PHYS 1443 – Section 001
Lecture #2

Tuesday, June 7, 2011

Dr. Jaehoon Yu

- Dimensional Analysis
- Fundamentals
- One Dimensional Motion: Average Velocity; Acceleration; Motion under constant acceleration; Free Fall

Announcements

- Homework registration and submissions
  - 18/27 registered but only 1 submitted the answer!
  - I will then have to approve your enrollment request
    - So please go ahead and take an action as soon as possible
    - The roster closes tomorrow, Wednesday!
- Quiz tomorrow at the beginning of the class!
  - Problems will be on Appendices A and B!

Special Problems for Extra Credit

- Derive the quadratic equation for $Bx^2 - Cx + A = 0$ ➔ 5 points
- Derive the kinematic equation $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$ from first principles and the known kinematic equations ➔ 10 points
- You must **show your work in detail** to obtain full credit
- Due at the start of the class, Thursday, June 9

Dimension and Dimensional Analysis

- An extremely useful concept in solving physical problems
- Good to write physical laws in mathematical expressions
- No matter what units are used the base quantities are the same
  - *Length* (distance) is length whether meter or inch is used to express the size: Usually denoted as *[L]*
  - The same is true for *Mass ([M])* and *Time ([T])*
  - One can say "Dimension of Length, Mass or Time"
  - Dimensions are treated as algebraic quantities: Can perform two algebraic operations; multiplication or division

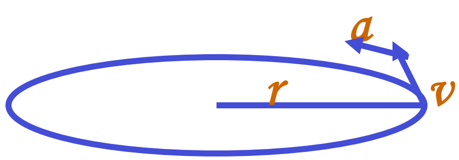
Dimension and Dimensional Analysis

- One can use dimensions only to check the validity of one's expression: Dimensional analysis
  - Eg: Speed $[v] = [L]/[T]=[L][T^{-1}]$
    - *Distance (L) traveled by a car running at the speed V in time T*
      - $L = V*T = [L/T]*[T]=[L]$
- More general expression of dimensional analysis is using exponents: eg. $[v]=[L^nT^m] =[L][T^{-1}]$ *where* $n = 1$ *and* $m = -1$

Examples

- Show that the expression *[v] = [at]* is dimensionally correct
  - Speed: *[v]* =[L]/[T]
  - Acceleration: *[a]* =$[L]/[T]^2$
  - Thus, *[at]* = $(L/T^2)xT=LT^{(-2+1)}=LT^{-1}=[L]/[T]=$ *[v]*

•Suppose the acceleration a of a circularly moving particle with speed v and radius r is proportional to r^n and v^m. What are n and m?



$$L^1T^{-2} = (L)^n\left(\frac{L}{T}\right)^m = L^{n+m}T^{-m}$$

$$a = kr^n v^m$$

Dimensionless constant → k; Length → r; Speed → v

$$-m = -2 \Rightarrow m = 2$$

$$n + m = n + 2 = 1 \Rightarrow n = -1$$

$$a = kr^{-1}v^2 = \frac{v^2}{r}$$

Some Fundamentals

- Kinematics: Description of Motion without understanding the cause of the motion
- Dynamics: Description of motion accompanied with understanding the cause of the motion
- Vector and Scalar quantities:
  - Scalar: Physical quantities that require magnitude but no direction
    - Speed, length, mass, height, volume, area, energy, heat, etc
  - Vector: Physical quantities that require both magnitude and direction
    - Velocity, Acceleration, Force, Momentum
    - It does not make sense to say "I ran with velocity of 10miles/hour."
- Objects can be treated as point-like if their sizes are smaller than the scale in the problem
  - Earth can be treated as a point like object (or a particle) in celestial problems
    - Simplification of the problem (The first step in setting up to solve a problem…)
  - Any other examples?

Some More Fundamentals

- Motions:Can be described as long as the position is known at any given time (or position is expressed as a function of time)
  - Translation: Linear motion along a line
  - Rotation: Circular or elliptical motion
  - Vibration: Oscillation
- Dimensions
  - 0 dimension: A point
  - 1 dimension: Linear drag of a point, resulting in a line ➔ Motion in one-dimension is a motion on a line
  - 2 dimension: Linear drag of a line resulting in a surface
  - 3 dimension: Perpendicular Linear drag of a surface, resulting in a stereo object

Displacement, Velocity and Speed

One dimensional displacement is defined as:

$$\Delta x \equiv x_f - x_i$$

A vector quantity

Displacement is the difference between initial and final potions of the motion and is a vector quantity. How is this different than distance?

