PHYS 1442 – Section 001
Lecture #12

Monday & Wednesday, July 20 and 22, 2009

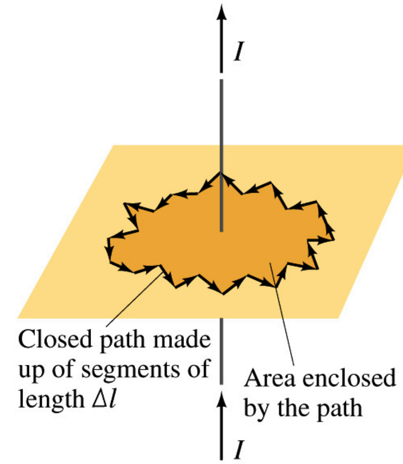
Dr. Jaehoon Yu

- Chapter 21
  - Ampere's Law recap
  - EMF Induction
  - Faraday's Law
  - Lentz' Law
  - EMF Induction in Moving Conductor
  - Generation of Electricity
  - Energy Stored in Magnetic Field
  - LR Circuits
  - Alternating Current

Ampére's Law

- What is the relationship between magnetic field strength and the current? $\boxed{B = \frac{\mu_0 I}{2\pi r}}$
  - Does this work in all cases?
    - Nope!
    - OK, then when?
    - Only valid for a long straight wire
- Then what would be the more generalized relationship between the current and the magnetic field for any shape of the wire?
  - French scientist André Marie Ampére proposed such a relationship soon after Oersted's discovery

Ampére's Law



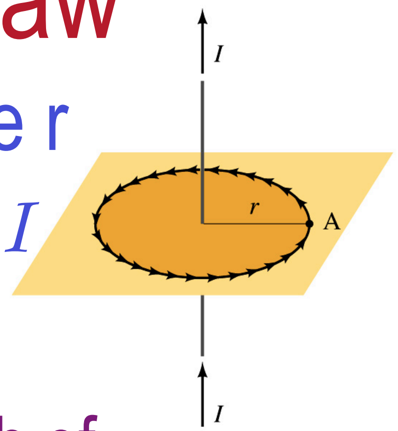
- Let's consider an arbitrary closed path around the current as shown in the figure.
  - Let's split this path with small segments each of Δl long.
  - The sum of all the products of the length of each segment and the component of B parallel to that segment is equal to μ_0 times the net current I_{encl} that passes through the surface enclosed by the path
  - $$\boxed{\sum B_{\|}\Delta l = \mu_0 I_{encl}}$$
  - In the limit $\Delta l \rightarrow 0$, this relation becomes
  - $$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}}$$

Ampére's Law

Looks very similar to a law in the electricity. Which law is it?

Gauss' Law

Verification of Ampére's Law



- Let's find the magnitude of B at a distance r away from a long straight wire w/ current I
  - This is a verification of Ampere's Law
  - We can apply Ampere's law to a circular path of radius r.

$$\mu_0 I_{encl} = 2\pi r B$$

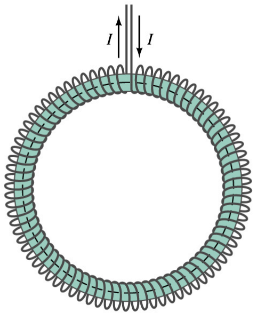
Solving for $\mathcal{B}$

$$B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0}{2\pi}\frac{I}{r}$$

  - We just verified that Ampere's law works in a simple case
  - Experiments verified that it works for other cases too
  - The importance, however, is that it provides means to relate magnetic field to current

