PHYS 1442 – Section 001 Lecture #11

Wednesday, July 15, 2009

Dr. Jaehoon Yu

- Chapter 20
  - Ampére's Law and Its Verification
  - Solenoid and Toroidal Magnetic Field
- Chapter 21
  - Induced EMF and Electromagnetic Induction
  - Faraday's Law of Induction
  - Magnetic Flux
  - Lenz's Law
  - Electric Generators

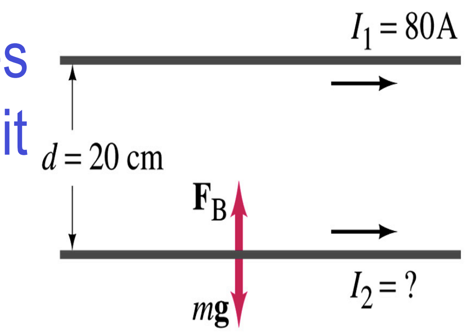
Today's homework is #6, due 9pm, Thursday, July 23!!

Announcements

- Second non-comprehensive exam
  - Date and time: 6 – 7:30pm, Monday, July 27
  - Covers from CH19 – CH21
  - There will be a help session Wednesday, July 22, bottom half of the class
- Reading assignments
  - CH 21.6, CH21.11 – 21.14
- Mid-term grade discussion
  - Bottom half of today's class

Example 20 – 11

Suspending a wire with current. A horizontal wire carries a current I_1=80A DC. A second parallel wire 20cm below it must carry how much current I_2 so that it doesn't fall due to the gravity? The lower has a mass of 0.12g per meter of length.



Which direction is the gravitational force? Downward

This force must be balanced by the magnetic force exerted on the wire by the first wire.

$$\frac{F_g}{l} = \frac{mg}{l} = \frac{F_M}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

Solving for I_2

$$I_2 = \frac{mg}{l}\frac{2\pi d}{\mu_0 I_1} =$$

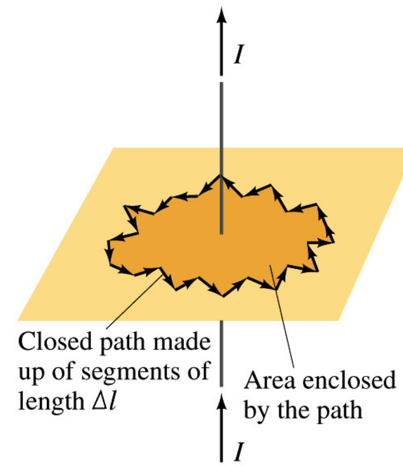
$$\frac{2\pi\left(9.8\,m/s^2\right)\cdot\left(0.12\times10^{-3}\,kg\right)\cdot\left(0.20m\right)}{\left(4\pi\times10^{-7}\,T\cdot m/A\right)\cdot\left(80A\right)} = 15A$$

Ampére's Law

- What is the relationship between magnetic field strength and the current? $B = \frac{\mu_0 I}{2\pi r}$
  - Does this work in all cases?
    - Nope!
    - OK, then when?
    - Only valid for a long straight wire
- Then what would be the more generalized relationship between the current and the magnetic field for any shape of the wire?
  - French scientist André Marie Ampére proposed such a relationship soon after Oersted's discovery

Ampére's Law

- Let's consider an arbitrary closed path around the current as shown in the figure.
  - Let's split this path with small segments each of Δl long.
  - The sum of all the products of the length of each segment and the component of B parallel to that segment is equal to μ_0 times the net current I_{encl} that passes through the surface enclosed by the path
  - $$\sum B_{\parallel} \Delta l = \mu_0 I_{encl}$$
  - In the limit $\Delta l \rightarrow 0$, this relation becomes
  - $$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

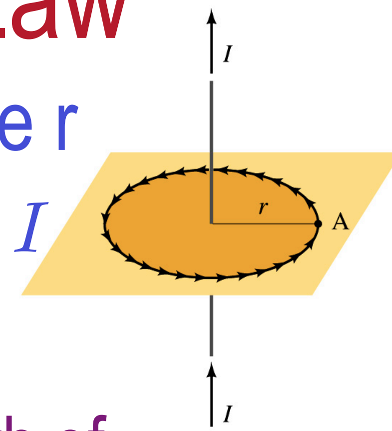


Ampére's Law

Looks very similar to a law in the electricity. Which law is it?

