PHYS 1443 – Section 001
Lecture #4

Thursday, June 9, 2011

Dr. Jaehoon Yu

- Motion in Two Dimensions:
  - Motion under a constant acceleration
  - Projectile Motion
  - Maximum range and Maximum height
- Newton's Laws of Motion
  - Force; Mass; Newton's 1st – 3rd Laws

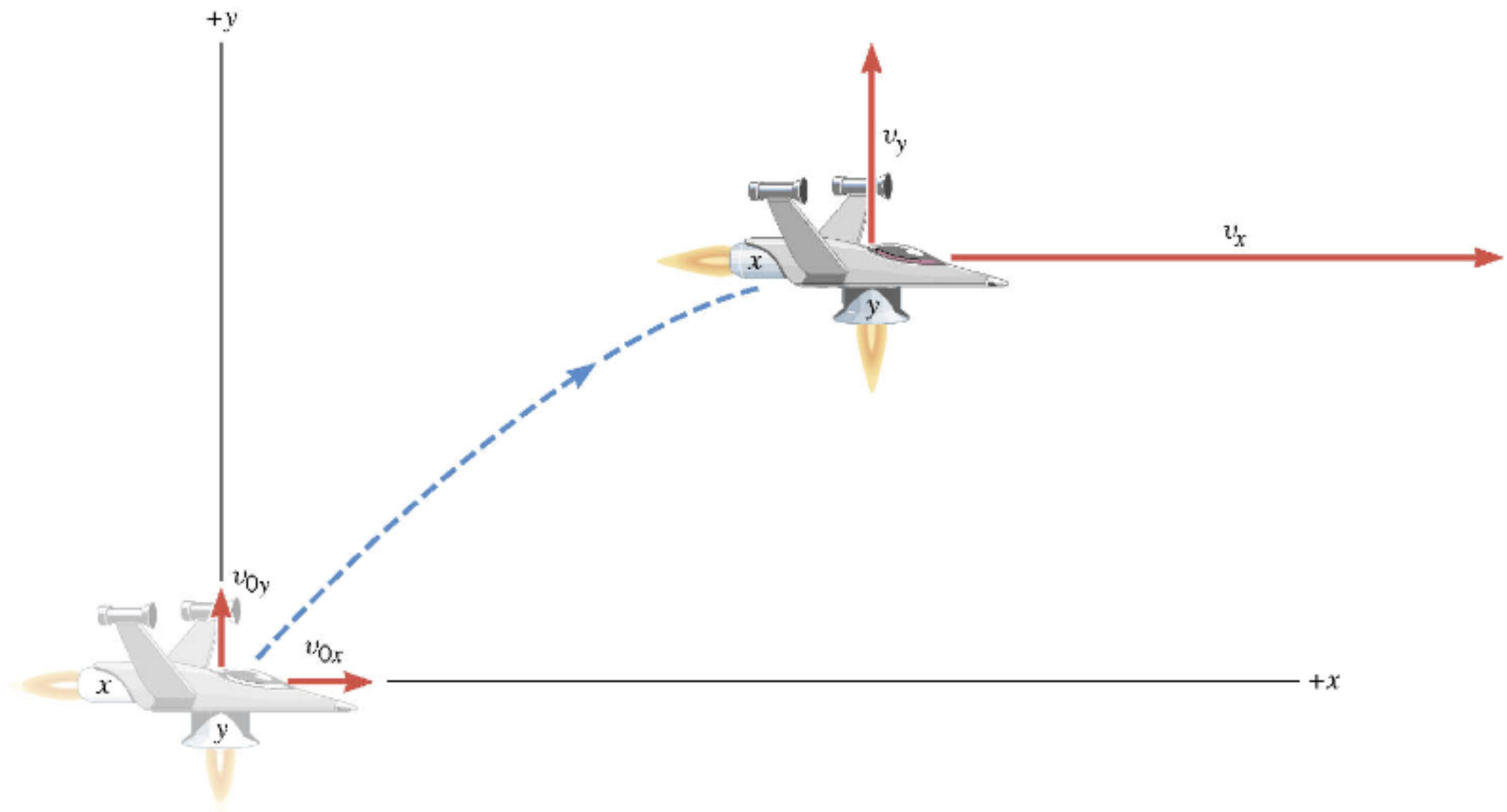
Announcements

- Homework
  - 100% of you have registered and submitted!!
  - Excellent job!
  - 6/27 already started working on HW#2, Great!!
- Quiz 1 results
  - Class average: 7.5/12
