PHYS 1443 – Section 001
Lecture #3

Wednesday, June 8, 2011

Dr. Jaehoon Yu

- One Dimensional Motion: Free Fall
- Coordinate System
- Vectors and Scalars and their operations
- Motion in Two Dimensions: Motion under constant acceleration; Projectile Motion; Maximum ranges and heights
- Newton's Laws of Motion

Today's homework is homework #2, due 10pm, Monday, June 13!!

Announcements

- Homework registration and submissions
  - 27/28 registered but only 9 completed the submission
  - The due for homework #1, the freebee, is 10pm tonight!
- Quiz #2 coming Tuesday, June 14
  - Covers: CH 1.1 – what we finish on Monday, June 13
- Reading assignment
  - CH3.9

Reminder: Special Problems for Extra Credit

- Derive the quadratic equation for $Bx^2-Cx+A=0$ ➔ 5 points
- Derive the kinematic equation $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$ from first principles and the known kinematic equations ➔ 10 points
- You must **show your work in detail** to obtain full credit
- Due at the start of the class, Thursday, June 9

Special Project for Extra Credit

- Show that the trajectory of a projectile motion is a parabola!!
  - 20 points
  - Due: next Tuesday, June 14
  - You MUST show full details of your OWN computations to obtain any credit
    - Much beyond what was covered in page 40 of this lecture note!!

Free Fall

- Free fall motion is a motion under the influence of the gravitational pull (gravity) only; Which direction is a freely falling object moving?

  Yes, down to the center of the earth!!

  - A motion under constant acceleration
  - All kinematic formula we learned can be used to solve for falling motions.
- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The magnitude of the gravitational acceleration is $g=9.80m/s^2$ on the surface of the earth, most of the time.
- The direction of gravitational acceleration is **<u>ALWAYS</u>** toward the center of the earth, which we normally call (-y); where up and down direction are indicated as the variable "y"
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80m/s^2$ when +y points upward

Example for Using 1D Kinematic Equations on a Falling object (similar to Ex. 2.16)

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

What is the acceleration in this motion? $g=-9.80m/s^2$

(a) Find the time the stone reaches at the maximum height.

What is so special about the maximum height? V=0

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00 m/s$$

Solve for t

$$t = \frac{20.0}{9.80} = 2.04s$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

(e) Find the velocity and position of the stone at t=5.00s.

Velocity

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0(m/s)$$

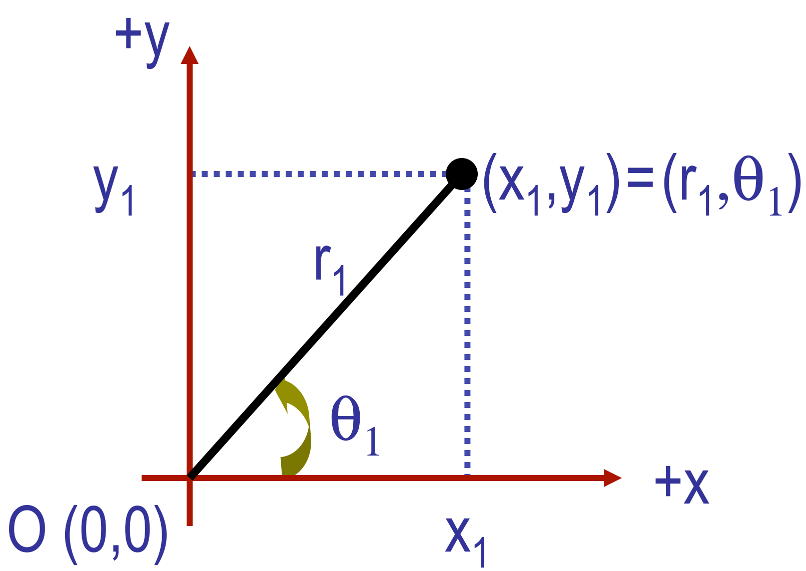
Position

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$

2D Coordinate Systems

- They make it easy and consistent to express locations or positions
- Two commonly used systems, depending on convenience, are
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in distance from the origin ® and the angle measured from the x-axis, θ (r,θ)
- Vectors become a lot easier to express and compute



How are Cartesian and Polar coordinates related?