Summary on Solenoid and Toroid

- The magnitude of the solenoid magnetic field without any material inside of the loop $$B = \mu_0 n I$$
  - n is the number of loops per unit length
  - I is the current going through the loop
- If the loop has some material inside of it: $$B = \mu n I$$
- The magnitude of the Toroid magnetic field with radius r:



$$B = \frac{\mu_0 N I}{2\pi r}$$

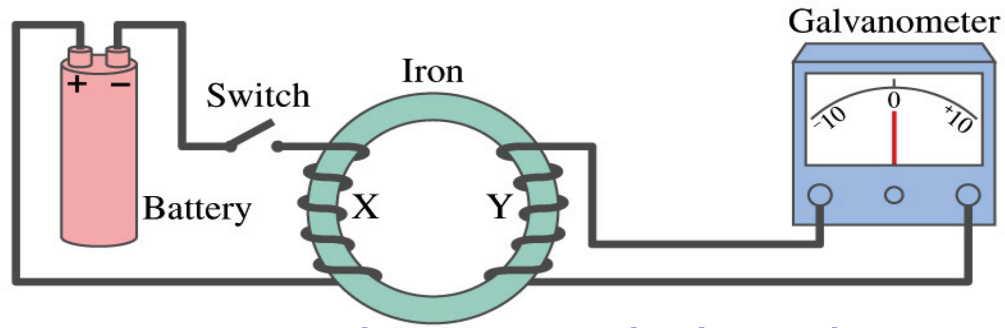
$$B = \frac{\mu N I}{2\pi r}$$

Induced EMF

- It has been discovered by Oersted and company in early 19th century that
  - Magnetic field can be produced by the electric current
  - Magnetic field can exert force on electric charge
- So if you were scientists at that time, what would you wonder?
  - Yes, you are absolutely right. You would wonder if the magnetic field can create the electric current.
  - An American scientist Joseph Henry and an English scientist Michael Faraday independently found that it was possible
    - Though, Faraday was given the credit since he published his work before Henry did
      - He also did a lot of detailed studies on magnetic induction

Electromagnetic Induction

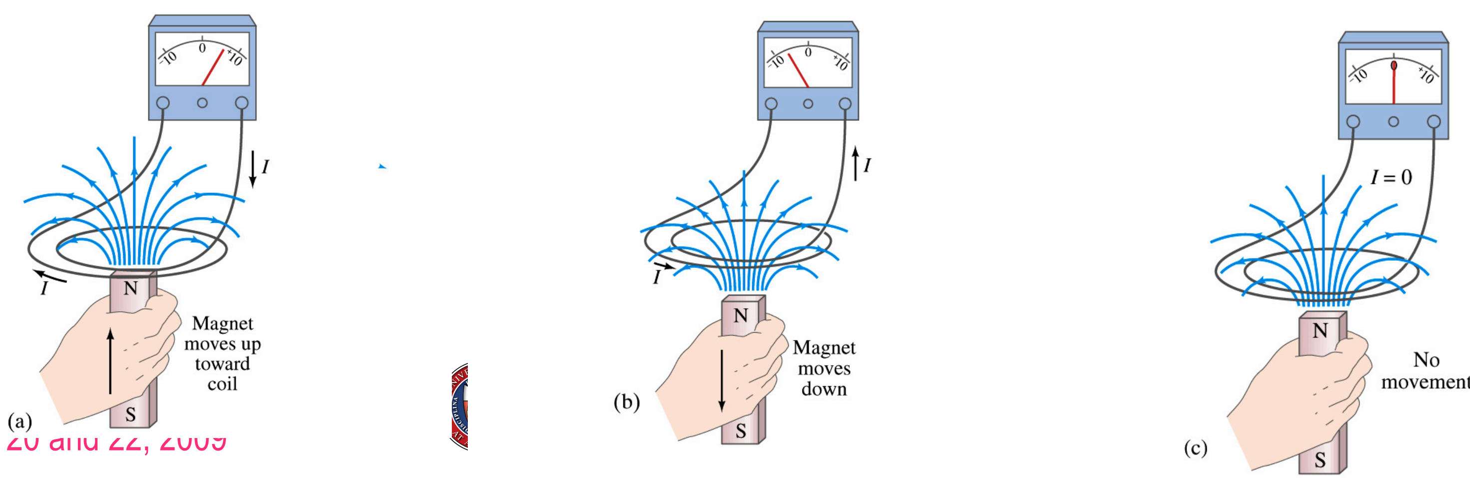
- Faraday used an apparatus below to show that magnetic field can induce current