Gauss' Law

Verification of Ampére's Law



- Let's find the magnitude of B at a distance r away from a long straight wire w/ current I
  - This is a verification of Ampere's Law
  - We can apply Ampere's law to a circular path of radius r.

$$\mu_0 I_{encl} = 2\pi r B$$

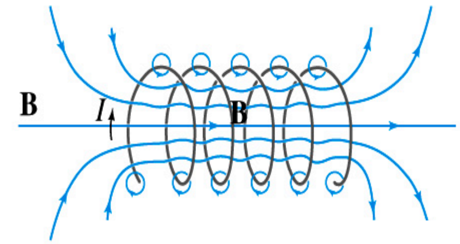
Solving for $\mathcal{B}$

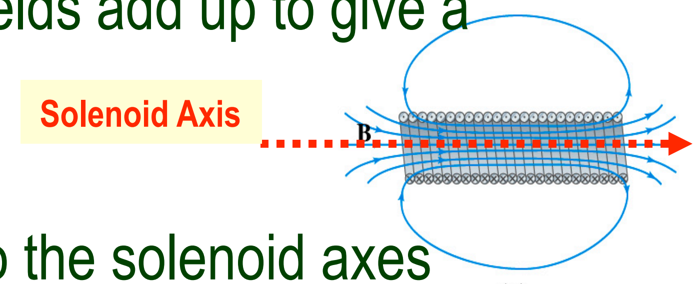
$$B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0}{2\pi}\frac{I}{r}$$

  - We just verified that Ampere's law works in a simple case
  - Experiments verified that it works for other cases too
  - The importance, however, is that it provides means to relate magnetic field to current

Solenoid and Its Magnetic Field

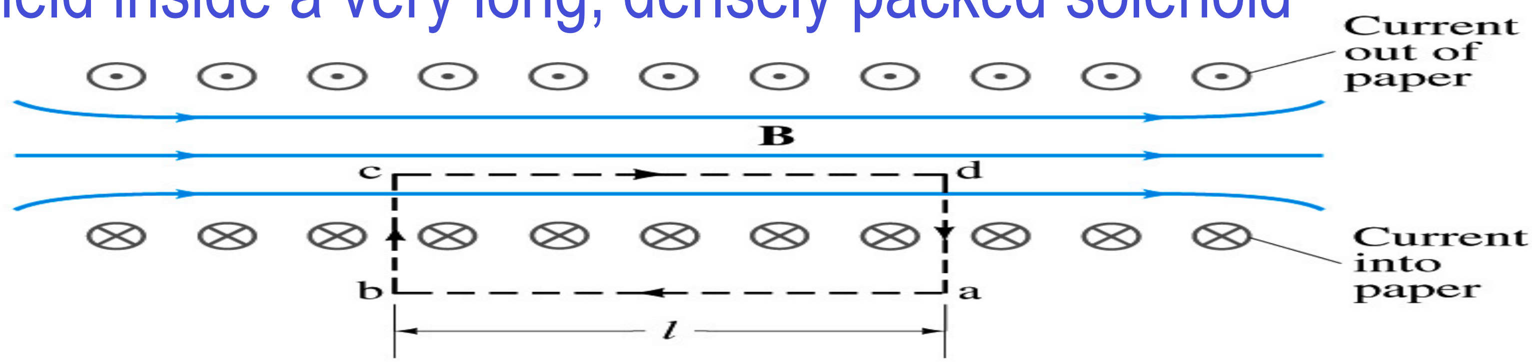
- What is a solenoid?
  - A long coil of wire consisting of many loops
  - If the space between loops are wide
    - The field near the wires are nearly circular
    - Between any two wires, the fields due to each loop cancel
    - Toward the center of the solenoid, the fields add up to give a field that can be fairly large and uniform
  - For a long, densely packed loops
    - The field is nearly uniform and parallel to the solenoid axes within the entire cross section
    - The field outside the solenoid is very small compared to the field inside, except the ends
      - The same number of field lines spread out to an open space





Solenoid Magnetic Field

- Now let's use Ampere's law to determine the magnetic field inside a very long, densely packed solenoid



- Let's choose the path *abcd*, far away from the ends
  - We can consider four segments of the loop for integral
  - $\oint \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} \quad \int_d^a \vec{B}\cdot d\vec{l}$
  - The field outside the solenoid is negligible. So the integral on $a \rightarrow b$ is 0.
  - Now the field B is perpendicular to the *bc* and *da* segments. So these integrals become 0, also.