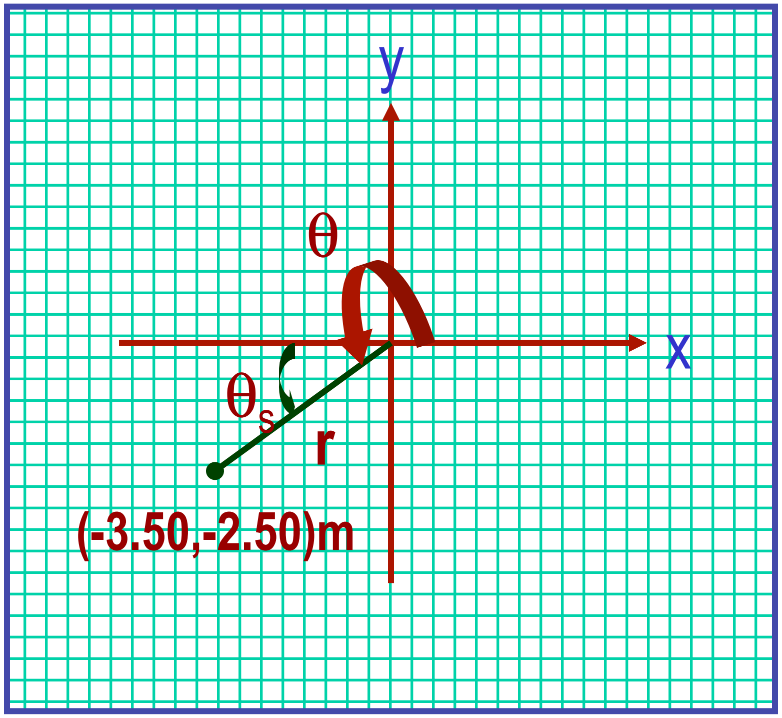
$$x_1 = r_1 \cos\theta_1 \qquad r_1 = \sqrt{\left(x_1^2 + y_1^2\right)}$$

$$y_1 = r_1 \sin\theta_1 \qquad \tan\theta_1 = \frac{y_1}{x_1}$$

$$\theta_1 = \tan^{-1}\left(\frac{y_1}{x_1}\right)$$

Example

Cartesian Coordinate of a point in the xy plane are (x,y)= (-3.50,-2.50)m. Find the equivalent polar coordinates of this point.



$$r = \sqrt{\left(x^2 + y^2\right)}$$

$$= \sqrt{\left(\left(-3.50\right)^2 + \left(-2.50\right)^2\right)}$$

$$= \sqrt{18.5} = 4.30(m)$$

$$\theta = 180 + \theta_s$$

$$\tan\theta_s = \frac{-2.50}{-3.50} = \frac{5}{7}$$

$$\theta_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$$

$$\therefore \theta = 180 + \theta_s = 180^\circ + 35.5^\circ = 216^\circ$$

Vector and Scalar

Vector quantities have both magnitudes (sizes) and directions

Force, gravitational acceleration, momentum

Normally denoted in **BOLD** letters, $\mathbf{F}$, or a letter with arrow on top $\vec{F}$

Their sizes or magnitudes are denoted with normal letters, F, or absolute values: $\left|\vec{F}\right|$ or $\left|\mathbf{F}\right|$