- Despite his hope he did not see steady current induced on the other side when the switch is thrown
- But he did see that the needle on the Galvanometer turns strongly when the switch is initially thrown and is opened
  - When the magnetic field through coil Y changes, a current flows as if there were a source of emf
- Thus he concluded that **<u>an induced emf is produced by a changing magnetic field</u>** ➔ **<u>Electromagnetic Induction</u>**

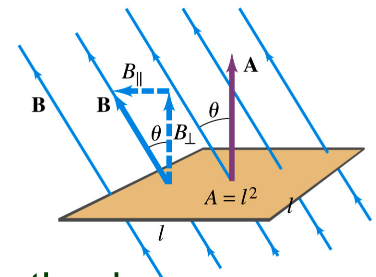
Electromagnetic Induction

- Further studies on electromagnetic induction taught
  - If magnet is moved quickly into a coil of wire, a current is induced in the wire.
  - If the magnet is removed from the coil, a current is induced in the wire in the opposite direction
  - By the same token, current can also be induced if the magnet stays put but the coil moves toward or away from the magnet
  - Current is also induced if the coil rotates.
- In other words, it does not matter whether the magnet or the coil moves. It is the relative motion that counts.



Magnetic Flux

- So what do you think is the induced emf proportional to?
  - The rate of changes of the magnetic field?
    - the higher the changes the higher the induction
  - Not really, it rather depends on the rate of change of the **magnetic flux**, F_B.
  - Magnetic flux is defined as (just like the electric flux)
  - $$\boxed{\Phi_B = B_\perp A = BA\cos\theta = \vec{B}\cdot\vec{A}}$$
    - q is the angle between **B** and the area vector **A** whose direction is perpendicular to the face of the loop based on the right-hand rule
  - What kind of quantity is the magnetic flux?
    - Scalar. Unit?
    - $T \cdot m^2$ or weber $\boxed{1Wb = 1T \cdot m^2}$



- If the area of the loop is not simple or B is not uniform, the magnetic flux can be written as $\boxed{\Phi_B = \sum \vec{B}\cdot\Delta\vec{A}}$

Faraday's Law of Induction

- In terms of magnetic flux, we can formulate Faraday's findings
  - The emf induced in a circuit is equal to the rate of change of magnetic flux through the circuit

$$\boxed{\varepsilon = -\frac{\Delta \Phi_B}{\Delta t}}$$

Faraday's Law of Induction

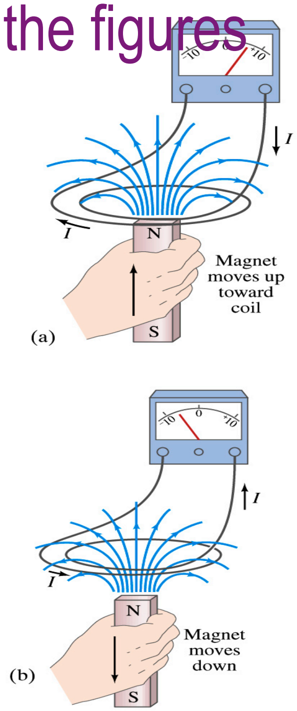
- If the circuit contains N closely wrapped loops, the total induced emf is the sum of emf induced in each loop

$$\boxed{\varepsilon = -N\frac{\Delta \Phi_B}{\Delta t}}$$

  - Why negative?
    - Has got a lot to do with the direction of induced emf…

Lenz's Law

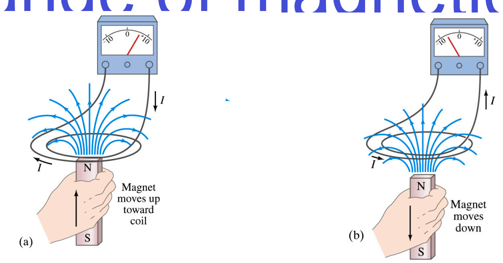
- It is experimentally found that
  - An induced emf gives rise to a current whose magnetic field opposes the original change in flux ➔ This is known as **<u>Lenz's Law</u>**
  - In other words, an induced emf is always in a direction that opposes the original change in flux that caused it.
  - We can use Lenz's law to explain the following cases in the figures
    - When the magnet is moving into the coil
      - Since the external flux increases, the field inside the coil takes the opposite direction to minimize the change and causes the current to flow clockwise
    - When the magnet is moving out
      - Since the external flux decreases, the field inside the coil takes the opposite direction to compensate the loss, causing the current to flow counter-clockwise