    - Equivalent to 62.5/100
  - Top score: 11.2/12
- Quiz #2 coming Tuesday, June 14
  - Covers: CH 1.1 – what we finish on Monday, June 13

Reminder: Special Project for Extra Credit

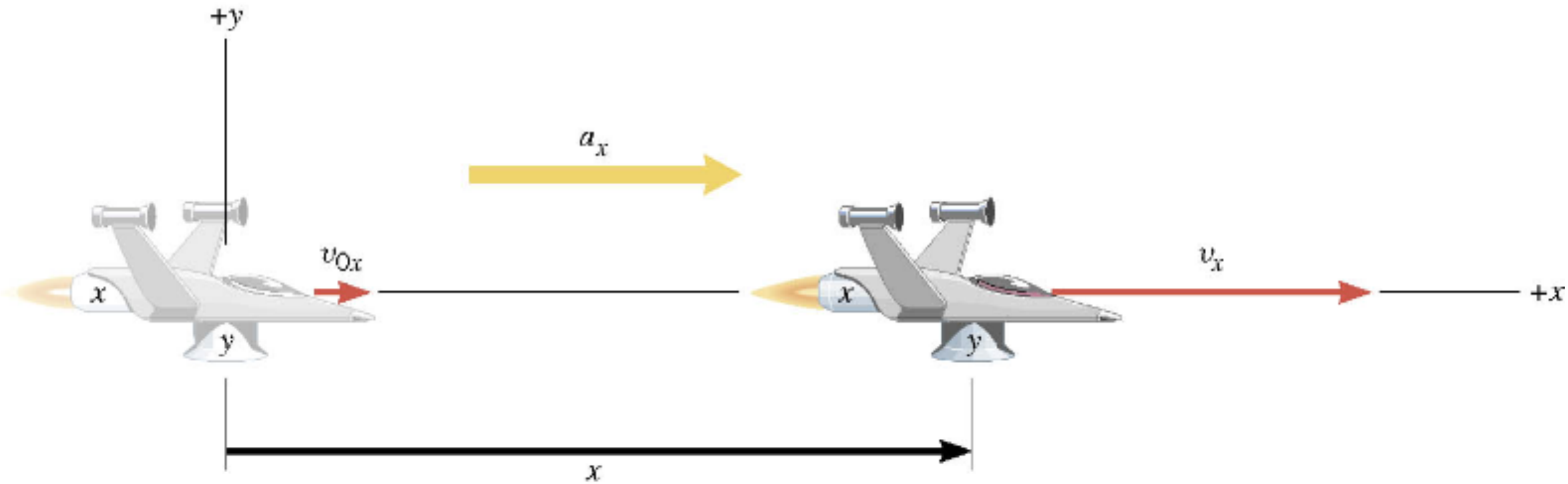
- Show that the trajectory of a projectile motion is a parabola!!
  - 20 points
  - Due: next Tuesday, June 14
  - You MUST show full details of your OWN computations to obtain any credit
    - Much beyond what was covered in page 21 of this lecture note!!

