

PHYS 1442 – Section 001

Lecture #13

Wednesday, July 29, 2009

*Dr. **Jaehoon** **Yu***

- EMF induction recap
- Chapter 22
 - Gauss' Law of Magnetism
 - Maxwell's Equations
 - Production of Electromagnetic Waves
 - EM Waves from Maxwell's Equations
 - Speed of EM Waves

Today's homework is #7, due 9pm, Thursday, August 6!!

Wednesday, July 29, 2009



PHYS 1442-001, Summer 2009, Dr.
Jaehoon Yu

Announcements

- Final comprehensive exam
 - Date and time: 6 – 7:30pm, Wednesday, Aug. 12
 - Covers from CH16 – what we finish Wednesday, Aug. 5 + Appendix A.1 – A.8
 - There will be a help session Monday, Aug. 10
- Reading assignments
 - CH22.5 – 22.7
- Term Exam Results
 - Class Average: 70/100
 - Previous exam: 59/100
 - Top score: 98/100
- Quiz next Wednesday, Aug. 5
 - Beginning of the class
 - Covers CH 22.1 – what we learn next Monday, Aug. 3



Special Projects

1. Derive the unit of speed from the product of the permittivity and permeability, starting from their original units. (5 points)
 2. Derive and compute the speed of light in free space from the four Maxwell's equations. (20 points for derivation and 3 points for computation.)
 3. Compute the speed of the EM waves in copper, water and one other material which is different from other students. (3 points each)
- Due of these projects are the start of the class Monday, Aug. 10!



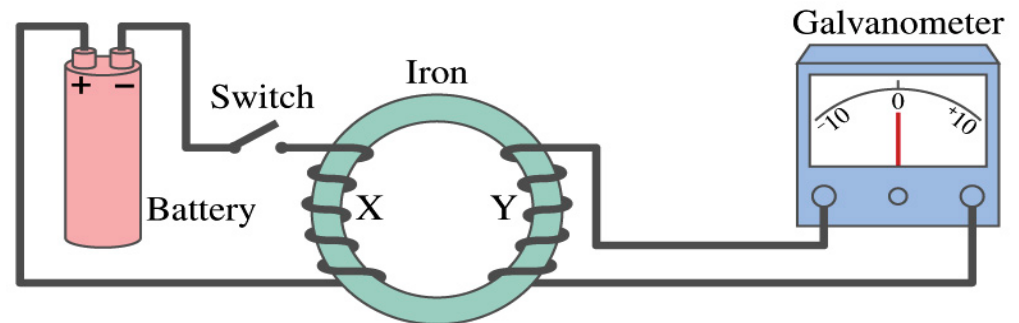
Induced EMF

- It has been discovered by Oersted and company in early 19th century that
 - Magnetic field can be produced by the electric current
 - Magnetic field can exert force on the electric charge
- So if you were scientists at that time, what would you wonder?
 - Yes, you are absolutely right. You would wonder if the magnetic field can create the electric current.
 - An American scientist Joseph Henry and an English scientist Michael Faraday independently found that it was possible
 - Faraday was given the credit since he published his work before Henry did



Electromagnetic Induction

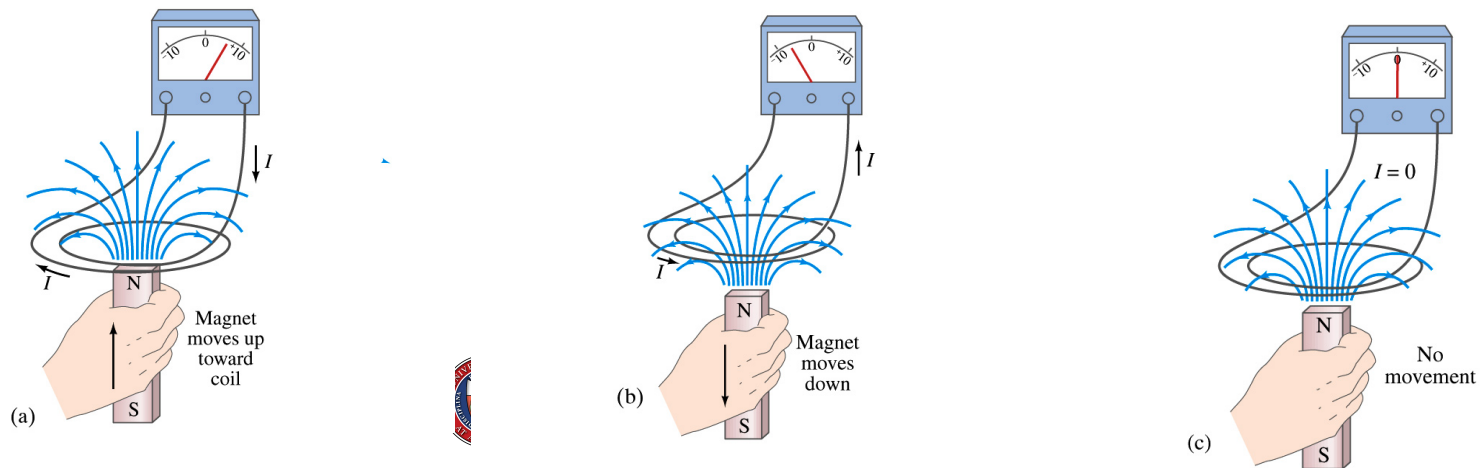
- Faraday used an apparatus below to show that magnetic field can induce current



