

PHYS 1442 – Section 001

Lecture #10

Monday, July 13, 2009

Dr. Jaehoon Yu

- Chapter 20
 - Magnetic Forces on a Moving Charge
 - Charged Particle Path in a Magnetic Field
 - Sources of Magnetic Field
 - Magnetic Field Due to Straight Wire
 - Forces Between Two Parallel Wires
 - Ampère's Law and Its Verification
 - Solenoid and Toroidal Magnetic Field



Announcements

- Quiz Results
 - Class average: 33/45
 - Equivalent to 74/100
 - Previous results: 56/100 and 63/100
 - Continued upward trend!!! Excellent!!
 - Top score: 45/45
- Second non-comprehensive exam
 - Date and time: 6 – 7:30pm, Monday, July 27
 - Covers from CH19 – what we finish next Monday, July 20
 - There will be a help session Wednesday, July 22, in class
- Mid-term grade discussion
 - Bottom half of Wednesday's class
 - Please do not miss it



Magnetic Forces on a Moving Charge

- Will moving charge in a magnetic field experience force?
 - Yes
 - Why?
 - Since the wire carrying a current (moving charge) experience force in a magnetic field, a free moving charge must feel the same kind of force...☺
- OK, then how much force would it experience?
 - Let's consider N moving particles with charge q each, and they pass by the given point in a time interval t .
 - What is the current? $I = Nq/t$
 - Let t be the time for a charge q to travel a distance ℓ in a magnetic field $\mathbf{B}$
 - Then, the displacement vector ℓ becomes $\vec{\ell} = \vec{v}t$
 - Where $\mathbf{v}$ is the velocity of the incident particle with charge q
- Thus the force on N particles by the field is $\vec{F} = I\vec{\ell} \times \vec{B} = Nq\vec{v} \times \vec{B}$
- The force on one particle with charge q , $\vec{F} = q\vec{v} \times \vec{B}$



Magnetic Forces on a Moving Charge

- This can be an alternative way of defining the magnetic field.

- How?

- The magnitude of the force on a particle with charge q moving with a velocity v in the field is

- $F = qvB \sin \theta$

- What is θ ?

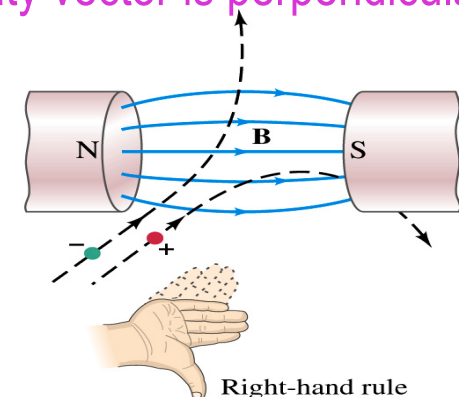
- The angle between the magnetic field and the direction of particle's movement

- When is the force maximum?

- When the angle between the field and the velocity vector is perpendicular.

- $F_{\max} = qvB \rightarrow B = \frac{F_{\max}}{qv}$

- The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field



Example 20 – 4

Magnetic force on a proton. A proton having a speed of $5 \times 10^6 \text{ m/s}$ in a magnetic field feels a force of $F = 8.0 \times 10^{-14} \text{ N}$ toward the west when it moves vertically upward. When moving horizontally in a northerly direction, it feels zero force. What is the magnitude and direction of the magnetic field in this region?

What is the charge of a proton? $q_p = +e = 1.6 \times 10^{-19} \text{ C}$

What does the fact that the proton does not feel any force in a northerly direction tell you about the magnetic field?

The field is along the north-south direction. Why?

Because the particle does not feel any magnetic force when it is moving along the direction of the field.