Unit? **m**

The average velocity is defined as: $v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} \equiv \frac{\text{Displacement}}{\text{Elapsed Time}}$

Unit? **m/s**

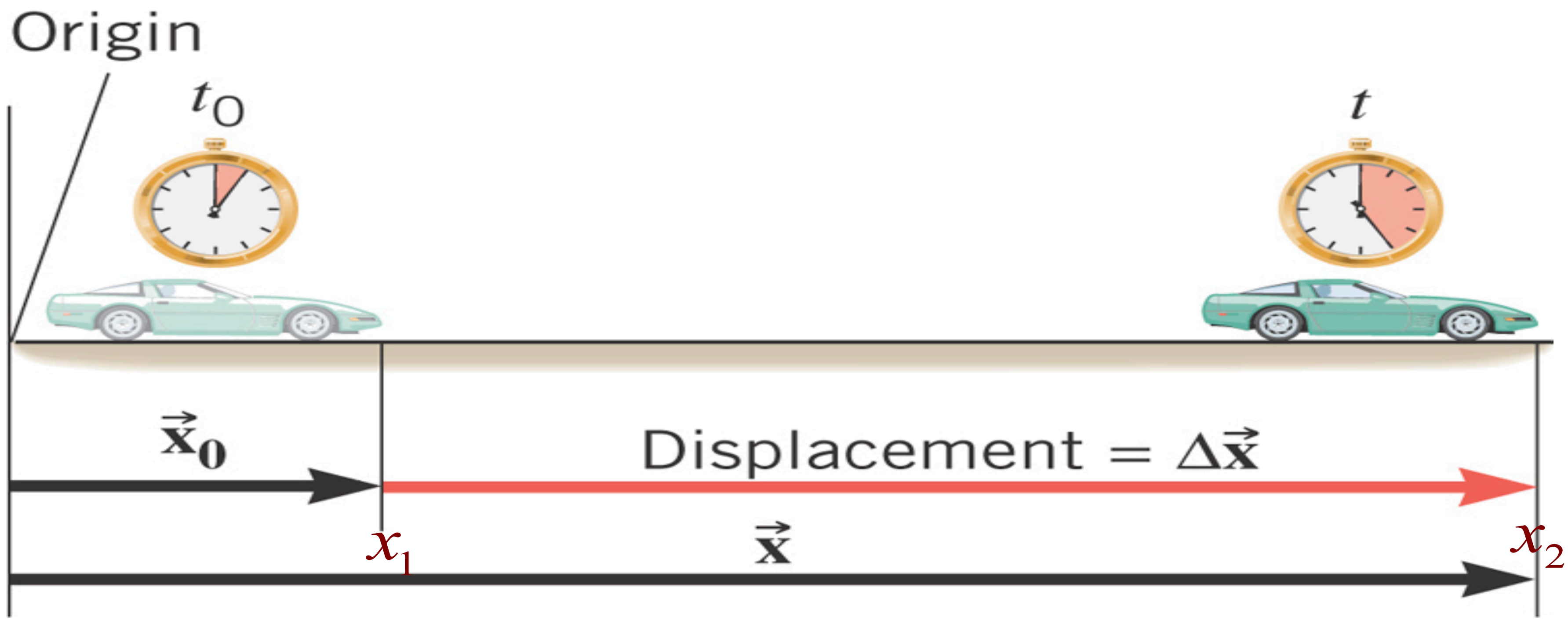
A vector quantity

Displacement per unit time in the period throughout the motion

The average speed is defined as: $v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Elapsed Time}}$

Unit? **m/s**

A scalar quantity



What is the displacement? $\Delta x = x_2 - x_1$

How much is the elapsed time? $\Delta t = t - t_0$

Displacement, Velocity and Speed

One dimensional displacement is defined as:

$$\Delta x \equiv x_f - x_i$$

Displacement is the difference between initial and final potions of the motion and is a vector quantity. How is this different than distance?

Unit? m

The average velocity is defined as: $$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} \equiv \frac{\text{Displacement}}{\text{Elapsed Time}}$$

Unit? m/s

Displacement per unit time in the period throughout the motion

The average speed is defined as: $$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Elapsed Time}}$$

Unit? m/s

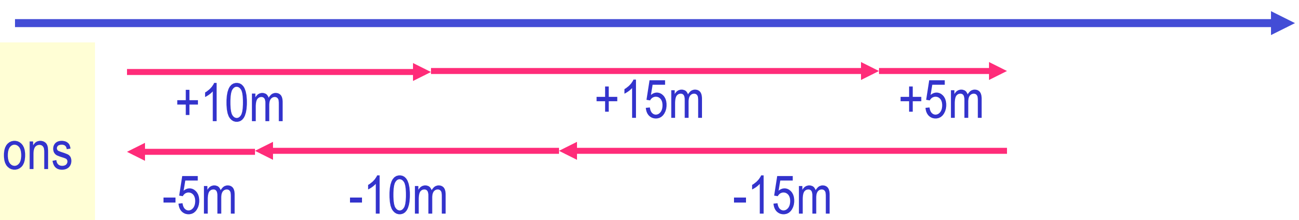
Can someone tell me what the difference between speed and velocity is?

Difference between Speed and Velocity

- Let's take a simple one dimensional translation that has many steps:

Let's call this line as X-axis

Let's have a couple of motions in a total time interval of 20 sec.