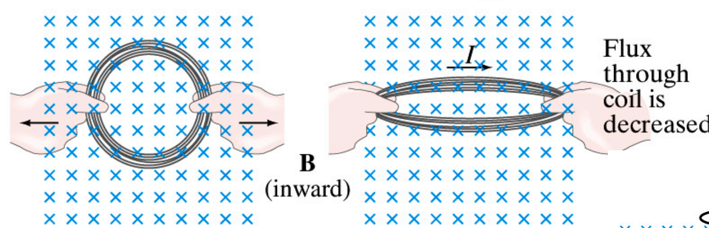
- Which law is Lenz's law result of?
  - Energy conservation. Why?

Induction of EMF

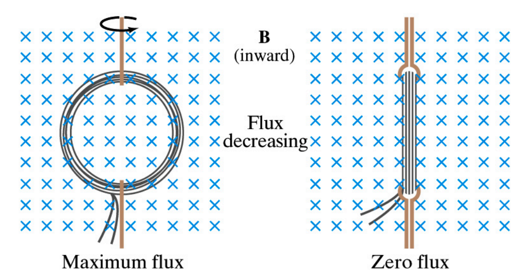
- How can we induce emf?
- Let's look at the formula for magnetic flux
- $$\Phi_B = \sum \vec{B} \cdot \Delta\vec{A} = \sum B\cos\theta \Delta A$$
- What do you see? What are the things that can change with time to result in change of magnetic flux?
  - Magnetic field



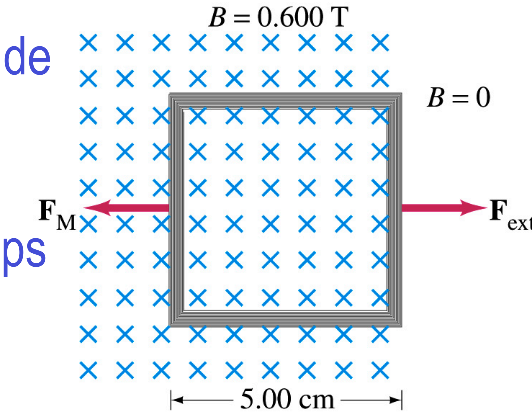
  - The area of the loop



  - The angle θ between the field and the area vector



Example 21 – 5



Pulling a coil from a magnetic field. A square coil of wire with side 5.00cm contains 100 loops and is positioned perpendicular to a uniform 0.600-T magnetic field. It is quickly and uniformly pulled from the field (moving perpendicular to B) to a region where B drops abruptly to zero. At t=0, the right edge of the coil is at the edge of the field. It takes 0.100s for the whole coil to reach the field-free region. Find (a) the rate of change in flux through the coil, (b) the emf and current induced, and (c) how much energy is dissipated in the coil if its resistance is 100W. (d) what was the average force required?

What should be computed first? The initial flux at t=0.

The flux at t=0 is $\Phi_B = \vec{B}\cdot\vec{A} = BA = 0.600T\cdot\left(5\times10^{-2}\,m\right)^2 = 1.50\times10^{-3}\,Wb$

The change of flux is $\Delta\Phi_B = 0 - 1.50\times10^{-3}\,Wb = -1.50\times10^{-3}\,Wb$

Thus the rate of change of the flux is

$$\frac{\Delta\Phi_B}{\Delta t} = \frac{-1.50\times10^{-3}\,Wb}{0.100s} = -1.50\times10^{-2}\,Wb/s$$