A Motion in 2 Dimension



This is a motion that could be viewed as two motions combined into one. (superposition…)

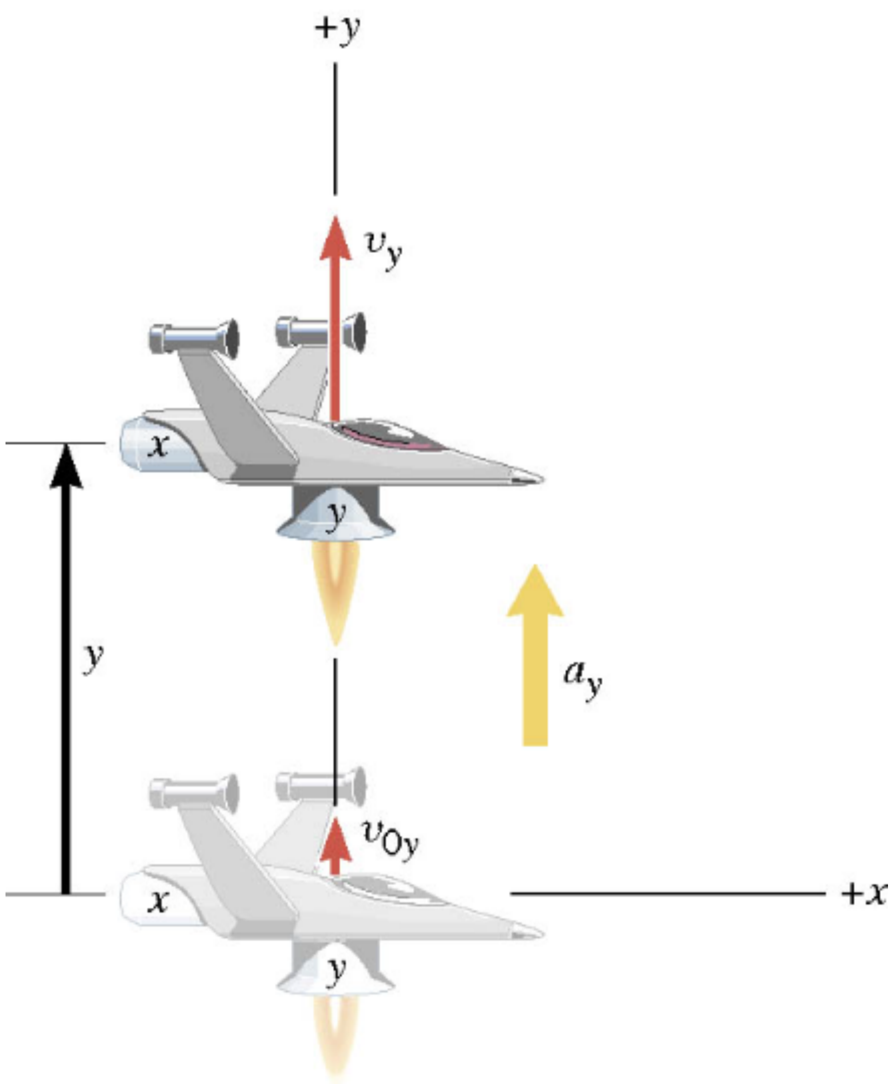
Motion in horizontal direction (x)



$$v_x = v_{xo} + a_x t \qquad x = \frac{1}{2}\left(v_{xo} + v_x\right)t$$

$$v_x^2 = v_{xo}^2 + 2a_x x \qquad x = v_{xo}t + \frac{1}{2}a_x t^2$$

Motion in vertical direction (y)