Total Displacement: $\Delta x \equiv x_f - x_i = x_i - x_i = 0(m)$

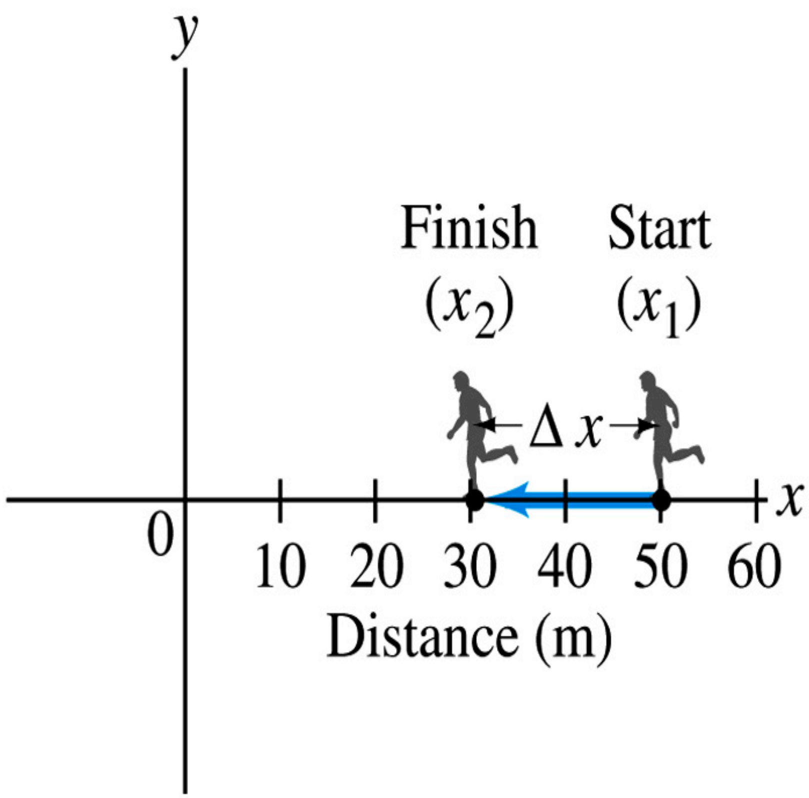
Average Velocity: $v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} = \frac{0}{20} = 0(m/s)$

Total Distance Traveled: $D = 10 + 15 + 5 + 15 + 10 + 5 = 60(m)$

Average Speed: $v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}} = \frac{60}{20} = 3(m/s)$

Example 2.1

The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from x_1=50.0m to x_2=30.5 m, as shown in the figure. What was the runner's average velocity? What was the average speed?



- Displacement:

$$\Delta x \equiv x_f - x_i = x_2 - x_1 = 30.5 - 50.0 = -19.5(m)$$

- Average Velocity:

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{-19.5}{3.00} = -6.50(m/s)$$

- Average Speed:

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}}$$

$$= \frac{|50.0 - 30.5|}{3.00} = \frac{19.5}{3.00} = 6.50(m/s)$$

Instantaneous Velocity and Speed

- Can average quantities tell you the detailed story of the whole motion? **NO!!**
- Instantaneous velocity is defined as:
  - What does this mean?
    - Displacement in an infinitesimal time interval
    - Average velocity over a very, very short amount of time