Scalar quantities have magnitudes only

Can be completely specified with a value and its unit

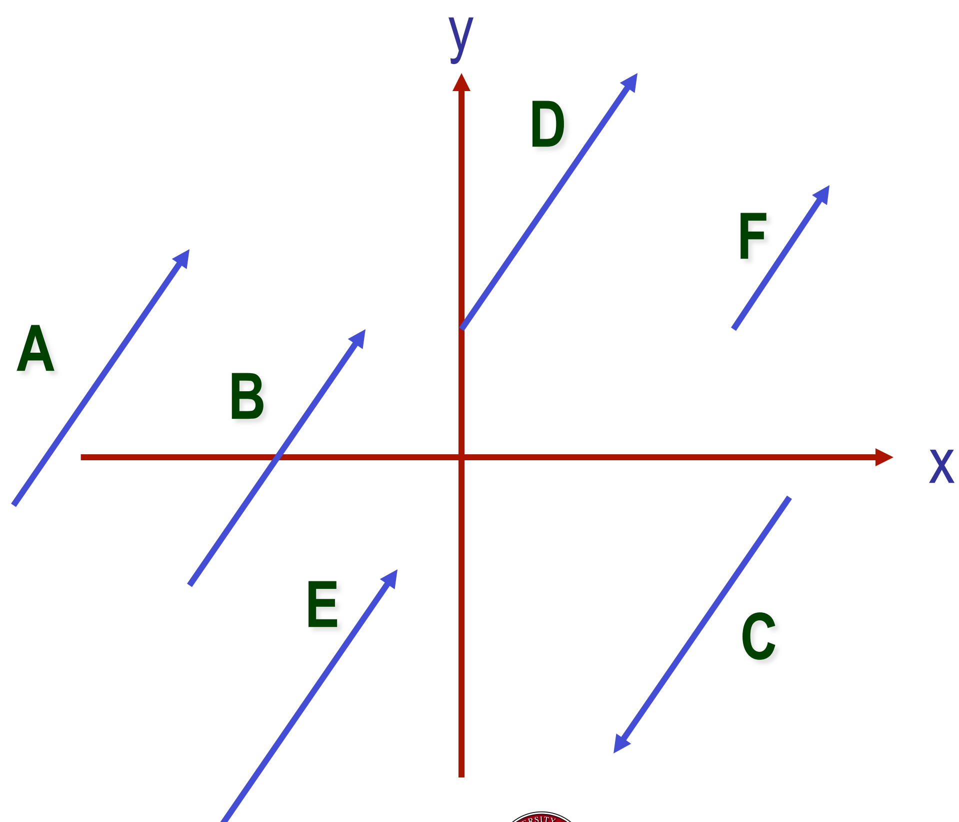
Energy, heat, mass, time

Normally denoted in normal letters, E

Both have units!!!

Properties of Vectors

- Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system!!



Which ones are the same vectors?

A=B=E=D

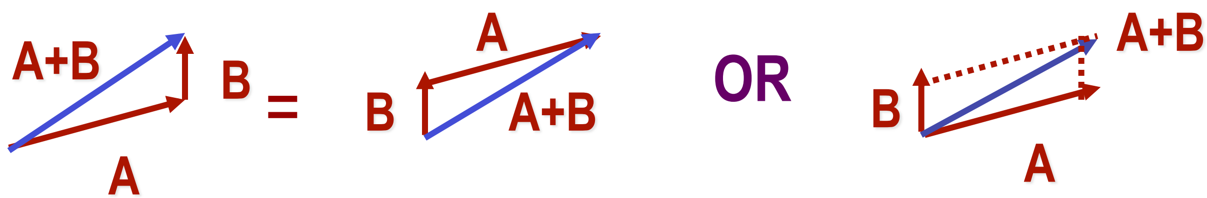
Why aren't the others?