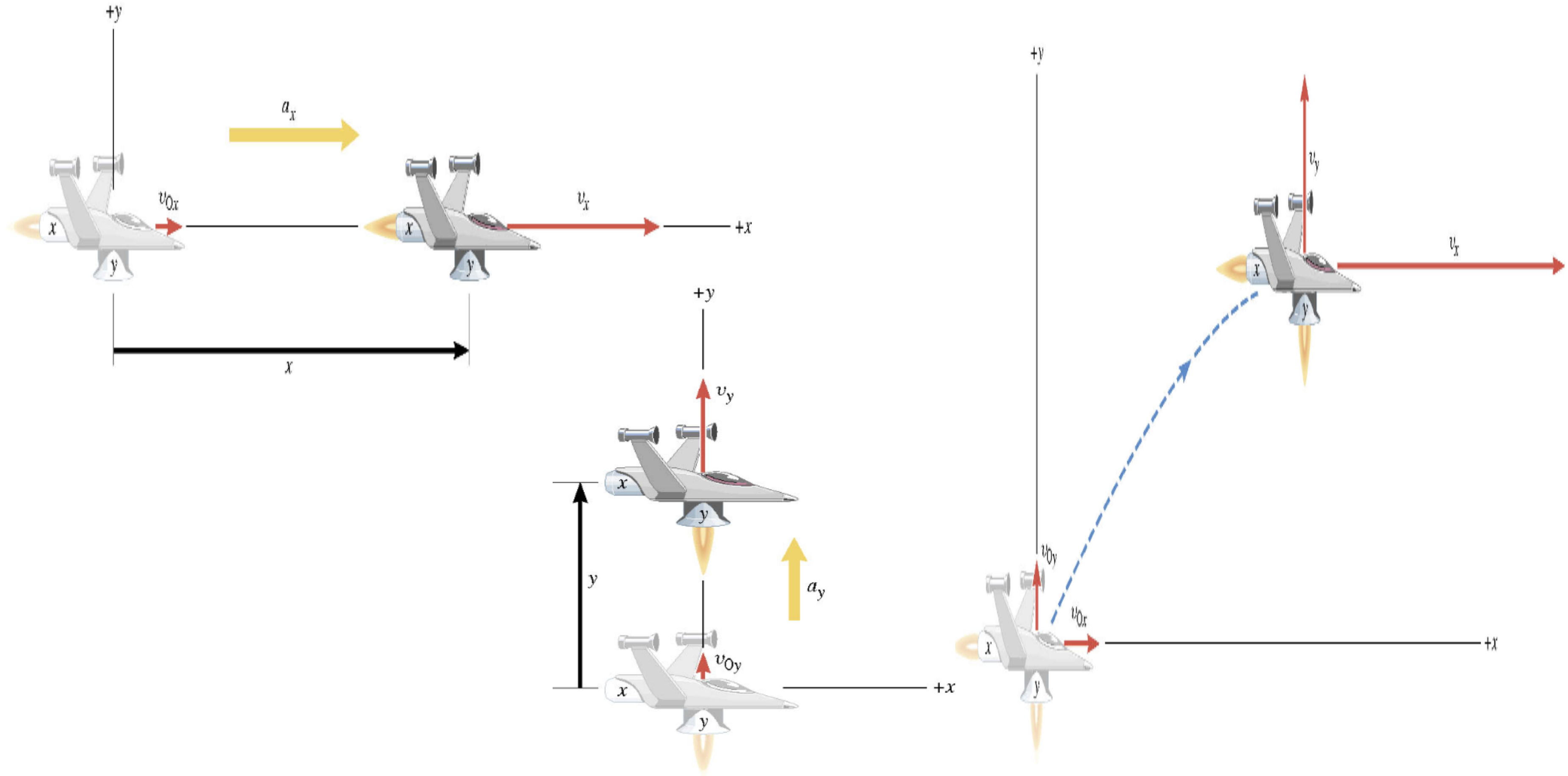
$$v_y = v_{yo} + a_y t$$

$$y = \frac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$y = v_{yo}t + \frac{1}{2}a_y t^2$$

A Motion in 2 Dimension



Imagine you add the two 1 dimensional motions on the left.
It would make up a one 2 dimensional motion on the right.