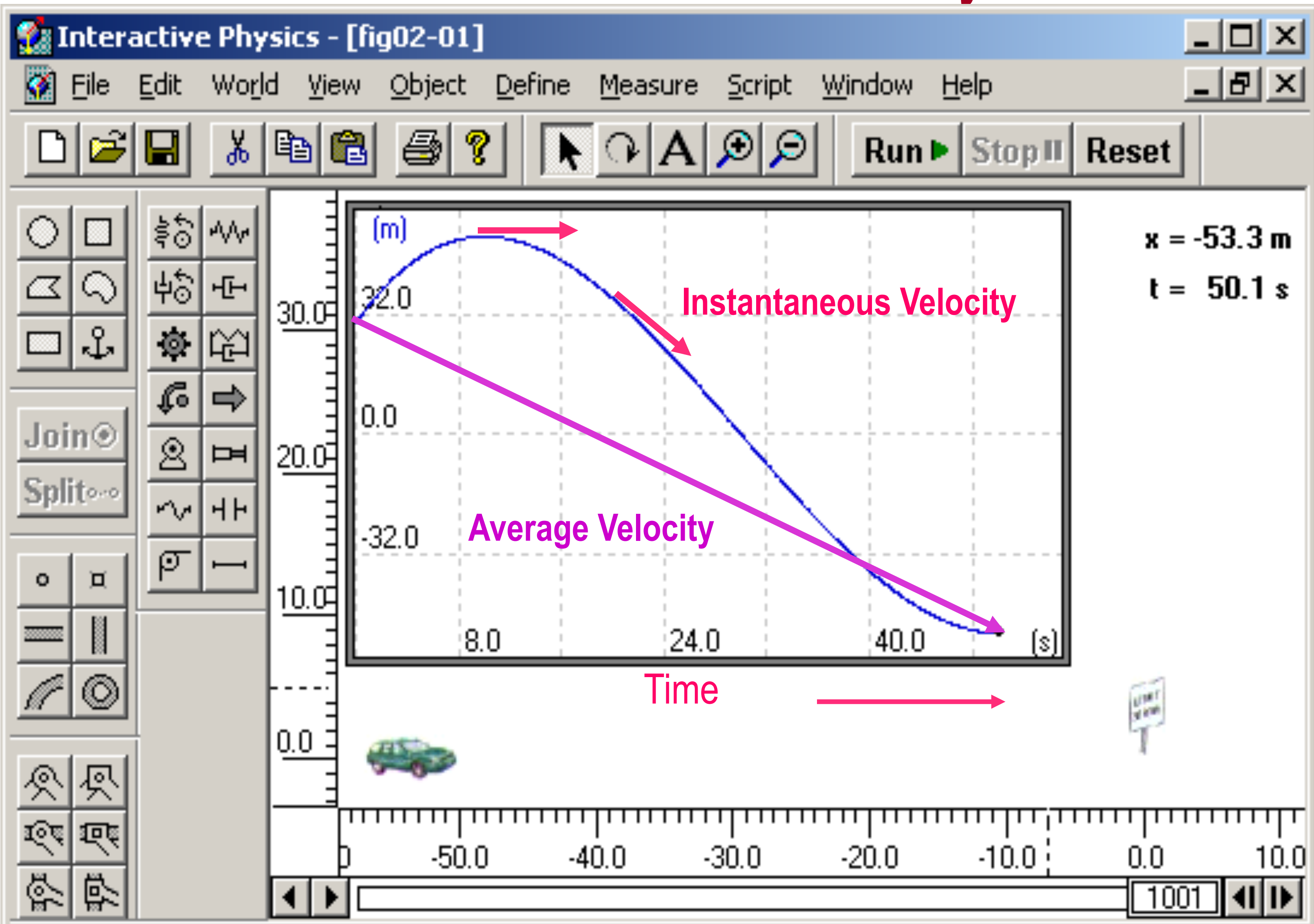
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

•Instantaneous speed is the size (magnitude) of the velocity vector:

$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right|$$

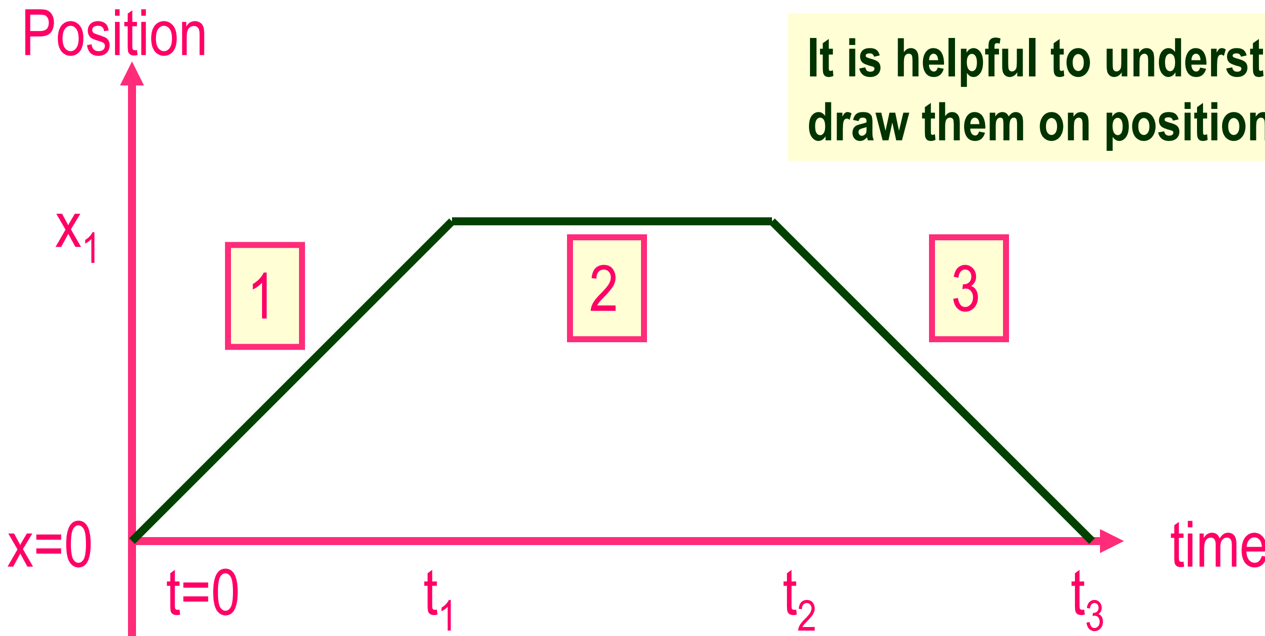
*Magnitude of Vectors are Expressed in absolute values