C: The same magnitude but opposite direction: **C=-A:**A negative vector

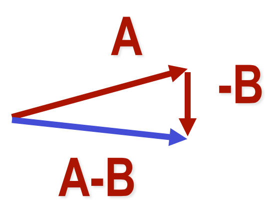
F: The same direction but different magnitude

Vector Operations

- Addition:
  - Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  - Parallelogram method: Connect the tails of the two vectors and extend
  - Addition is commutative: Changing order of operation does not affect the results **A+B=B+A**, **A+B+C+D+E=E+C+A+B+D**



- Subtraction:
  - The same as adding a negative vector: **A** - **B** = **A** + (-**B**)



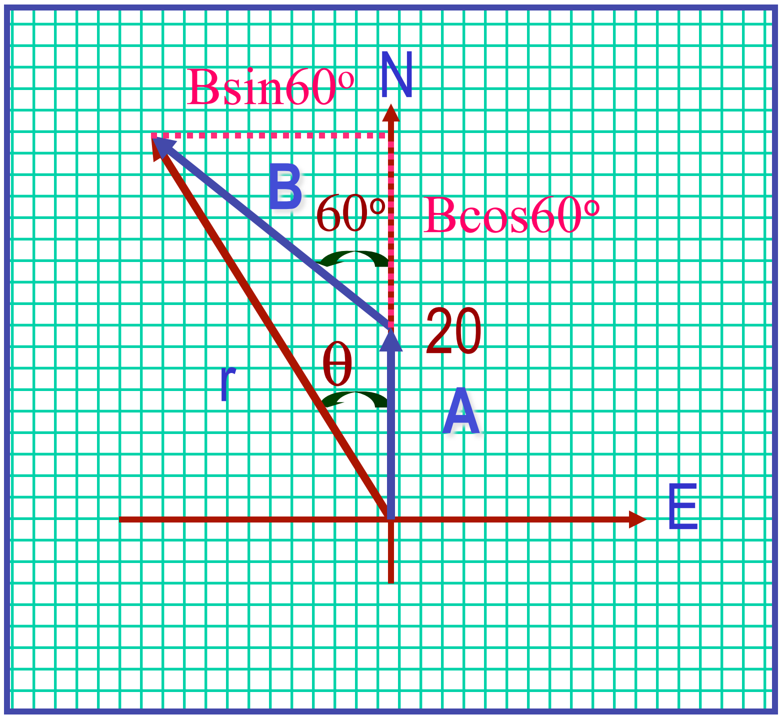
Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude **A**, **B**=2**A**

$$\boxed{|\mathcal{B}| = 2|\mathcal{A}|}$$

Example for Vector Addition

A car travels 20.0km due north followed by 35.0km in a direction 60.0° west of north. Find the magnitude and direction of resultant displacement.



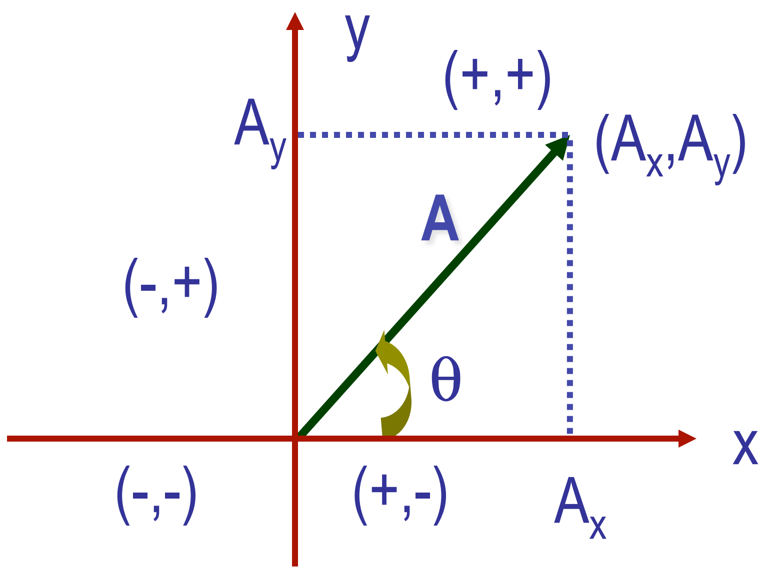
$$
\begin{aligned}
r &= \sqrt{\left(A + B\cos\theta\right)^2 + \left(B\sin\theta\right)^2} \\
&= \sqrt{A^2 + B^2\left(\cos^2\theta + \sin^2\theta\right) + 2AB\cos\theta} \\
&= \sqrt{A^2 + B^2 + 2AB\cos\theta} \\
&= \sqrt{(20.0)^2 + (35.0)^2 + 2\times 20.0\times 35.0\cos 60} \\
&= \sqrt{2325} = 48.2(km)
\end{aligned}
$$