Example 21 – 5, cnt'd

Thus the total emf induced in this period is

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -100\cdot\left(-1.50\times10^{-2}\,Wb/s\right) = 1.5V$$

The induced current in this period is

$$I = \frac{\varepsilon}{R} = \frac{1.5V}{100\Omega} = 1.50\times10^{-2}\,A = 15.0mA$$

Which direction would the induced current flow? Clockwise

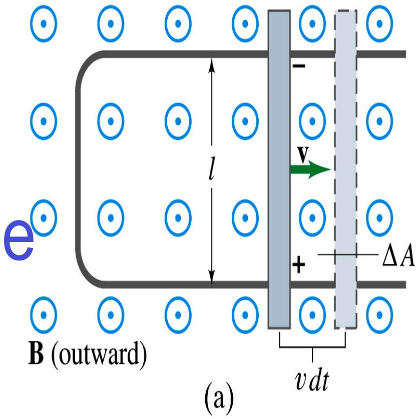
The total energy dissipated is

$$E = Pt = I^2Rt = \left(1.50\times10^{-2}\,A\right)^2\cdot100\Omega\cdot0.100s = 2.25\times10^{-3}\,J$$

Force for each coil is $\vec{F} = I\vec{l}\times\vec{B}$ Force for N coil is $\vec{F} = NI\vec{l}\times\vec{B}$

$$\left|F\right| = NIlB = 100\cdot\left(1.50\times10^{-2}\,A\right)\cdot\left(4\times5\times10^{-2}\right)\cdot0.600T = 0.045N$$

EMF Induced on a Moving Conductor



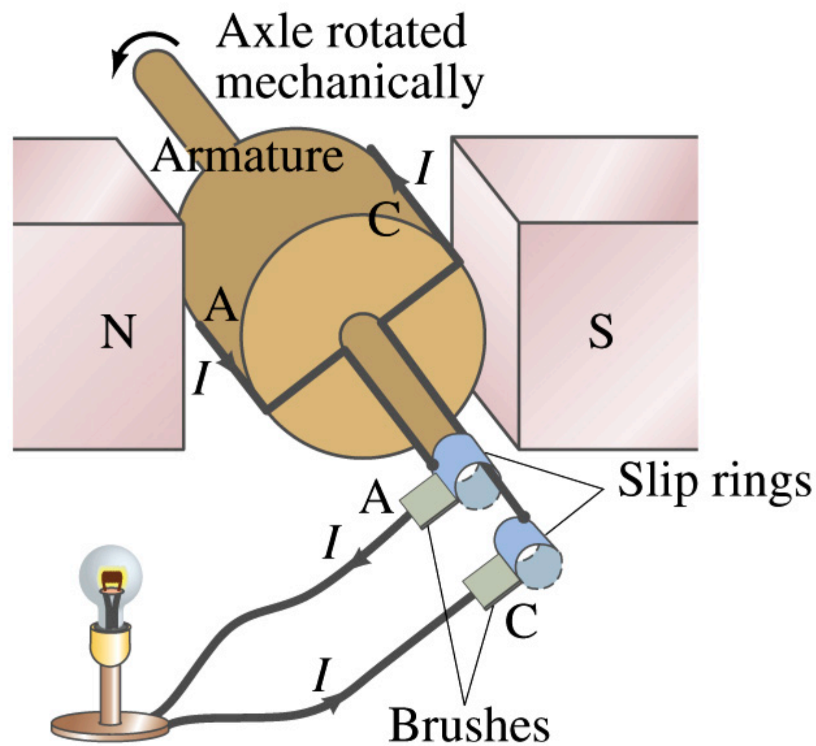
- Another way of inducing emf is using a U shaped conductor with a movable rod resting on it.
- As the rod moves at a speed v, it travels $v$dt in time dt, changing the area of the loop by ΔA=$lv\Delta$ t.
- Using Faraday's law, the induced emf for this loop is

$$\left|\varepsilon\right| = \frac{\Delta\Phi_B}{\Delta t} = \frac{B\Delta A}{\Delta t} = \frac{Blv\Delta t}{\Delta t} = Blv$$