- Despite his hope he did not see steady current induced on the other side when the switch is thrown
- But he did see that the needle on the Galvanometer turns strongly when the switch is initially thrown and is opened
 - When the magnetic field through coil Y changes, a current flows as if there were a source of emf
- Thus he concluded that **an induced emf is produced by a changing magnetic field** → **Electromagnetic Induction**

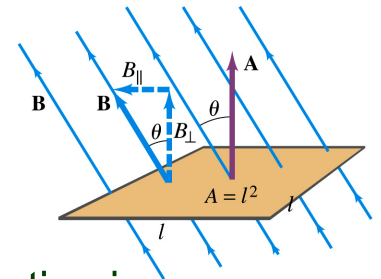
Electromagnetic Induction

- Further studies on electromagnetic induction taught
 - If magnet is moved quickly into a coil of wire, a current is induced in the wire.
 - If the magnet is removed from the coil, a current is induced in the wire in the opposite direction
 - By the same token, current can also be induced if the magnet stays put but the coil moves toward or away from the magnet
 - Current is also induced if the coil rotates.
- In other words, it does not matter whether the magnet or the coil moves. It is the relative motion that counts.



Magnetic Flux

- So what is the induced emf proportional to?
 - The rate of changes of the magnetic field?
 - the higher the changes the higher the induction
 - Not really, it rather depends on the rate of change of the **magnetic flux**, Φ_B .
 - Magnetic flux is defined as (just like the electric flux)
 - $\Phi_B = B_{\perp} A = BA \cos \theta = \vec{B} \cdot \vec{A}$
 - θ is the angle between $\mathbf{B}$ and the area vector $\mathbf{A}$ whose direction is perpendicular to the face of the loop based on the right-hand rule
 - What kind of quantity is the magnetic flux?
 - Scalar. Unit?
 - $T \cdot m^2$ or weber $1Wb = 1T \cdot m^2$
 - If the area of the loop is not simple or B is not uniform, the magnetic flux can be written as



$$\Phi_B = \sum_{loop} \vec{B} \cdot \vec{A}$$

Faraday's Law of Induction

- In terms of magnetic flux, we can formulate Faraday's findings
 - The emf induced in a circuit is equal to the rate of change of magnetic flux through the circuit

$$\mathcal{E} = - \frac{\Delta \Phi_B}{\Delta t}$$

Faraday's Law of Induction

- If the circuit contains N closely wrapped loops, the total induced emf is the sum of emf induced in each loop

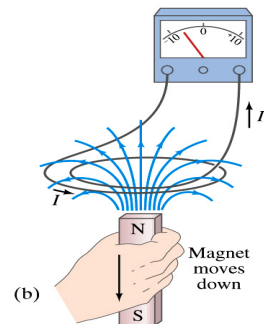
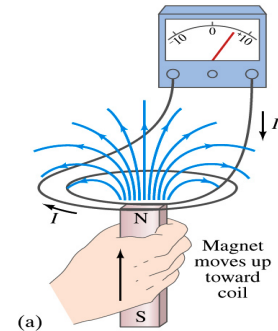
$$\mathcal{E} = -N \frac{\Delta \Phi_B}{\Delta t}$$

- Why negative?
 - Has got a lot to do with the direction of induced emf...



Lenz's Law

- It is experimentally found that
 - An induced emf gives rise to a current whose magnetic field opposes the original change in flux → This is known as Lenz's Law
 - In other words, an induced emf is always in the direction that opposes the original change in flux that caused it.
 - We can use Lenz's law to explain the following cases in the figures
 - When the magnet is moving into the coil
 - Since the external flux increases, the field inside the coil takes the opposite direction to minimize the change and causes the current to flow clockwise
 - When the magnet is moving out
 - Since the external flux decreases, the field inside the coil takes the opposite direction to compensate the loss, causing the current to flow counter-clockwise
- Which law is Lenz's law result of?
 - Energy conservation. Why?