$$
\begin{aligned}
\theta &= \tan^{-1}\frac{\left|\vec{B}\right|\sin 60}{\left|\vec{A}\right| + \left|\vec{B}\right|\cos 60} \\
&= \tan^{-1}\frac{35.0\sin 60}{20.0 + 35.0\cos 60} \\
&= \tan^{-1}\frac{30.3}{37.5} = 38.9^\circ \text{ to W wrt N}
\end{aligned}
$$

Do this using components!!

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$\left.\begin{array}{l} A_x = \left|\vec{A}\right|\cos\theta \\ A_y = \left|\vec{A}\right|\sin\theta \end{array}\right\}\text{Components}$$

$$\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2}\ \Big\}\ \text{Magnitude}$$

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right|\cos\theta\right)^2 + \left(\left|\vec{A}\right|\sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2\left(\cos^2\theta + \sin^2\theta\right)} = \left|\vec{A}\right|$$

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
- **Dimensionless**
- **Magnitudes these vectors are exactly 1**
- Unit vectors are usually expressed in **i, j, k** or

$$\vec{i},\ \vec{\mathrm{j}},\ \vec{\mathrm{k}}$$

So a vector **A** can be expressed as

$$\vec{A} = A_x \vec{i} + A_y \vec{j} = \left|\vec{A}\right| \cos\theta \vec{i} + \left|\vec{A}\right| \sin\theta \vec{j}$$

Examples of Vector Operations

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C}=\vec{A}+\vec{B}=\left(2.0\vec{i}+2.0\vec{j}\right)+\left(2.0\vec{i}-4.0\vec{j}\right)$$

$$=\left(2.0+2.0\right)\vec{i}+\left(2.0-4.0\right)\vec{j}=4.0\vec{i}\ -2.0\vec{j}\left(m\right)$$

$$\left|\vec{C}\right|=\sqrt{\left(4.0\right)^2+\left(-2.0\right)^2}$$

$$=\sqrt{16+4.0}=\sqrt{20}=4.5(m)$$

$$\theta=\tan^{-1}\frac{C_y}{C_x}=\tan^{-1}\frac{-2.0}{4.0}=-27^\circ$$

Find the resultant displacement of three consecutive displacements: **d**$_1$=(15**i**+30**j** +12**k**)cm, **d**$_2$=(23**i**+14**j** -5.0**k**)cm, and **d**$_3$=(-13**i**+15**j**)cm

$$\overrightarrow{D}=\vec{d}_1+\vec{d}_2+\vec{d}_3=\left(15\vec{i}+30\vec{j}+12\vec{k}\right)+\left(23\vec{i}+14\vec{j}-5.0\vec{k}\right)+\left(-13\vec{i}+15\vec{j}\right)$$

$$=\left(15+23-13\right)\vec{i}+\left(30+14+15\right)\vec{j}+\left(12-5.0\right)\vec{k}=25\vec{i}\ +59\vec{j}+7.0\vec{k}(cm)$$

Magnitude

$$\left|\overrightarrow{D}\right|=\sqrt{\left(25\right)^2+\left(59\right)^2+\left(7.0\right)^2}=65(cm)$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$
- Average Acceleration: $$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