Since the particle feels force toward the west, the field should be pointing to North

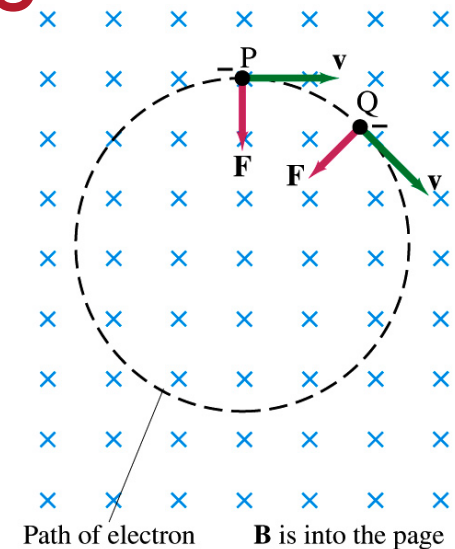
Using the formula for the magnitude of the field B , we obtain

$$B = \frac{F}{qv} = \frac{8.0 \times 10^{-14} \text{ N}}{1.6 \times 10^{-19} \text{ C} \cdot 5.0 \times 10^6 \text{ m/s}} = 0.10 \text{ T}$$

We can use magnetic field to measure the momentum of a particle. How?

Charged Particle's Path in Magnetic Field

- What shape do you think is the path of a charged particle on a plane perpendicular to a uniform magnetic field?
 - Circle!! Why?
 - An electron moving to right at the point P in the figure will be pulled downward
 - At a later time, the force is still perpendicular to the velocity
 - Since the force is always perpendicular to the velocity, the magnitude of the velocity is constant
 - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field
 - Thus, the electron moves on a circular path with a centripetal force F .



Example 20 – 5

Electron's path in a uniform magnetic field. An electron travels at a speed of $2.0 \times 10^7 \text{ m/s}$ in a plane perpendicular to a 0.010-T magnetic field. Describe its path.

What is formula for the centripetal force? $F = ma = m \frac{v^2}{r}$

Since the magnetic field is perpendicular to the motion of the electron, the magnitude of the magnetic force is

$$F = evB$$

Since the magnetic force provides the centripetal force, we can establish an equation with the two forces

$$F = evB = m \frac{v^2}{r}$$

 $r = \frac{mv}{eB} = \frac{(9.1 \times 10^{-31} \text{ kg}) \cdot (2.0 \times 10^7 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C}) \cdot (0.010 \text{ T})} = 1.1 \times 10^{-2} \text{ m}$



Cyclotron Frequency

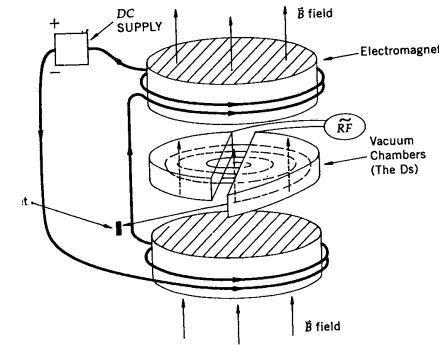
- The time required for a particle of charge q moving w/ constant speed v to make one circular revolution in a uniform magnetic field, $\vec{B} \perp \vec{v}$, is

$$T = \frac{2\pi r}{v} = \frac{2\pi m v}{v q B} = \frac{2\pi m}{q B}$$

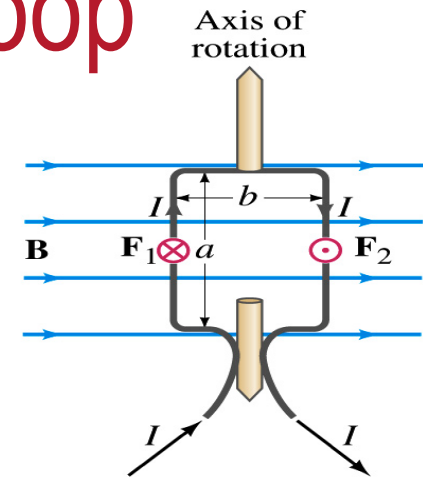
- Since T is the period of rotation, the frequency of the rotation is

$$f = \frac{1}{T} = \frac{q B}{2\pi m}$$

- This is the cyclotron frequency, the frequency of a particle with charge q in a cyclotron accelerator
 - While r depends on v , the frequency is independent of v and r .