Displacement Current

- Maxwell predicted the second term in the generalized Ampere's law and interpreted it as equivalent to an electric current
 - If a change of magnetic field induces an electric current, a change of electric field must do the same to magnetic field...
 - He called this term as the displacement current, I_D
 - While the original term is called the conduction current, I
- Ampere's law then can be written as

$$\sum \vec{B} \cdot \Delta \vec{l} = \mu_0 (I + I_D)$$

- Where

$$I_D = \epsilon_0 \frac{\Delta \Phi_E}{\Delta t}$$

- While it is in effect equivalent to an electric current, a flow of electric charge, this actually does not have anything to do with the flow of electric charge itself



Gauss' Law for Magnetism

- What was the Gauss' law in the electric case?
 - The electric flux through a closed surface is equal to the total net charge Q enclosed by the surface divided by ϵ_0 .

$$\sum_{\text{Close surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Gauss' Law for electricity

- Similarly, we can write Gauss' law for magnetism as

$$\sum_{\text{Closed surface}} \vec{B} \cdot \vec{A} = 0$$

Gauss' Law for magnetism

- Why is result of the integral zero?
 - There is no isolated magnetic poles, the magnetic equivalent of single electric charges

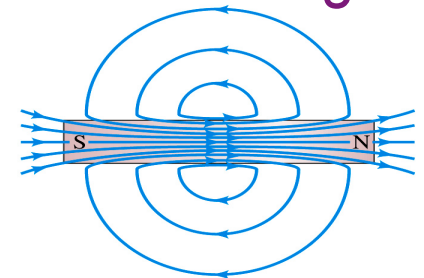


Gauss' Law for Magnetism

- What does the Gauss' law in magnetism mean physically?

$$\sum_{\text{Closed surface}} \vec{B} \cdot \vec{A} = 0$$

- There are as many magnetic flux lines that enter the enclosed volume as leave it
- If magnetic monopole does not exist, there is no starting or stopping point of the flux lines
 - Electricity do have the source and the sink
- Magnetic field lines must be continuous
- Even for bar magnets, the field lines exist both inside and outside of the magnet



Maxwell's Equations

- In the absence of dielectric or magnetic materials, the four equations developed by Maxwell are:

$$\sum_{\text{Closed surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Gauss' Law for electricity

A generalized form of Coulomb's law relating electric field to its sources, the electric charge

$$\sum_{\text{Closed surface}} \vec{B} \cdot \vec{A} = 0$$

Gauss' Law for magnetism

A magnetic equivalent of Coulomb's law, relating magnetic field to its sources. This says there are no magnetic monopoles.

$$\sum_{\text{Closed loop}} \vec{E} \cdot \vec{l} = -\frac{\Delta\Phi_B}{\Delta t}$$

Faraday's Law

An electric field is produced by a changing magnetic field

$$\sum_{\text{Closed loop}} \vec{B} \cdot \vec{l} = \mu_0 I_{\text{encl}} + \mu_0 \epsilon_0 \frac{\Delta\Phi_E}{\Delta t}$$

Ampère's Law

A magnetic field is produced by an electric current or by a changing electric field

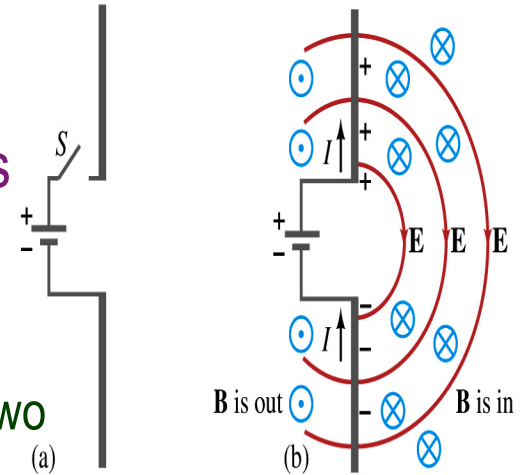
Maxwell's Amazing Leap of Faith

- According to Maxwell, a magnetic field will be produced even in an empty space if there is a changing electric field
 - He then took this concept one step further and concluded that
 - If a changing magnetic field produces an electric field, the electric field is also changing in time.
 - This changing electric field in turn produces the magnetic field that also changes
 - This changing magnetic field then in turn produces the electric field that changes
 - This process continues
 - With the manipulation of his equations, Maxwell found that the net result of this interacting changing fields is a wave of electric and magnetic fields that can actually propagate (travel) through the space



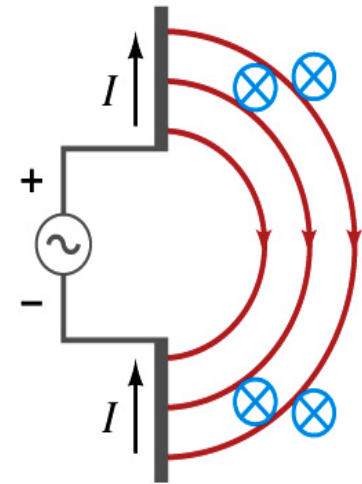
Production of EM Waves

- Consider two conducting rods that will serve as an antenna are connected to a DC power source
 - What do you think will happen when the switch is closed?
 - The rod connected to the positive terminal is charged positive and the other negative
 - Then the electric field will be generated between the two rods
 - Since there is current that flows through the rods, a magnetic field around them will be generated
- How far would the electric and magnetic fields extend?
 - In static case, the field extends indefinitely
 - When the switch is closed, the fields are formed near the rods quickly but
 - The stored energy in the fields won't propagate w/ infinite speed