Instantaneous Velocity



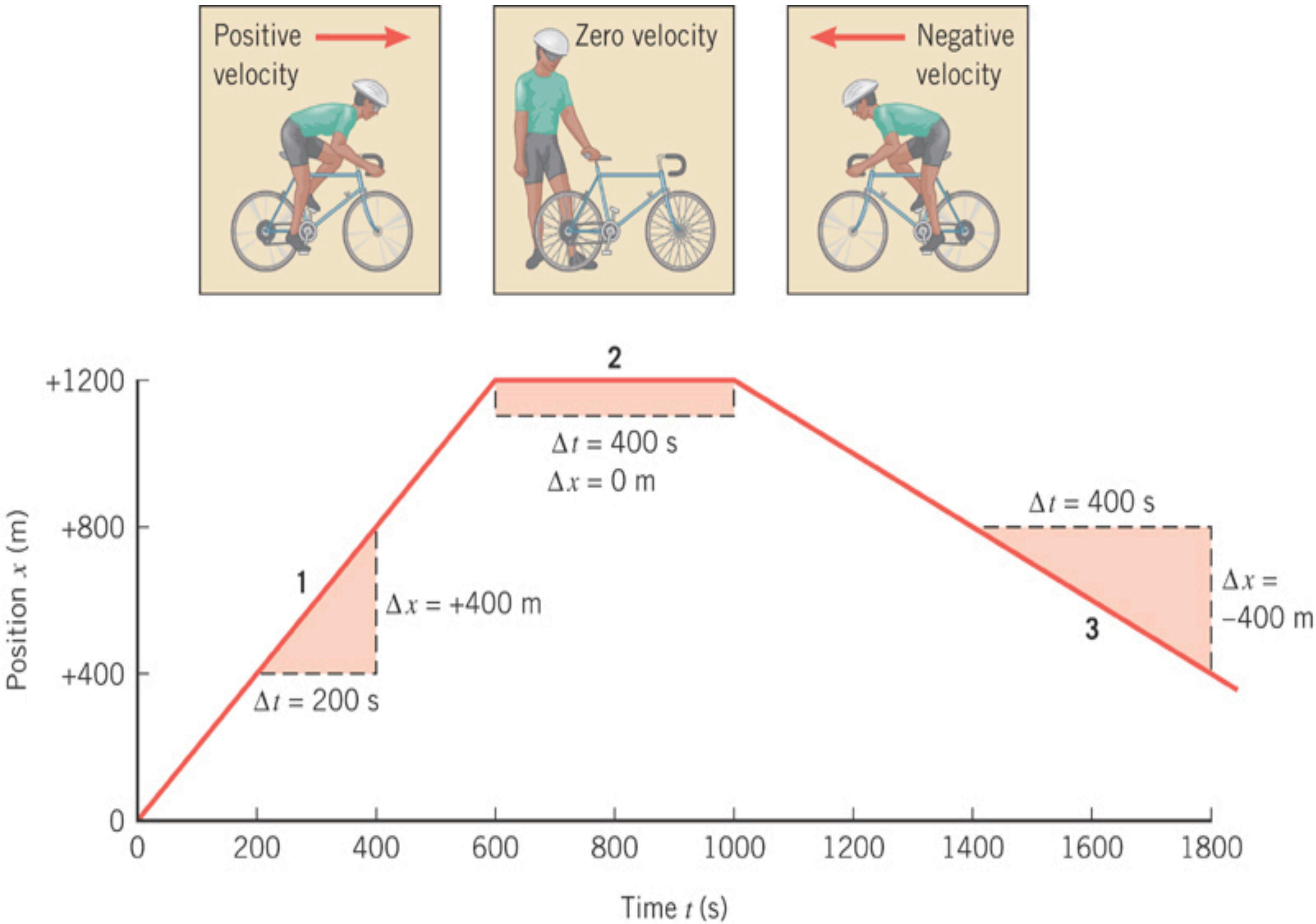
Position vs Time Plot

It is helpful to understand motions to draw them on position vs time plots.



1. Running at a constant velocity (go from x=0 to $x=x_1$ in t_1, Displacement is + x_1 in t_1 time interval)
2. Velocity is 0 (go from x_1 to x_1 no matter how much time changes)
3. Running at a constant velocity but in the reverse direction as 1. (go from x_1 to x=0 in t_3-t_2 time interval, Displacement is - x_1 in t_3-t_2 time interval)

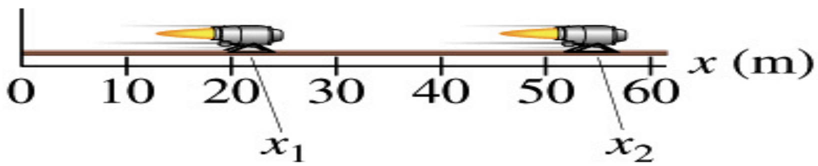
Does this motion physically make sense?