  – This equation is valid as long as B, l and v are perpendicular to each other. What do we do if not?
    - Use the scalar product of vector quantities
- An emf induced on a conductor moving in a magnetic field is called a **motional emf**

Electric Generators

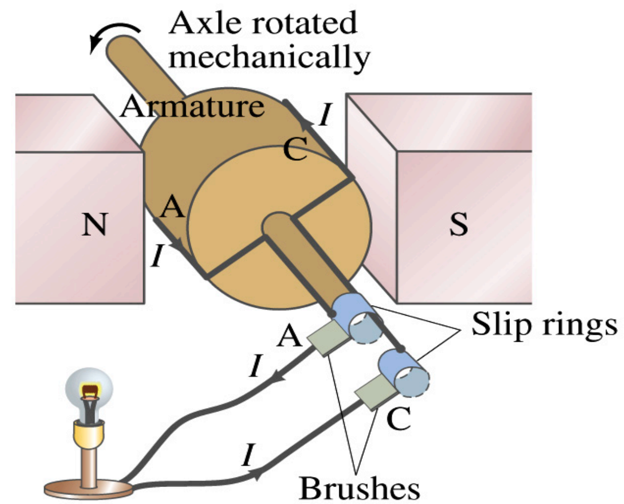
- What does a generator do?
  - Transforms mechanical energy into the electrical energy
  - What does this look like?
    - An inverse of an electric motor which transforms electrical energy to mechanical energy
  - An electric generator is also called a dynamo



- Whose law does the generator based on?
  - Faraday's law of induction

How does an Electric Generator work?

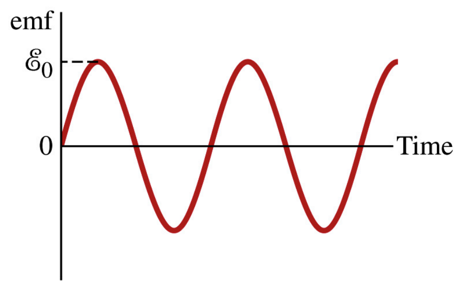
- An electric generator consists of
  - Many coils of wires wound on an armature that can rotate by mechanical means in a magnetic field
- An emf is induced in the rotating coil
- Electric current is the output of a generator



- Which direction does the output current flow when the armature rotates counterclockwise?
  - The conventional current flows outward on wire A toward the brush
  - After half the revolution the wire A will be where the wire C is and the current flow on A is reversed
- Thus the current produced is alternating its direction

How does an Electric Generator work?

- Let's assume the loop is rotating in a uniform B field w/ constant angular velocity w. The induced emf is
- $$\varepsilon = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{\Delta}{\Delta t}\left(\vec{B}\cdot\vec{A}\right) = -\frac{\Delta}{\Delta t}\left[BA\cos\theta\right]$$
- What is the variable that changes above?
  - The angle θ. What is Δθ/Δt?
    - The angular speed ω.
  - So $\theta=\theta_0+\omega t$
  - If we choose $\theta_0=0$, we obtain
  - $$\varepsilon = -BA\frac{\Delta}{\Delta t}\left[\cos\varpi t\right] = BA\varpi\sin\varpi t$$
  - If the coil contains N loops: $$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t} = NBA\varpi\sin\varpi t = \varepsilon_0\sin\varpi t$$
  - What is the shape of the output?
    - Sinusoidal w/ amplitude $\mathscr{E}_0$=NBAω
- US AC frequency is 60Hz. Europe is at 50Hz
  - Most the U.S. power is generated at steam plants



Example

An AC generator. The armature of a 60-Hz AC generator rotates in a 0.15-T magnetic field. If the area of the coil is $2.0\times10^{-2}m^2$, how many loops must the coil contain if the peak output is to be e_0=170V?