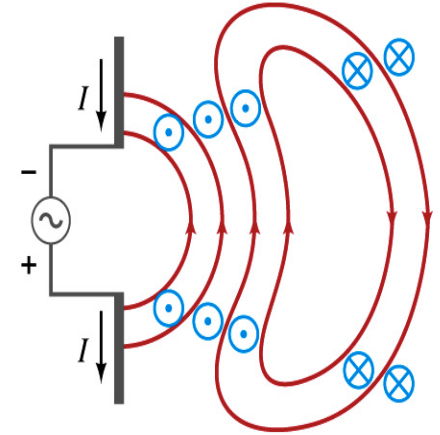
Production of EM Waves, cnt'd

- What happens if the antenna is connected to an AC power source?
 - When the connection was initially made, the rods are charging up quickly w/ the current flowing in one direction as shown in the figure
 - The field lines form as in the DC case
 - The field lines propagate away from the antenna
 - Then the direction of the voltage reverses



Production of EM Waves, cnt'd

- New field lines in the opposite direction forms
- While the original field lines still propagates away from the rod reaching out far
 - Since the original field propagates through an empty space, the field lines must form a closed loop (no charge exist)
 - Since changing electric and magnetic fields produce changing magnetic and electric fields, the fields moving outward is self supporting and do not need antenna with flowing charge
- The fields far from the antenna is called the **radiation field**
- Both electric and magnetic fields form closed loops perpendicular to each other



Properties of Radiation Fields

- The fields travel on the other side of the antenna as well
- The field strength are the greatest in the direction perpendicular to the oscillating charge while along the parallel direction is 0
- The magnitude of **E** and **B** in the radiation field decrease with distance from the source as $1/r$
- The energy carried by the EM wave is proportional to the square of the amplitude, E^2 or B^2
 - So the intensity of wave decreases from the source as $1/r^2$



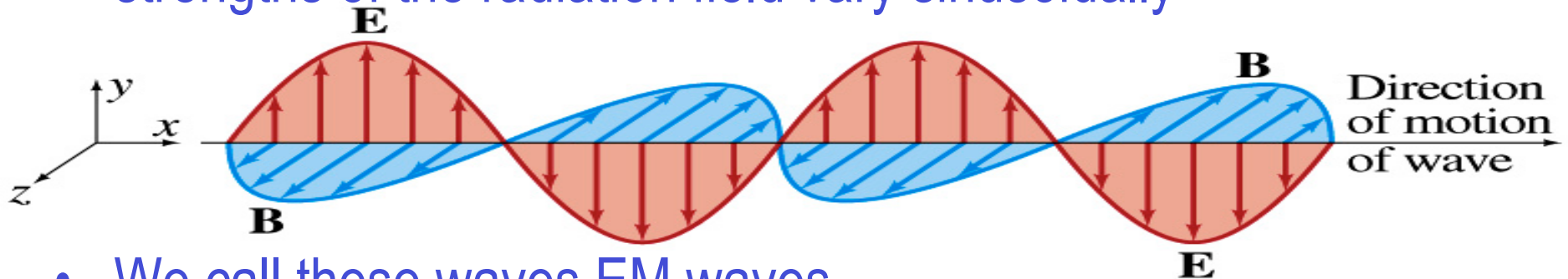
Properties of Radiation Fields, cnt'd

- The electric and magnetic fields at any point are perpendicular to each other and to the direction of motion
- The fields alternate in direction
 - The field strengths vary from maximum to 0 in one direction to 0 to maximum in the opposite direction
- The electric and magnetic fields are in phase
- Very far from the antenna, the field lines are pretty flat over a reasonably large area
 - Called plane waves



EM Waves

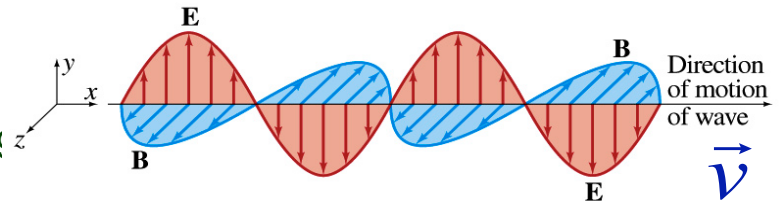
- If the voltage of the source varies sinusoidally, the field strengths of the radiation field vary sinusoidally



- We call these waves EM waves
- They are transverse waves
- EM waves are always waves of fields
 - Since these are fields, they can propagate through an empty space
- In general accelerating electric charges give rise to electromagnetic waves
- This prediction from Maxwell's equations was experimentally proven by Heinrich Hertz through the discovery of radio waves

