_C=\varepsilon$.
- In how much time?
  - The quantity RC is called the time constant, τ, of the circuit
    - τ=RC, What is the unit? $\boxed{\text{Sec.}}$
  - What is the physical meaning?
    - The time required for the capacitor to reach $(1-e^{-1})=0.63$ or 63% of the full charge
- The current is $I = \frac{dQ}{dt} = \frac{\varepsilon}{R} e^{-t/RC}$

Discharging RC Circuits

- When a capacitor is already charged, it is allowed to discharge through a resistance R.
  - When the switch S is closed, the voltage across the resistor at any instant equals that across the capacitor. Thus IR=Q/C.
  - The rate at which the charge leaves the capacitor equals the negative of the current flows through the resistor
    - I= - dQ/dt. Why negative?
    - Since the current is leaving the capacitor
  - Thus the voltage equation becomes a differential equation



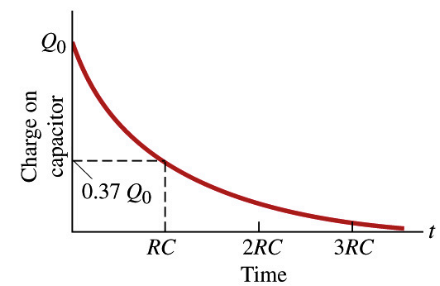
$$-\frac{dQ}{dt}R=\frac{Q}{C} \quad \text{Rearrange terms} \quad \frac{dQ}{Q}=-\frac{dt}{RC}$$

Discharging RC Circuits

– Now, let's integrate from t=0 when the charge is Q_0 to t when the charge is Q $\int_{Q_0}^{Q}\frac{dQ}{Q}=-\int_0^t\frac{dt}{RC}$

– The result is $\ln Q\Big|_{Q_0}^{Q}=\ln\frac{Q}{Q_0}=-\frac{t}{RC}$

– Thus, we obtain $\boxed{Q(t)=Q_0e^{-t/RC}}$



(a) (b)

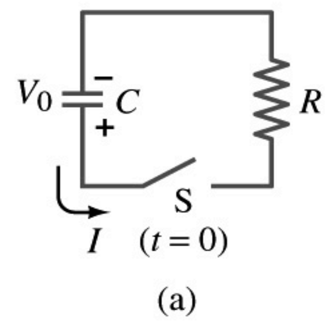
– What does this tell you about the charge on the capacitor?
  - It decreases exponentially with time at a time constant RC
  - Just like the case of charging

– The current is: $\boxed{I(t)=I_0e^{-t/RC}}$

– The voltage is: $\boxed{V(t)=V_0e^{-t/RC}}$

Ex. 19 – 12

Discharging RC circuit. If a charged capacitor C=35μF is connected to a resistance R=120Ω as in the figure, how much time will elapse until the voltage falls to 10% of its original (maximum) value?



What is the RC time of this circuit?

The RC time $\tau = RC = 120 \cdot 35 \times 10^{-6} = 4.2ms$

Since we are looking for the time it takes for V_c=10% of V_0, we obtain

$$V(t) = V_0 e^{-t/RC}$$

For $0.1V_0$

$$0.1V_0 = V_0 e^{-t/RC}$$

Rearrange terms

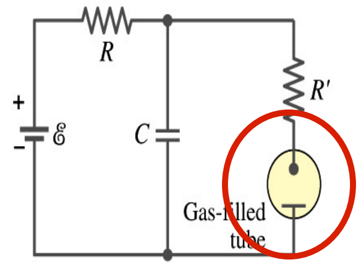
$$-t/RC = \ln 0.1 = -\ln 10 = -2.3$$

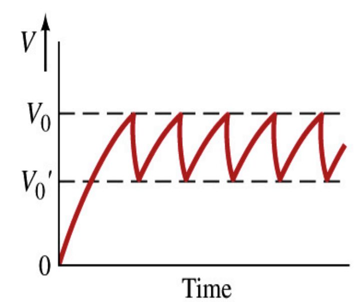
Solve for t

$$t = RC \cdot (2.3) = 4.2 \times 10^{-3} \cdot (2.3) = 9.7 \times 10^{-3} (\sec)$$

Application of RC Circuits

- What do you think the charging and discharging characteristics of RC circuits can be used for?
  - To produce voltage pulses at a regular frequency
  - How?
    - The capacitor charges up to a particular voltage and discharges
    - A simple way of doing this is to use breakdown of voltage in a gas filled tube
      - The discharge occurs when the voltage breaks down at V_0
      - After the completion of discharge, the tube no longer conducts
      - Then the voltage is at V_0' and it starts charging up
      - How do you think the voltage as a function of time look?
        - » A sawtooth shape
    - Pace maker, intermittent windshield wiper, etc





Electric Hazards: Leakage Currents

- How does one feel shock by electricity?
  - Electric current stimulates nerves and muscles, and we feel a shock
  - The severity of the shock depends on the amount of current, how long it acts and through what part of the body it passes
  - Electric current heats tissues and can cause burns
- Currents above 70mA on a torso for a second or more is fatal, causing heart to function irregularly, “ventricular fibrillation”
- A dry human body between two points on opposite side of the body is about 10^4 to 10^6 Ω.
- When wet, it could be 10^3Ω.
- A person in good contact with the ground who touches 120V DC line with wet hands can get the current:

$$I = \frac{V}{R} = \frac{120V}{1000\Omega} = 120mA$$

  - Could be lethal