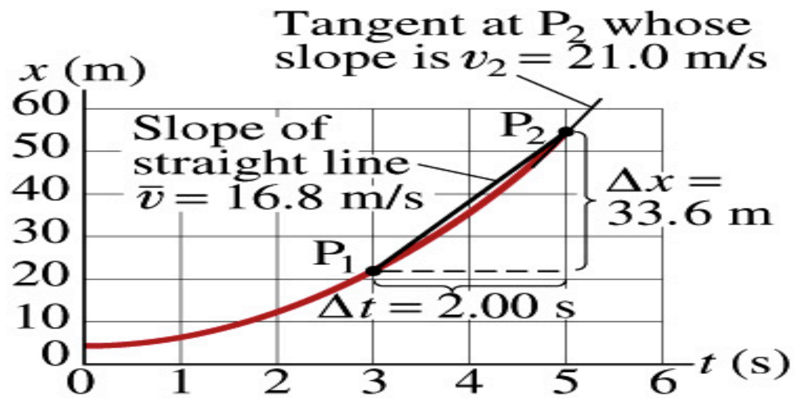


Positive velocity
Zero velocity
Negative velocity
2
+1200
Δt = 400 s
Δx = 0 m
+800
Δt = 400 s
1
Δx = +400 m
Δx = –400 m
3
+400
Δt = 200 s
0
Position x (m)
0
200
400
600
800
1000
1200
1400
1600
1800
Time t (s)

Example 2.3

A jet engine moves along a track. Its position as a function of time is given by the equation $x=At^2+B$ where A=2.10m/s^2 and B=2.80m.





(a) Determine the displacement of the engine during the interval from t_1=3.00s to t_2=5.00s.

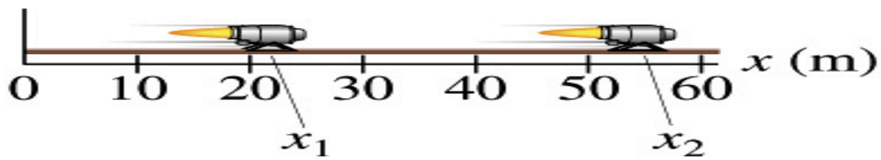
$$x_1 = x_{t_1=3.00} = 2.10\times(3.00)^2 + 2.80 = 21.7m \qquad x_2 = x_{t_2=5.00} = 2.10\times(5.00)^2 + 2.80 = 55.3m$$

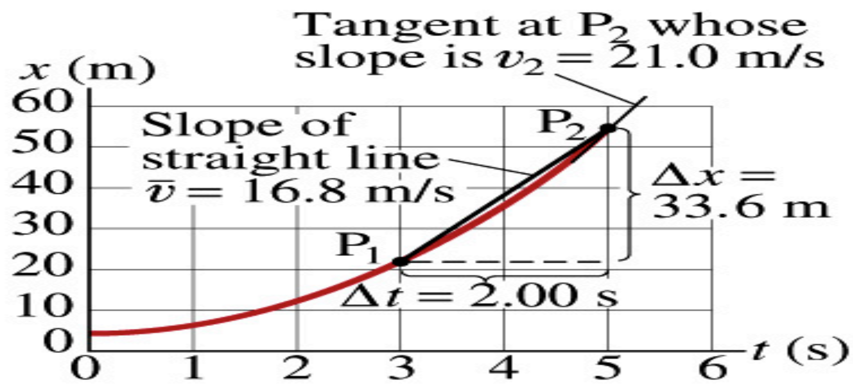
Displacement is, therefore: $$\Delta x = x_2 - x_1 = 55.3 - 21.7 = +33.6(m)$$

(b) Determine the average velocity during this time interval.

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{33.6}{5.00-3.00} = \frac{33.6}{2.00} = 16.8(m/s)$$

Example 2.3 cont'd





(c) Determine the instantaneous velocity at $t=t_2=5.00$s.

Calculus formula for derivative $\frac{d}{dt}\left(Ct^n\right) = nCt^{n-1}$ and $\frac{d}{dt}\left(C\right) = 0$

The derivative of the engine's equation of motion is

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt}\left(At^2 + B\right) = 2At$$

The instantaneous velocity at t=5.00s is

$$v_x\left(t = 5.00s\right) = 2A \times 5.00 = 2.10 \times 10.0 = 21.0\left(m/s\right)$$

Displacement, Velocity and Speed

Displacement
$$\Delta x \equiv x_f - x_i$$

Average velocity
$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Average speed
$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

Instantaneous velocity
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous speed
$$\left|v_x\right| = \left|\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\right| = \left|\frac{dx}{dt}\right|$$

Acceleration

Change of velocity in time (what kind of quantity is this?) **Vector**

- Definition of the average acceleration:

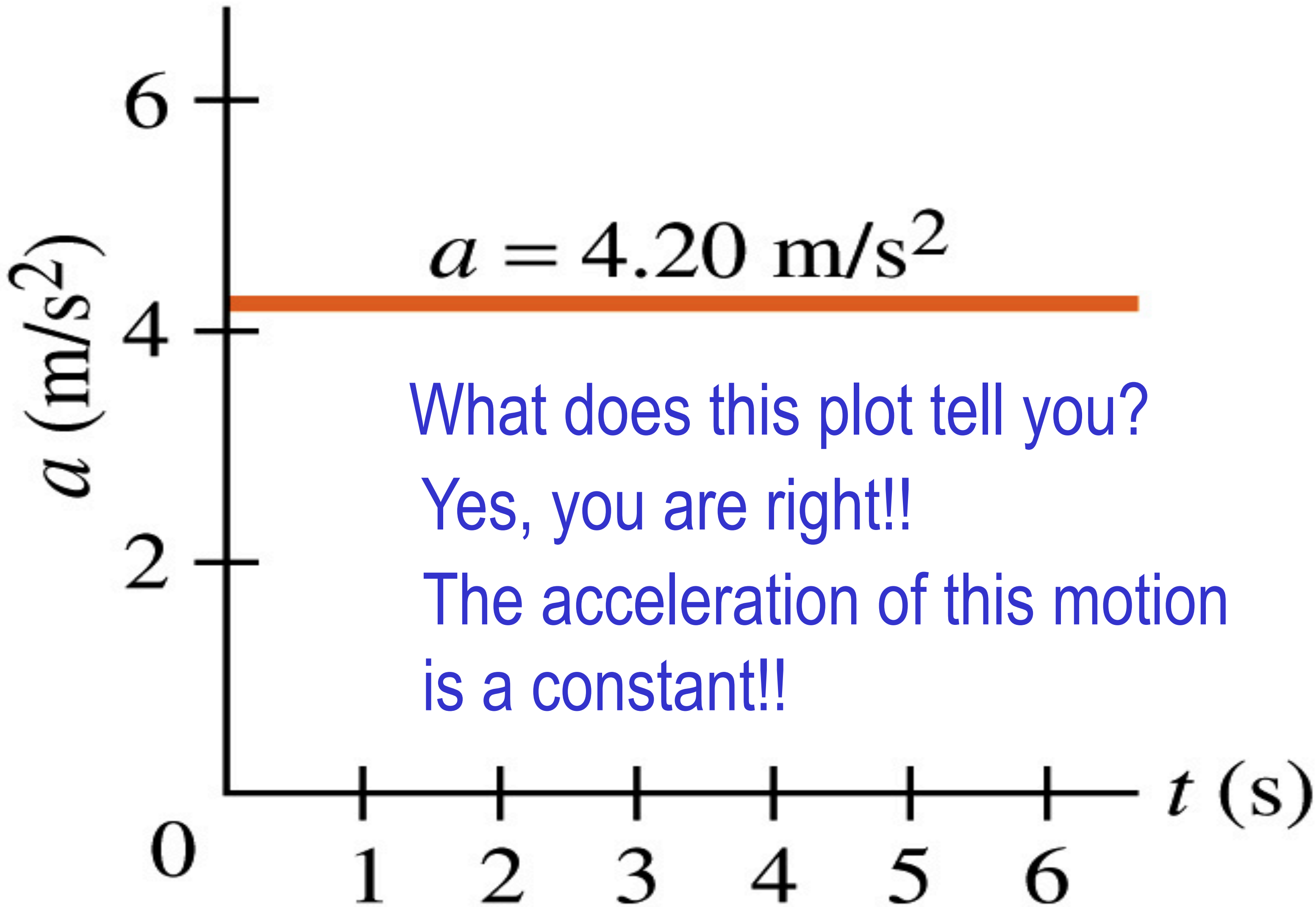
$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} \quad \text{analogous to} \quad v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

- Definition of the instantaneous acceleration:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \quad \text{analogous to} \quad v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- In calculus terms: The slope (derivative) of the velocity vector with respect to time or the change of slopes of position as a function of time