EM Waves and Their Speeds

- Let's consider a region of free space. What's a free space?
 - An area of space where there is no charges or conduction currents
 - In other words, far from emf sources so that the wave fronts are essentially flat or not distorted over a reasonable area
 - What are these flat waves called?
 - Plane waves
 - At any instance **E** and **B** are uniform over a large plane perpendicular to the direction of propagation
 - So we can also assume that the wave is traveling in the x-direction w/ velocity, $\mathbf{v} = v\mathbf{i}$, and that **E** is parallel to y axis and **B** is parallel to z axis



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Maxwell's Equations w/ $Q=I=0$

- In free space where $Q=0$ and $I=0$, the four Maxwell's equations become

$$\sum_{\text{Closed surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

$$Q_{\text{encl}}=0$$

$$\sum_{\text{Closed surface}} \vec{E} \cdot \vec{A} = 0$$

$$\sum_{\text{Closed surface}} \vec{B} \cdot \vec{A} = 0$$

No Changes

$$\sum_{\text{Closed surface}} \vec{B} \cdot \vec{A} = 0$$

$$\sum_{\text{Closed loop}} \vec{E} \cdot \vec{l} = -\frac{\Delta\Phi_B}{\Delta t}$$

No Changes

$$\sum_{\text{Closed loop}} \vec{E} \cdot \vec{l} = -\frac{\Delta\Phi_B}{\Delta t}$$

$$\sum_{\text{Closed loop}} \vec{B} \cdot \vec{l} = \mu_0 I_{\text{encl}} + \mu_0 \epsilon_0 \frac{\Delta\Phi_E}{\Delta t}$$

$$I_{\text{encl}}=0$$

$$\sum_{\text{Closed loop}} \vec{B} \cdot \vec{l} = \mu_0 \epsilon_0 \frac{\Delta\Phi_E}{\Delta t}$$

One can observe the symmetry between electricity and magnetism.

The last equation is the most important one for EM waves and their propagation!!



EM Waves from Maxwell's Equations

- If the wave is sinusoidal w/ wavelength λ and frequency f , such traveling wave can be written as

$$E = E_y = E_0 \sin(kx - \omega t)$$

$$B = B_z = B_0 \sin(kx - \omega t)$$

– Where

$$k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f \quad \text{Thus} \quad f\lambda = \frac{\omega}{k} = v$$

– What is v ?

- It is the speed of the traveling wave

– What are E_0 and B_0 ?

- The amplitudes of the EM wave. Maximum values of **E** and **B** field strengths.



From Faraday's Law

- Let's apply Faraday's law

$$\sum_{loop} \vec{E} \cdot \Delta \vec{l} = - \frac{\Delta \Phi_B}{\Delta t}$$

- to the rectangular loop of height Δy and width Δx

- $\vec{E} \cdot \Delta \vec{l}$ along the top and bottom of the loop is 0. Why?

- Since $\vec{E}$ is perpendicular to $\Delta \vec{l}$

- So the result of the sum through the loop counterclockwise

becomes

$$\begin{aligned} \sum_{loop} \vec{E} \cdot \Delta \vec{l} &= \vec{E} \cdot d\vec{x} + (\vec{E} + \Delta \vec{E}) \cdot \Delta \vec{y} + \vec{E} \cdot d\vec{x}' + \vec{E} \cdot \Delta \vec{y}' = \\ &= 0 + (E + \Delta E) \Delta y - 0 - E \Delta y = \Delta E \Delta y \end{aligned}$$

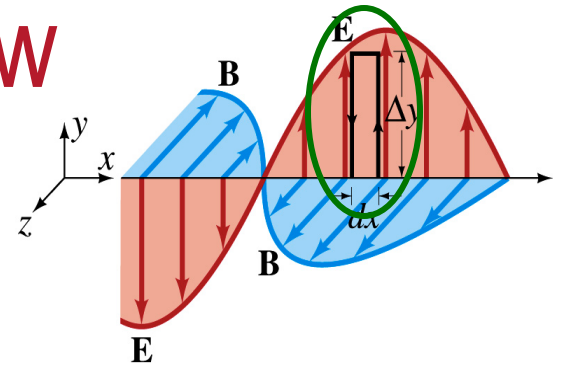
- For the right-hand side of Faraday's law, the magnetic flux through the loop changes as

$$- \frac{\Delta \Phi_B}{\Delta t} = \frac{\Delta B}{\Delta t} \Delta x \Delta y$$

Thus

$$\Delta E \Delta y = - \frac{\Delta B}{\Delta t} \Delta x \Delta y$$

$$\frac{\Delta E}{\Delta x} = - \frac{\Delta B}{\Delta t}$$



From Modified Ampère's Law

- Let's apply Maxwell's 4th equation

$$\sum_{loop} \vec{B} \cdot \Delta \vec{l} = \mu_0 \epsilon_0 \frac{\Delta \Phi_E}{\Delta t}$$