Torque on a Current Loop



- What do you think will happen to a closed rectangular loop of wire with electric current as shown in the figure?
 - It will rotate! Why?
 - The magnetic field exerts a force on both vertical sections of wire.
 - Where is this principle used in?
 - Ammeters, motors, volt-meters, speedometers, etc
- The two forces on the different sections of the wire exerts net torque to the same direction about the rotational axis along the symmetry axis of the wire.
- What happens when the wire turns 90 degrees?
 - It will not turn unless the direction of the current changes

Torque on a Current Loop

- So what would be the magnitude of this torque?

- What is the magnitude of the force on the section of the wire with length a ?

- $F_a = IaB$
- The moment arm of the coil is $b/2$

- So the total torque is the sum of the torques by each of the forces

$$\tau = IaB \frac{b}{2} + IaB \frac{b}{2} = IabB = IAB$$

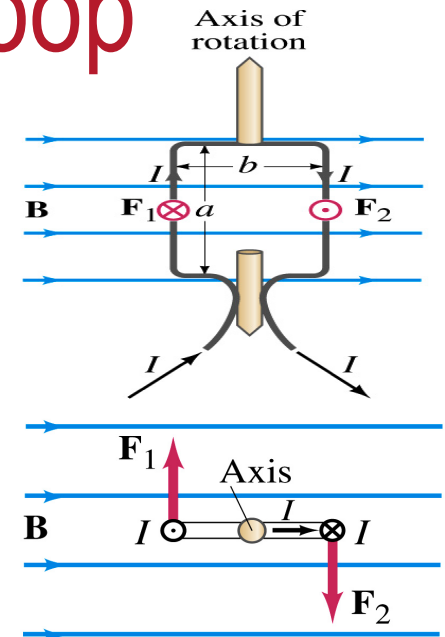
- Where $A = ab$ is the area of the coil

- What is the total net torque if the coil consists of N loops of wire?

$$\tau = NIAB$$

- If the coil makes an angle θ w/ the field

$$\tau = NIAB \sin \theta$$



Sources of Magnetic Field

- We have learned so far about the effects of magnetic field on electric currents and moving charge
- We will now learn about the dynamics of magnetism
 - How do we determine magnetic field strengths in certain situations?
 - How do two wires with electric current interact?
 - What is the general approach to finding the connection between current and magnetic field?



Magnetic Field due to a Straight Wire

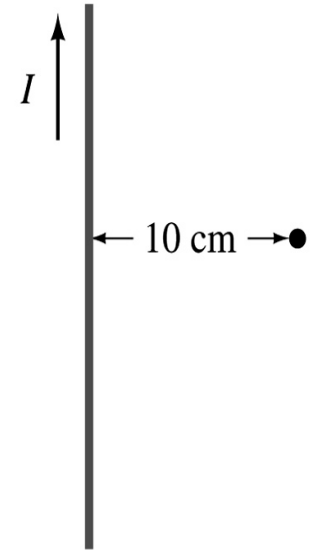
- The magnetic field due to the current flowing through a straight wire forms a circular pattern around the wire
 - What do you imagine the strength of the field is as a function of the distance from the wire?
 - It must be weaker as the distance increases
 - How about as a function of the current?
 - Directly proportional to the current
 - Indeed, the above are experimentally verified $B \propto \frac{I}{r}$
 - This is valid as long as $r \ll$ the length of the wire
 - The proportionality constant is $\mu_0/2\pi$, thus the field strength becomes
 - μ_0 is the permeability of free space $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A}$
 - Permeability: the degree in which magnetizable material modifies the magnetic flux in the region occupied by it in a magnetic field

$$B = \frac{\mu_0 I}{2\pi r}$$



Example 20 – 7

Calculation of B near wire. A vertical electric wire in the wall of a building carries a DC current of 25A upward. What is the magnetic field at a point 10cm due north of this wire?