Solenoid Magnetic Field

– So the sum becomes: $\oint \vec{B} \cdot d\vec{l} = \int_c^d \vec{B} \cdot d\vec{l} = Bl$

– If the current I flows in the wire of the solenoid, the total current enclosed by the closed path is $\mathcal{N}I$

  - Where $\mathcal{N}$ is the number of loops (or turns of the coil) enclosed

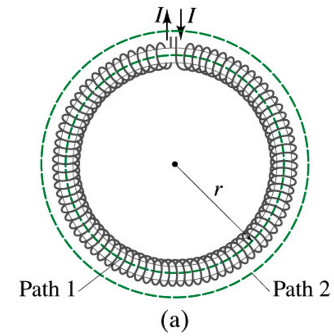
– Thus Ampere's law gives us $Bl = \mu_0 NI$

– If we let $n=\mathcal{N}/l$ be the number of loops per unit length, the magnitude of the magnetic field within the solenoid becomes

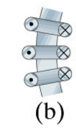
– $$B = \mu_0 nI$$

  - B depends on the number of loops per unit length, n, and the current I
    – Does not depend on the position within the solenoid but uniform inside it, like a bar magnet

Example Toroid



Toroid. Use Ampere's law to determine the magnetic field (a) inside and (b) outside a toroid, which is like a solenoid bent into the shape of a circle.



(a) How do you think the magnetic field lines inside the toroid look?

Since it is a bent solenoid, it should be a circle concentric with the toroid.

If we choose path of integration one of these field lines of radius r inside the toroid, path 1, to use the symmetry of the situation, making B the same at all points on the path, we obtain from Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r) = \mu_0 I_{encl} = \mu_0 NI$$

Solving for B

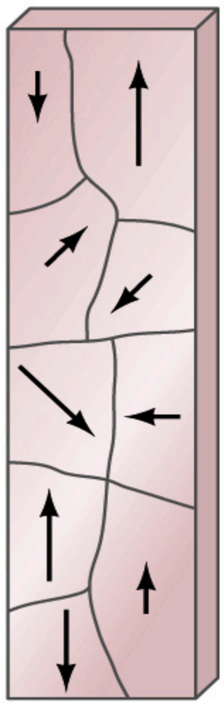
$$B = \frac{\mu_0 NI}{2\pi r}$$

So the magnetic field inside a toroid is not uniform. It is larger on the inner edge. However, the field will be uniform if the radius is large and the toroid is thin and $B = \mu_0 n I$.

(b) Outside the solenoid, the field is 0 since the net enclosed current is 0.

Magnetic Materials - Ferromagnetism

- Iron is a material that can turn into a strong magnet
  - This kind of material is called **ferromagnetic** material
- In a microscopic sense, ferromagnetic materials consists of many tiny regions called **domains**
  - Domains are like little magnets usually smaller than 1mm in length or width
- What do you think the alignment of domains are like when they are not magnetized?
  - Randomly arranged
- What if they are magnetized?
  - The size of the domains aligned with the external magnetic field direction grows while those of the domains not aligned reduce
  - This gives magnetization to the material
- How do we demagnetize a bar magnet?
  - Hit the magnet hard or heat it over the Curie temperature



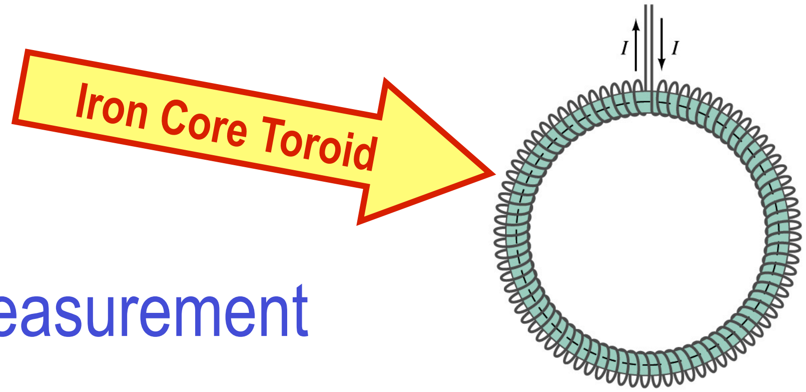
B in Magnetic Materials

- What is the magnetic field inside a solenoid?
- $$B_0 = \mu_0 n I$$
  - Magnetic field in a long solenoid is directly proportional to the current.
  - This is valid only if air is inside the coil
- What do you think will happen to B if we have something other than the air inside the solenoid?
  - It will be increased dramatically, when the current flows
    - Especially if a ferromagnetic material such as an iron is put inside, the field could increase by several orders of magnitude
- Why?
  - Since the domains in the iron aligns permanently by the external field.
  - The resulting magnetic field is the sum of that due to current and due to the iron