- to the rectangular loop of length Δz and width Δx

- $\vec{B} \cdot d\vec{l}$ along the x-axis of the loop is 0

- Since $\mathbf{B}$ is perpendicular to $d\mathbf{l}$

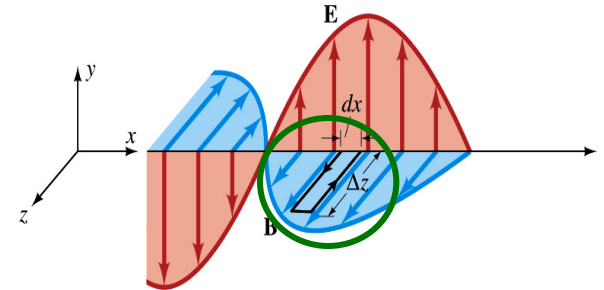
- So the result of the integral through the loop counterclockwise

becomes
$$\sum_{loop} \vec{B} \cdot \Delta \vec{l} = B \Delta Z - (B + \Delta B) \Delta Z = -\Delta B \Delta Z$$

- For the right-hand side of the equation is

$$\mu_0 \epsilon_0 \frac{\Delta \Phi_E}{\Delta t} = \mu_0 \epsilon_0 \frac{\Delta E}{\Delta t} \Delta x \Delta z \quad \text{Thus} \quad -\Delta B \Delta z = \mu_0 \epsilon_0 \frac{\Delta E}{\Delta t} \Delta x \Delta z$$

$$-\frac{\Delta B}{\Delta x} = -\mu_0 \epsilon_0 \frac{\Delta E}{\Delta t}$$



Relationship between **E**, **B** and **v**

- Let's now use the relationship from Faraday's law $\frac{\Delta E}{\Delta x} = -\frac{\Delta B}{\Delta t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\Delta E}{\Delta x} = kE_0 \cos(kx - \omega t)$$

$$\frac{\Delta B}{\Delta t} = -\omega B_0 \cos(kx - \omega t)$$

Since $\frac{\Delta E}{\Delta x} = -\frac{\Delta B}{\Delta t}$  We obtain $kE_0 \cos(kx - \omega t) = \omega B_0 \cos(kx - \omega t)$

 Thus $\frac{E_0}{B_0} = \frac{\omega}{k} = v$

- Since E and B are in phase, we can write $E/B = v$
 - This is valid at any point and time in space. What is v?
 - The velocity of the wave

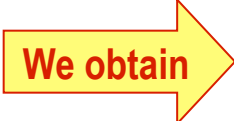


Speed of EM Waves

- Let's now use the relationship from Ampere's law $\frac{\Delta B}{\Delta x} = -\epsilon_0 \mu_0 \frac{\Delta E}{\Delta t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\Delta B}{\Delta x} = k B_0 \cos(kx - \omega t)$$

$$\frac{\Delta E}{\Delta t} = -\omega E_0 \cos(kx - \omega t)$$

Since $\frac{\Delta B}{\Delta x} = -\epsilon_0 \mu_0 \frac{\Delta E}{\Delta t}$  **We obtain** $k B_0 \cos(kx - \omega t) = \epsilon_0 \mu_0 \omega E_0 \cos(kx - \omega t)$

 **Thus** $\frac{B_0}{E_0} = \frac{\epsilon_0 \mu_0 \omega}{k} = \epsilon_0 \mu_0 v$

– However, from the previous page we obtain $E_0/B_0 = v = \frac{1}{\epsilon_0 \mu_0}$

– Thus $v^2 = \frac{1}{\epsilon_0 \mu_0}$

$$v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{1}{\sqrt{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \cdot (4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A})}} = 3.00 \times 10^8 \text{ m/s}$$

The speed of EM waves is the same as the speed of light. EM waves behaves like the light.

Light as EM Wave

- People knew some 60 years before Maxwell that light behaves like a wave, but ...
 - They did not know what kind of wave it is.
 - Most importantly what is it that oscillates in light?
- Heinrich Hertz first generated and detected EM waves experimentally in 1887 using a spark gap apparatus
 - Charge was rushed back and forth in a short period of time, generating waves with frequency about 10^9Hz (these are called radio waves)
 - He detected using a loop of wire in which an emf was produced when a changing magnetic field passed through
 - These waves were later shown to travel at the speed of light