Kinematic Equations in 2-Dim

x-component

y-component

$$v_x = v_{xo} + a_x t \qquad v_y = v_{yo} + a_y t$$

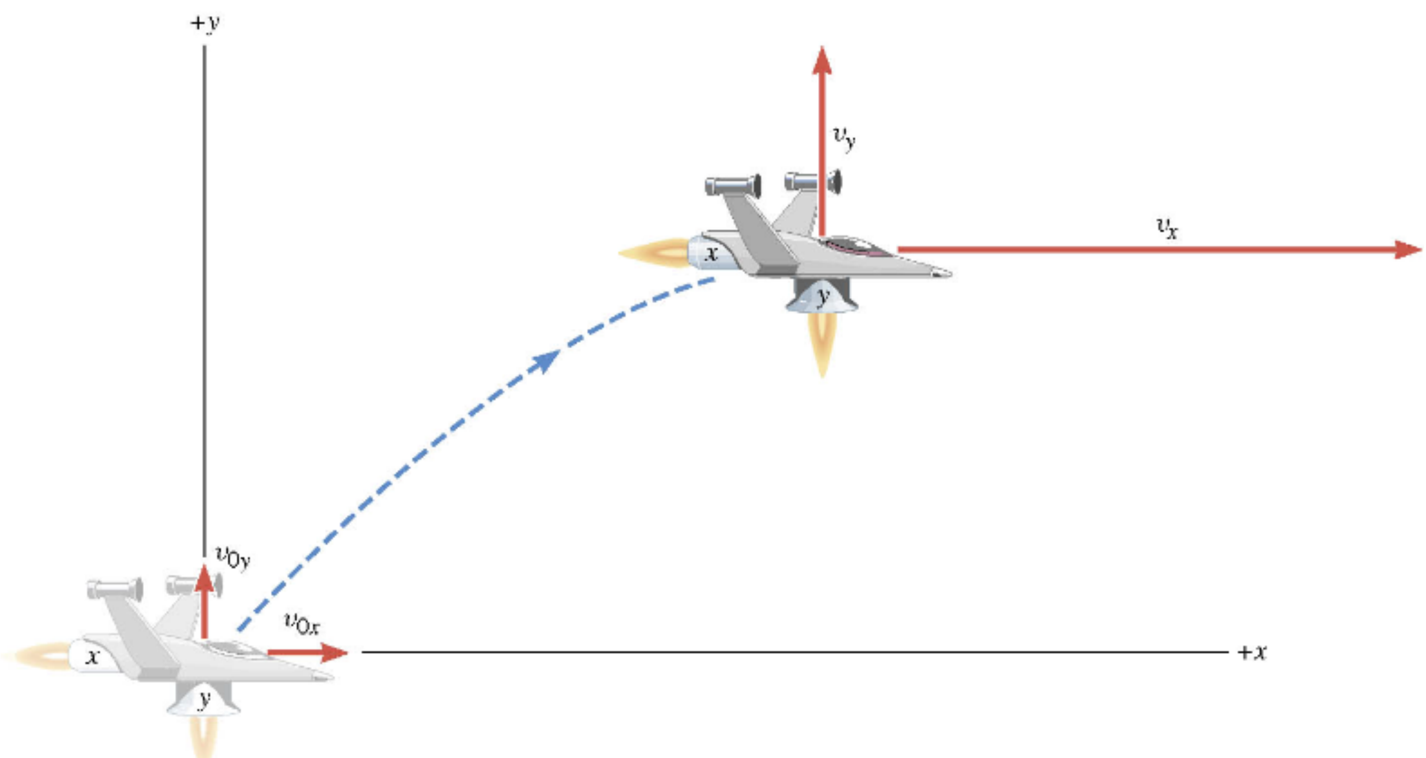
$$x = \frac{1}{2}\left(v_{xo} + v_x\right)t \qquad y = \frac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_x^2 = v_{xo}^2 + 2a_x x \qquad v_y^2 = v_{yo}^2 + 2a_y y$$

$$\Delta x = v_{xo}t + \frac{1}{2}a_x t^2 \qquad \Delta y = v_{yo}t + \frac{1}{2}a_y t^2$$

Ex. A Moving Spacecraft

In the *x* direction, the spacecraft in zero-gravity zone has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.



How do we solve this problem?