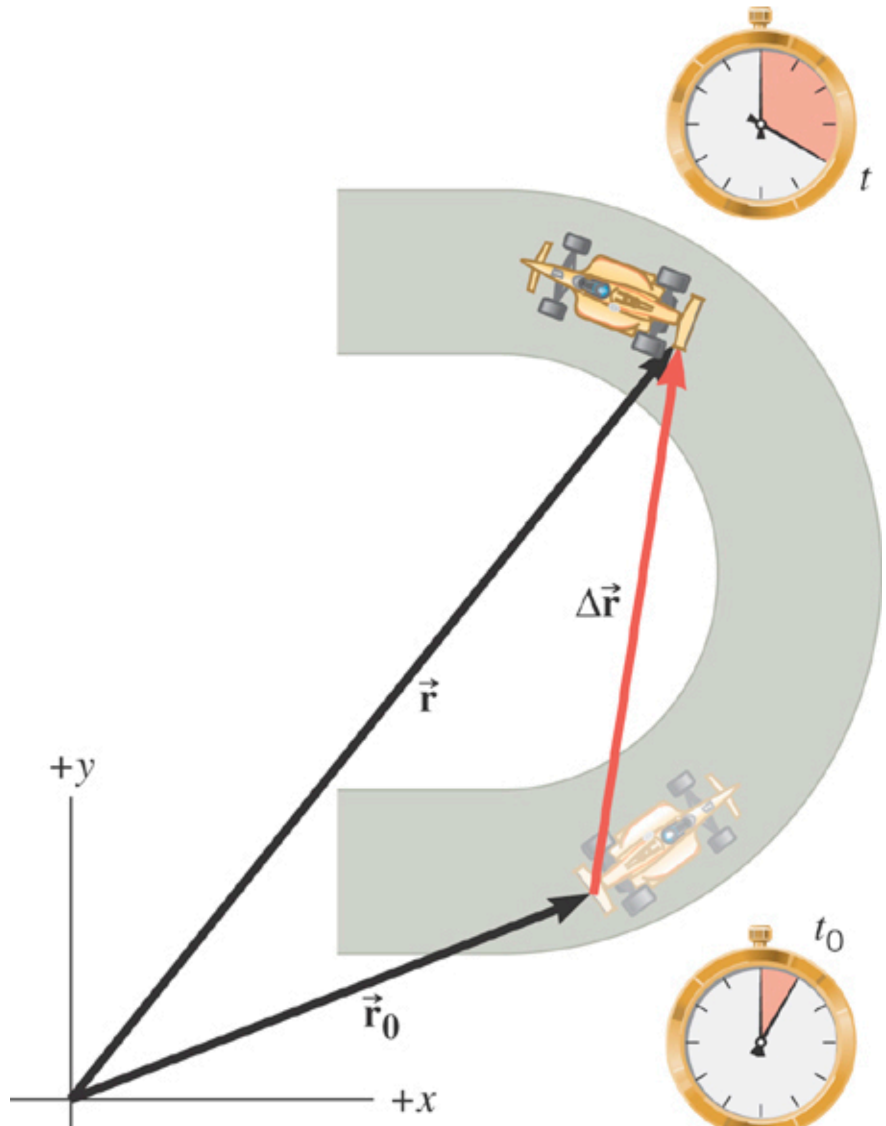
How is each of these quantities defined in 1-D?