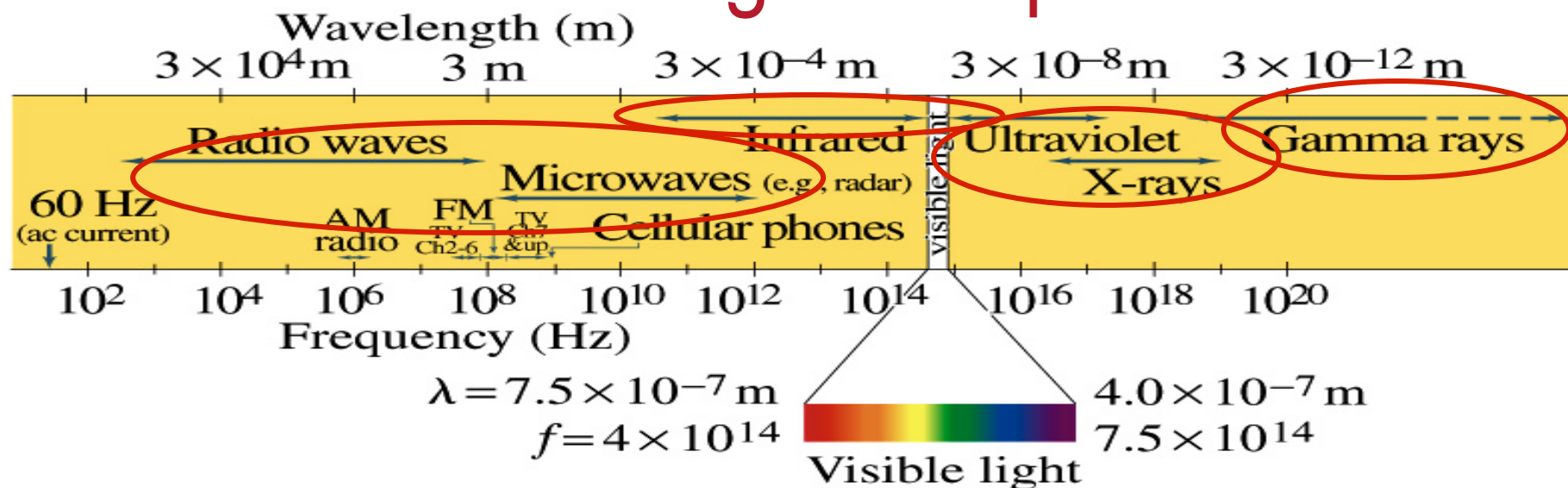


Light as EM Wave, cnt'd

- The wavelengths of visible light were measured in the first decade of the 19th century
 - The visible light wavelengths were found to be between $4.0 \times 10^{-7} \text{m}$ (400nm) and $7.5 \times 10^{-7} \text{m}$ (750nm)
 - The frequency of visible light is $f\lambda = c$
 - Where f and λ are the frequency and the wavelength of the wave
 - What is the range of visible light frequency?
 - $4.0 \times 10^{14} \text{Hz}$ to $7.5 \times 10^{14} \text{Hz}$
 - c is $3 \times 10^8 \text{m/s}$, the speed of light
- EM Waves, or EM radiation, are categorized using EM spectrum



Electromagnetic Spectrum



- Low frequency waves, such as radio waves or microwaves can be easily produced using electronic devices
- Higher frequency waves are produced natural processes, such as emission from atoms, molecules or nuclei
- Or they can be produced from acceleration of charged particles
- Infrared radiation (IR) is mainly responsible for the heating effect of the Sun
 - The Sun emits visible lights, IR and UV
 - The molecules of our skin resonate at infrared frequencies so IR is preferentially absorbed and thus warm up

Example 22 – 1

Wavelength of EM waves. Calculate the wavelength (a) of a 60-Hz EM wave, (b) of a 93.3-MHz FM radio wave and (c) of a beam of visible red light from a laser at frequency 4.74×10^{14} Hz.

What is the relationship between speed of light, frequency and the wavelength? $c = f\lambda$

Thus, we obtain $\lambda = \frac{c}{f}$

For $f=60\text{Hz}$
$$\lambda = \frac{3 \times 10^8 \text{ m/s}}{60 \text{ s}^{-1}} = 5 \times 10^6 \text{ m}$$

For $f=93.3\text{MHz}$
$$\lambda = \frac{3 \times 10^8 \text{ m/s}}{93.3 \times 10^6 \text{ s}^{-1}} = 3.22 \text{ m}$$

For $f=4.74 \times 10^{14}\text{Hz}$
$$\lambda = \frac{3 \times 10^8 \text{ m/s}}{4.74 \times 10^{14} \text{ s}^{-1}} = 6.33 \times 10^{-7} \text{ m}$$



EM Wave in the Transmission Lines

- Can EM waves travel through a wire?
 - Can it not just travel through the empty space?
 - Yes, it sure can travel through a wire.
- When a source of emf is connected to a transmission line, the electric field within the wire does not set up immediately at all points along the line
 - When two wires are separated via air, the EM wave travel through the air at the speed of light, c .
 - However, through medium w/ permittivity ϵ and permeability μ , the speed of the EM wave is given $v = 1/\sqrt{\epsilon\mu} < c$
 - Is this faster than c ? **Nope! It is slower.**



Energy in EM Waves

- Since $B=E/c$ and $c=1/\sqrt{\epsilon_0\mu_0}$, we can rewrite the energy density

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{\epsilon_0\mu_0 E^2}{\mu_0} = \epsilon_0 E^2 \quad \boxed{u = \epsilon_0 E^2}$$