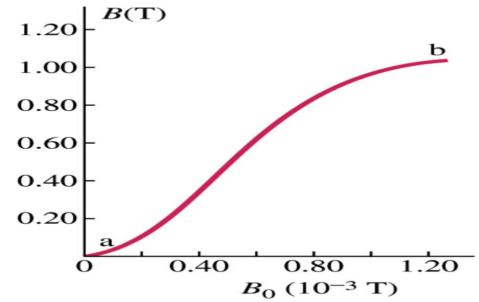
B in Magnetic Materials

- It is sometimes convenient to write the total field as the sum of two terms
- $$\vec{B} = \vec{B}_0 + \vec{B}_M$$
  - **B_0** is the field due only to the current in the wire, namely the external field
    - The field that would be present without a ferromagnetic material
  - **B_M** is the additional field due to the ferromagnetic material itself; often **B_M**>>**B_0**
- The total field in this case can be written by replacing μ_0 with another proportionality constant μ, the magnetic permeability of the material $B = \mu n I$
  - μ is a property of a magnetic material
  - μ is not a constant but varies with the external field

Hysteresis

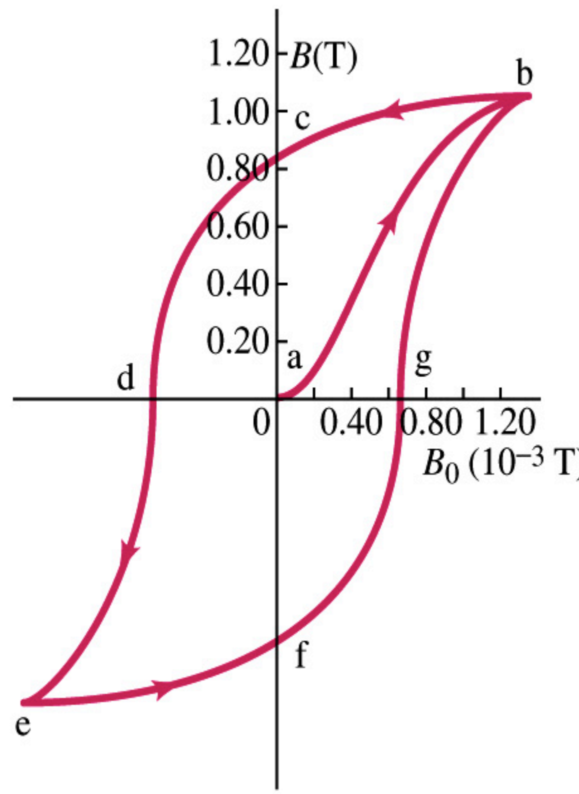


- What is a toroid?
  - A solenoid bent into a shape
- Toroid is used for magnetic field measurement
  - Why?
  - Since it does not leak magnetic field outside of itself, it fully contains all the magnetic field created within it.
- Consider an un-magnetized iron core toroid, without any current flowing in the wire
  - What do you think will happen if the current slowly increases?
  - B_0 increases linearly with the current.
  - And B increases also but follows the curved line shown in the graph
  - As B_0 increases, the domains become more aligned until nearly all are aligned (point b on the graph)
    - The iron is said to be approaching saturation
    - Point b is typically at 70% of the max



Hysteresis

- What do you think will happen to B if the external field B_0 is reduced to 0 by decreasing the current in the coil?
  - ~~Of course it goes to 0!!~~
  - Wrong! Wrong! Wrong! They do not go to 0. Why not?
  - The domains do not completely return to random alignment state
- Now if the current direction is reversed, the external magnetic field direction is reversed, causing the total field B pass 0, and the direction reverses to the opposite side
  - If the current is reversed again, the total field B will increase but never goes through the origin
- This kind of curve whose path does not retrace themselves and does not go through the origin is called the **Hysteresis**.



Magnetically Soft Material

- In a hysteresis cycle, much energy is transformed to thermal energy. Why?
  - Due to the microscopic friction between domains as they change directions to align with the external field
- The energy dissipated in the hysteresis cycle is proportional to the area of the hysteresis loop
- Ferromagnetic material with large hysteresis area is called magnetically hard while the small ones are called soft
  - Which ones do you think are preferred in electromagnets or transformers?
    - Soft. Why?
    - Since the energy loss is small and much easier to switch off the field
- Then how do we demagnetize a ferromagnetic material?
  - Keep repeating the Hysteresis loop, reducing the range of B_0.