Using the formula for the magnetic field near a straight wire

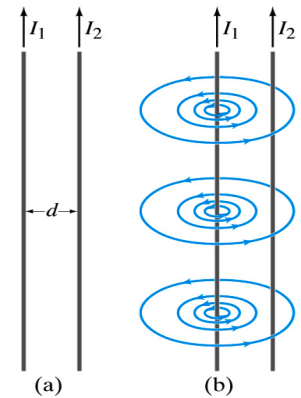
$$B = \frac{\mu_0 I}{2\pi r}$$

So we can obtain the magnetic field at 10cm away as

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) \cdot (25 \text{ A})}{(2\pi) \cdot (0.01 \text{ m})} = 5.0 \times 10^{-5} \text{ T}$$

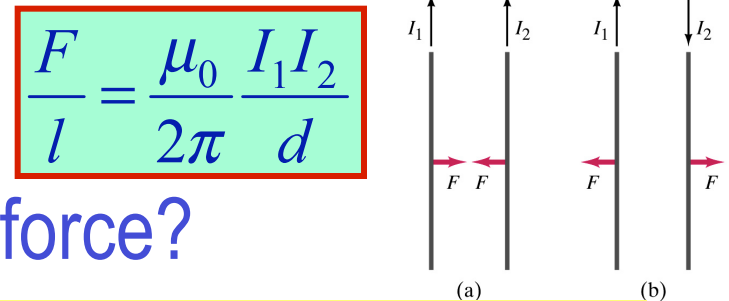
Force Between Two Parallel Wires

- We have learned that a wire carrying the current produces magnetic field
- Now what do you think will happen if we place two current carrying wires next to each other?
 - They will exert force onto each other. Repel or attract?
 - Depending on the direction of the currents
- This was first pointed out by Ampère.
- Let's consider two long parallel conductors separated by a distance d , carrying currents I_1 and I_2 .
- At the location of the second conductor, the magnitude of the magnetic field produced by I_1 is
$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$



Force Between Two Parallel Wires

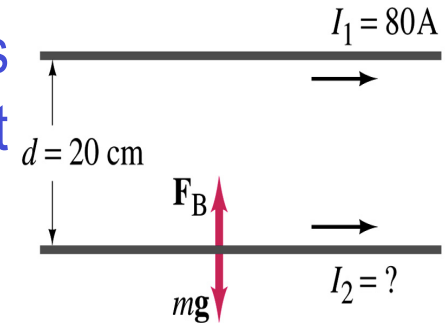
- The force F by a magnetic field B_1 on a wire of length l , carrying the current I_2 when the field and the current are perpendicular to each other is: $F = I_2 B_1 l$
 - So the force per unit length is $\frac{F}{l} = I_2 B_1 = I_2 \frac{\mu_0}{2\pi} \frac{I_1}{d}$
 - This force is only due to the magnetic field generated by the wire carrying the current I_1
 - There is the force exerted on the wire carrying the current I_1 by the wire carrying current I_2 of the same magnitude but in the opposite direction
- So the force per unit length is $\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}$
- How about the direction of the force?



If the currents are in the same direction, the attractive force. If opposite, repulsive.

Example 20 – 11

Suspending a wire with current. A horizontal wire carries a current $I_1=80\text{A}$ DC. A second parallel wire 20cm below it must carry how much current I_2 so that it doesn't fall due to the gravity? The lower has a mass of 0.12g per meter of length.



Which direction is the gravitational force? **Downward**

This force must be balanced by the magnetic force exerted on the wire by the first wire.

$$\frac{F_g}{l} = \frac{mg}{l} = \frac{F_M}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}$$

Solving for I_2

$$I_2 = \frac{mg}{l} \frac{2\pi d}{\mu_0 I_1} =$$

$$\frac{2\pi (9.8 \text{ m/s}^2) \cdot (0.12 \times 10^{-3} \text{ kg}) \cdot (0.20 \text{ m})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) \cdot (80 \text{ A})} = 15 \text{ A}$$

Operational Definition of Ampere and Coulomb

- The permeability of free space is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$$

- The unit of current, ampere, is defined using the definition of the force between two wires each carrying 1A of current and separated by 1m

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{2\pi} \frac{1\text{A} \cdot 1\text{A}}{1\text{m}} = 2 \times 10^{-7} \text{ N/m}$$

- So 1A is defined as: the current flowing each of two long parallel conductors 1m apart, which results in a force of exactly $2 \times 10^{-7} \text{ N/m}$.
- Coulomb is then defined as exactly $1\text{C} = 1\text{A} \cdot \text{s}$.
- We do it this way since current is measured more accurately and controlled more easily than charge.