2D Displacement

$$\vec{\mathbf{r}}_o = \text{ initial position}$$

$$\vec{\mathbf{r}} = \text{ final position}$$

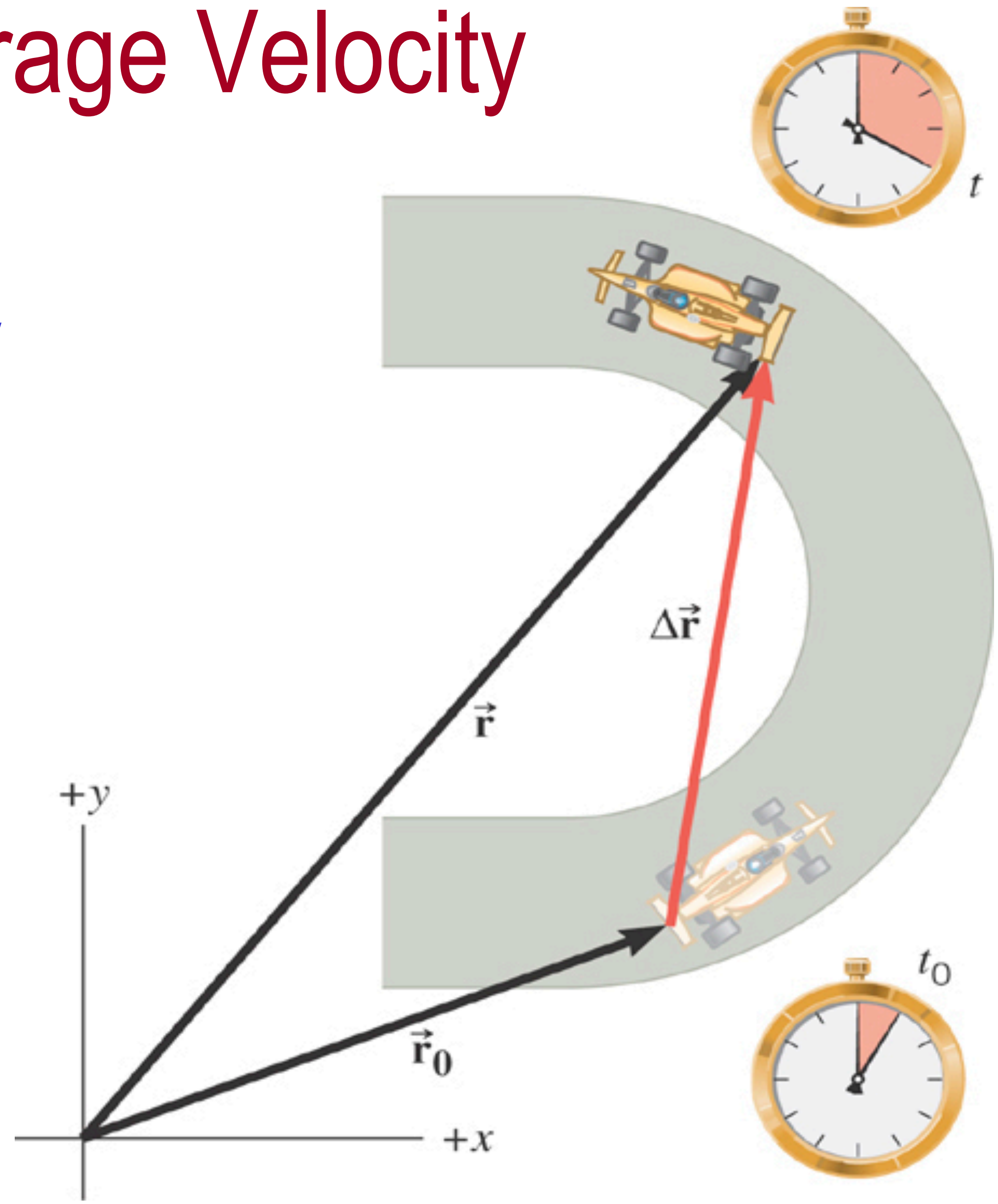
Displacement: $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_o$