- Note that the energy density associate with B field is the same as that associate with E
- So each field contribute half to the total energy

- By rewriting in B field only, we obtain

$$u = \frac{1}{2}\epsilon_0 \frac{B^2}{\epsilon_0\mu_0} + \frac{1}{2}\frac{B^2}{\mu_0} = \frac{B^2}{\mu_0} \quad \boxed{u = \frac{B^2}{\mu_0}}$$

- We can also rewrite to contain both E and B

$$u = \epsilon_0 E^2 = \epsilon_0 E c B = \frac{\epsilon_0 E B}{\sqrt{\epsilon_0\mu_0}} = \sqrt{\frac{\epsilon_0}{\mu_0}} E B \quad \boxed{u = \sqrt{\frac{\epsilon_0}{\mu_0}} E B}$$

Energy Transport

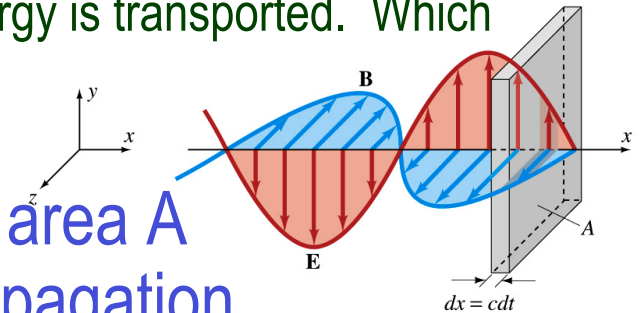
- What is the energy the wave transport per unit time per unit area?

- This is given by the vector **S**, the Poynting vector

- The unit of **S** is **W/m²**.
- The direction of **S** is the direction in which the energy is transported. Which direction is this?

- The direction the wave is moving

- Let's consider a wave passing through an area **A** perpendicular to the **x**-axis, the axis of propagation



- How much does the wave move in time Δt ?

- $\Delta x = c\Delta t$

- The energy that passes through **A** in time dt is the energy that occupies the volume ΔV , $\Delta V = A\Delta x = Ac\Delta t$

- Since the energy density is $u = \epsilon_0 E^2$, the total energy, dU , contained in the volume V is $\Delta U = u\Delta V = \epsilon_0 E^2 Ac\Delta t$

Energy Transport

- Thus, the energy crossing the area A per time dt is

$$S = \frac{1}{A} \frac{\Delta U}{\Delta t} = \epsilon_0 c E^2$$

- Since $E=cB$ and $c = 1/\sqrt{\epsilon_0 \mu_0}$, we can also rewrite

$$S = \epsilon_0 c E^2 = \frac{c B^2}{\mu_0} = \frac{EB}{\mu_0}$$

- Since the direction of S is along $\mathbf{v}$, perpendicular to $\mathbf{E}$ and $\mathbf{B}$, the Poynting vector $\mathbf{S}$ can be written

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$$

- This gives the energy transported per unit area per unit time at any instant



Average Energy Transport

- The average energy transport in an extended period of time since the frequency is so high we do not detect the rapid variation with respect to time.
- If E and B are sinusoidal, $\overline{E^2} = E_0^2 / 2$
- Thus we can write the magnitude of the average Poynting vector as

$$\overline{S} = \frac{1}{2} \epsilon_0 c E_0^2 = \frac{1}{2} \frac{c}{\mu_0} B_0^2 = \frac{E_0 B_0}{2 \mu_0}$$

- This time averaged value of S is the intensity, defined as the average power transferred across unit area. E_0 and B_0 are maximum values.

- We can also write

$$\overline{S} = \frac{E_{rms} B_{rms}}{\mu_0}$$

- Where E_{rms} and B_{rms} are the rms values ($E_{rms} = \sqrt{\overline{E^2}}$, $B_{rms} = \sqrt{\overline{B^2}}$)



Example 22 – 4

E and B from the Sun. Radiation from the Sun reaches the Earth (above the atmosphere) at a rate of about 1350W/m^2 . Assume that this is a single EM wave and calculate the maximum values of E and B.

What is given in the problem? The average S!!

$$\bar{S} = \frac{1}{2} \epsilon_0 c E_0^2 = \frac{1}{2} \frac{c}{\mu_0} B_0^2 = \frac{E_0 B_0}{2\mu_0}$$

For E_0 ,
$$E_0 = \sqrt{\frac{2\bar{S}}{\epsilon_0 c}} = \sqrt{\frac{2 \cdot 1350\text{W/m}^2}{(8.85 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2) \cdot (3.00 \times 10^8 \text{m/s})}} = 1.01 \times 10^3 \text{V/m}$$

For B_0
$$B_0 = \frac{E_0}{c} = \frac{1.01 \times 10^3 \text{V/m}}{3 \times 10^8 \text{m/s}} = 3.37 \times 10^{-6} \text{T}$$