The maximum emf of a generator is $\varepsilon_0 = NBA\varpi$

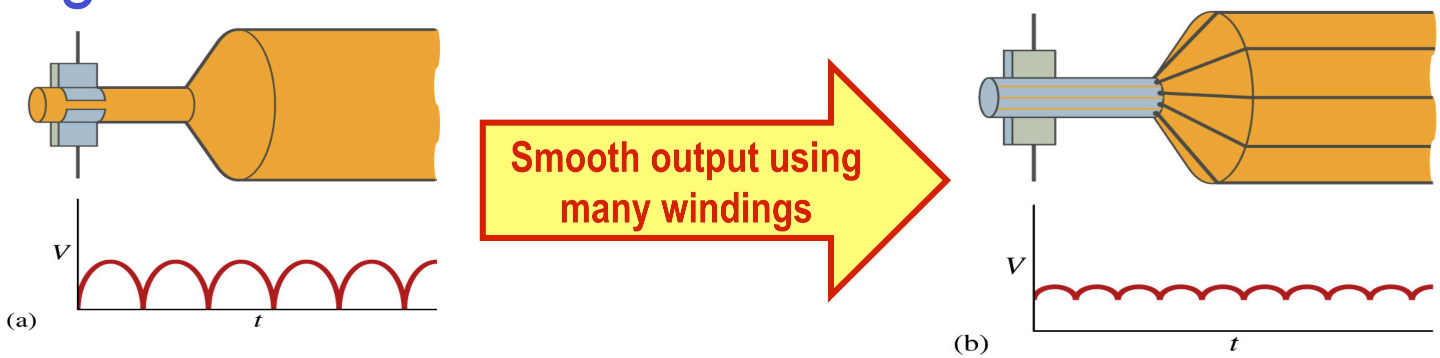
Solving for N $$N = \frac{\varepsilon_0}{BA\varpi}$$

Since $\varpi = 2\pi f$ We obtain

$$N = \frac{\varepsilon_0}{2\pi BAf} = \frac{170V}{2\pi\cdot(0.15T)\cdot\left(2.0\times10^{-2}m^2\right)\cdot\left(60s^{-1}\right)} = 150 turns$$

A DC Generator

- A DC generator is almost the same as an ac generator except the slip rings are replaced by split-ring commutators



- Output can be smoothed out by placing a capacitor on the output
  - More commonly done using many armature windings

Energy Stored in a Magnetic Field

- When an inductor of inductance $\mathcal{L}$ is carrying current I which is changing at a rate dI/dt, energy is supplied to the inductor at a rate
  - $P = I\varepsilon = IL\frac{\Delta I}{\Delta t}$
- What is the work needed to increase the current in an inductor from 0 to I?
  - The work, dW, done in time dt is $\Delta W = P\Delta t = LI\Delta I$
  - Thus the total work needed to bring the current from 0 to I in an inductor is

$$W = \sum \Delta W = \sum_I LI\Delta I = \frac{1}{2}LI^2$$

Energy Stored in a Magnetic Field

- The work done to the system is the same as the energy stored in the inductor when it is carrying current I
  - $\boxed{U = \frac{1}{2}LI^2}$

    Energy Stored in a magnetic field inside an inductor

  - This is compared to the energy stored in a capacitor, C, when the potential difference across it is V $U = \frac{1}{2}CV^2$
  - Just like the energy stored in a capacitor is considered to reside in the electric field between its plates
  - The energy in an inductor can be considered to be stored in its magnetic field

Stored Energy in terms of B

- So how is the stored energy written in terms of magnetic field B?
  - Inductance of an ideal solenoid without a fringe effect
    $$L = \mu_0 N^2 A/l$$
  - The magnetic field in a solenoid is $B = \mu_0 NI/l$
  - Thus the energy stored in an inductor is
    $$U = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{l}\left(\frac{Bl}{\mu_0 N}\right)^2 = \frac{1}{2}\frac{B^2}{\mu_0}Al$$
  - Thus the energy density is
    $$u = \frac{U}{V} = \frac{U}{Al} = \frac{1}{2}\frac{B^2}{\mu_0}$$
  - This formula is valid to any region of space
  - If a ferromagnetic material is present, m_0 becomes m.

What is this?