2D Average Velocity

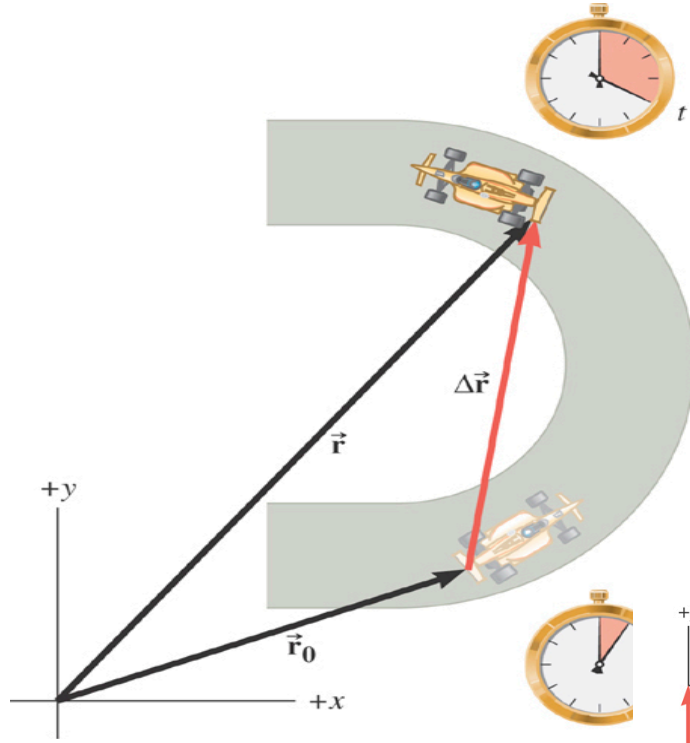
Average velocity is the displacement divided by the elapsed time.

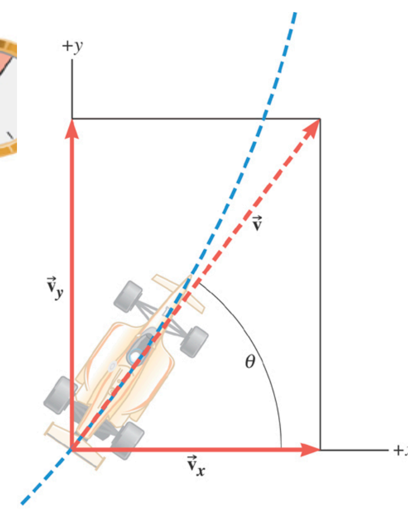
$$\overline{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_o}{t - t_o} = \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$



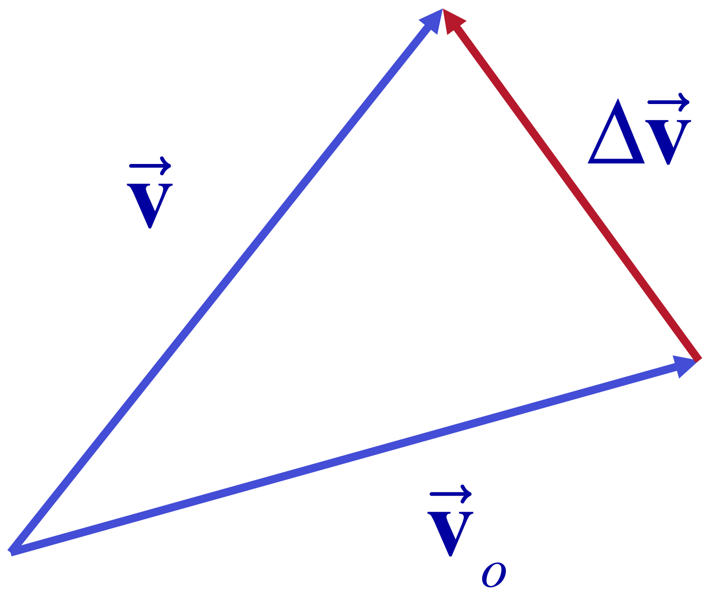
The ***instantaneous velocity*** indicates how fast the car moves and the direction of motion at each instant of time.

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$





2D Average Acceleration



$$\overline{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

Kinematic Quantities in 1D and 2D

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

What is the difference between 1D and 2D quantities?