Acceleration vs Time Plot



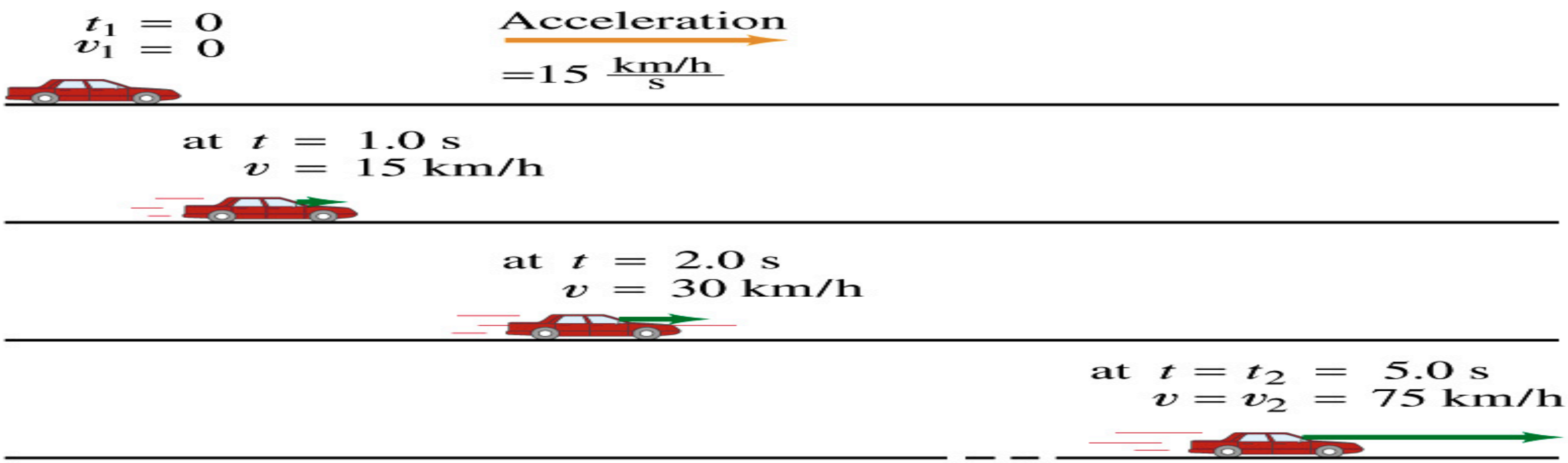
What does this plot tell you?

Yes, you are right!!

The acceleration of this motion is a constant!!

Example 2.4

A car accelerates along a straight road from rest to 75km/h in 5.0s.



What is the magnitude of its average acceleration?

$$v_{xi} = 0\ m/s$$

$$v_{xf} = \frac{75000m}{3600s} = 21\ m/s$$

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} = \frac{21-0}{5.0} = \frac{21}{5.0} = 4.2(m/s^2)$$

$$= \frac{4.2 \times (3600)^2}{1000} = 5.4 \times 10^4 (km/h^2)$$

Few Confusing Things on Acceleration

- When an object is moving in a constant velocity ($v=v_0$), there is no acceleration ($a=0$)
  - Is there any acceleration when an object is not moving?
- When an object is moving faster as time goes on, ($v=v(t)$), acceleration is positive ($a>0$).
  - Incorrect, since the object might be moving in negative direction initially
- When an object is moving slower as time goes on, ($v=v(t)$), acceleration is negative ($a<0$)
  - Incorrect, since the object might be moving in negative direction initially
- In all cases, velocity is positive, unless the direction of the movement changes.
  - Incorrect, since the object might be moving in negative direction initially
- Is there acceleration if an object moves in a constant speed but changes direction?

The answer is YES!!

Displacement, Velocity, Speed & Acceleration

Displacement

$$\Delta x \equiv x_f - x_i$$

Average velocity

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Average speed

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

Instantaneous velocity

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous speed

$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right| = \left| \frac{dx}{dt} \right|$$

Average acceleration

$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t}$$

Instantaneous acceleration

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$$

One Dimensional Motion

- Let's focus on the simplest case: acceleration is a constant ($a=a_0$)
- Using the definitions of average acceleration and velocity, we can derive equations of motion (description of motion, velocity and position as a function of time)

$$\bar{a}_x = a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$ (If $t_f=t$ and $t_i=0$) $$a_x = \frac{v_{xf} - v_{xi}}{t} \Rightarrow v_{xf} = v_{xi} + a_x t$$

For constant acceleration, average velocity is a simple numeric average

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2} a_x t$$

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i}$$ (If $t_f=t$ and $t_i=0$) $$\bar{v}_x = \frac{x_f - x_i}{t} \Rightarrow x_f = x_i + \bar{v}_x t$$

Resulting Equation of Motion becomes

$$x_f = x_i + \bar{v}_x t = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v_{xf}(t) = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t$$

Displacement as a function of velocities and time

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinematic equations, depending on the information given to you in specific physical problems!!

How do we solve a problem using a kinematic formula under constant acceleration?

- Identify what information is given in the problem.
  - Initial and final velocity?
  - Acceleration?
  - Distance, initial position or final position?
  - Time?
- Convert the units of all quantities to SI units to be consistent.
- Identify what the problem wants
- Identify which kinematic formula is **most appropriate and easiest** to solve for what the problem wants.
  - Frequently multiple formulae can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that can be easiest to solve.
- Solve the equations for the quantity or quantities wanted.

Example 2.11

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? ⇨ As long as it takes for it to crumple.

The initial speed of the car is $v_{xi} = 100km/h = \frac{100000m}{3600s} = 28m/s$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinematic formula $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$

The acceleration is $a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{0-(28m/s)^2}{2\times 1m} = -390m/s^2$

Thus the time for air-bag to deploy is $t = \frac{v_{xf} - v_{xi}}{a} = \frac{0-28m/s}{-390m/s^2} = 0.07s$