Volume V

$$\boxed{U = \frac{1}{2}\frac{B^2}{\mu_0}Al}$$

E

$$\boxed{u = \frac{1}{2}\frac{B^2}{\mu_0}}$$

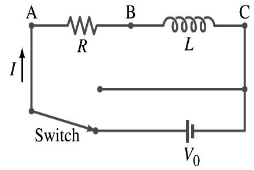
E density

What volume does Al represent?

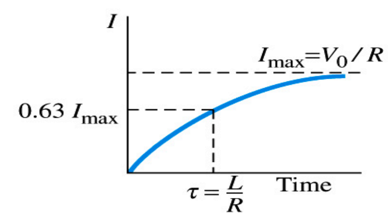
The volume inside a solenoid!!

LR Circuits

- What happens when an emf is applied to an inductor?
  - An inductor has some resistance, however negligible



    - So an inductor can be drawn as a circuit of separate resistance and coil. What is the name this kind of circuit? **LR Circuit**
  - What happens at the instance the switch is thrown to apply emf to the circuit?
    - The current starts to flow, gradually increasing from 0
    - This change is opposed by the induced emf in the inductor ➔ the emf at point B is higher than point C
    - However there is a voltage drop at the resistance which reduces the voltage across inductance
    - Thus the current increases less rapidly
    - The overall behavior of the current is gradual increase, reaching to the maximum current $I_{max}=V_0/R$.



Why do we care about circuits on AC?

- The circuits we've learned so far contain resistors, capacitors and inductors and have been connected to a DC source or a fully charged capacitor
  - What? This does not make sense.
  - The inductor does not work as an impedance unless the current is changing. So an inductor in a circuit with DC source does not make sense.
  - Well, actually it does. When does it impede?
    - Immediately after the circuit is connected to the source so the current is still changing. So?
      - It causes the change of magnetic flux.
  - Now does it make sense?
- Anyhow, learning the responses of resistors, capacitors and inductors in a circuit connected to an AC emf source is important. Why is this?
  - Since most the generators produce sinusoidal current
  - Any voltage that varies over time can be expressed in the superposition of sine and cosine functions

AC Circuits – the preamble

- Do you remember how the rms and peak values for current and voltage are related?

$$V_{rms} = \frac{V_0}{\sqrt{2}} \qquad I_{rms} = \frac{I_0}{\sqrt{2}}$$

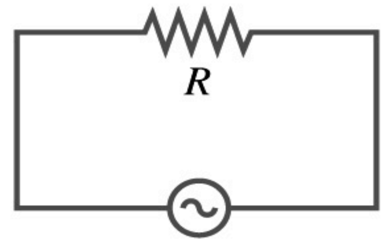
- The symbol for an AC power source is

- We assume that the voltage gives rise to current

$$I = I_0 \sin 2\pi f t = I_0 \sin \varpi t$$

  – where $\varpi = 2\pi f$

AC Circuit w/ Resistance only



- What do you think will happen when an ac source is connected to a resistor?
- From Kirchhoff's loop rule, we obtain

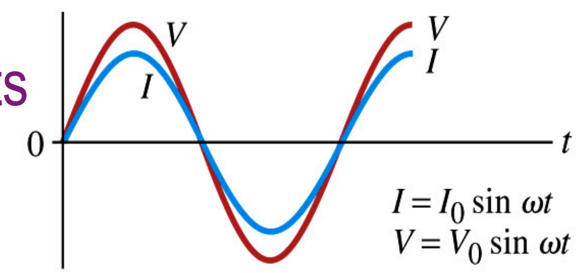
$$V - IR = 0$$

- Thus

$$V = I_0 R \sin \varpi t = V_0 \sin \varpi t$$

  – where $V_0 = I_0 R$

- What does this mean?
  – Current is 0 when voltage is 0 and current is in its peak when voltage is in its peak.
  – Current and voltage are "in phase"



- Energy is lost via the transformation into heat at an average rate $\overline{P} = \overline{I}\,\overline{V} = I_{rms}^2 R = V_{rms}^2 / R$
-