1. Visualize the problem ➔ Draw a picture!
2. Decide which directions are to be called positive (+) and negative (-).
3. Write down the values that are given for any of the five kinematic variables associated with each direction.
4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y* *separately.* Select the appropriate equation.
5. When the motion is divided into segments in time, remember that the final velocity of one time segment is the initial velocity for the next.
6. Keep in mind that there may be two possible answers to a kinematics problem.

Ex. continued

In the x direction, the spacecraft in a zero gravity zone has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the y direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) x and v_x, (b) y and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22.0 m/s	7.0 s

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14.0 m/s	7.0 s

First, the motion in x-direciton…

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$\Delta x = v_{ox}t + \tfrac{1}{2}a_x t^2$$

$$= (22\,\text{m/s})(7.0\text{ s}) + \tfrac{1}{2}(24\,\text{m/s}^2)(7.0\text{ s})^2 = +740\text{ m}$$

$$v_x = v_{ox} + a_x t$$

$$= (22\,\text{m/s}) + (24\,\text{m/s}^2)(7.0\text{ s}) = +190\,\text{m/s}$$

Now, the motion in y-direction…

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

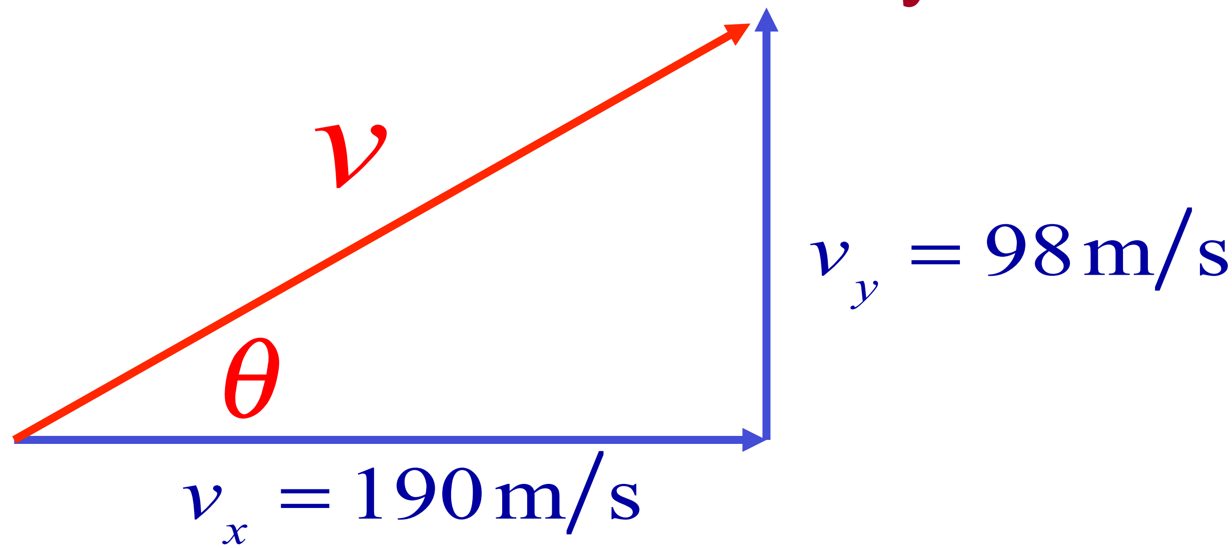
$$\Delta y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

$$= (14\,\mathrm{m/s})(7.0\ \mathrm{s}) + \tfrac{1}{2}(12\,\mathrm{m/s^2})(7.0\ \mathrm{s})^2 = +390\ \mathrm{m}$$

$$v_y = v_{oy} + a_y t$$

$$= (14\,\mathrm{m/s}) + (12\,\mathrm{m/s^2})(7.0\ \mathrm{s}) = +98\,\mathrm{m/s}$$

The final velocity…



$$v = \sqrt{(190\,\mathrm{m/s})^2 + (98\,\mathrm{m/s})^2} = 210\,\mathrm{m/s}$$

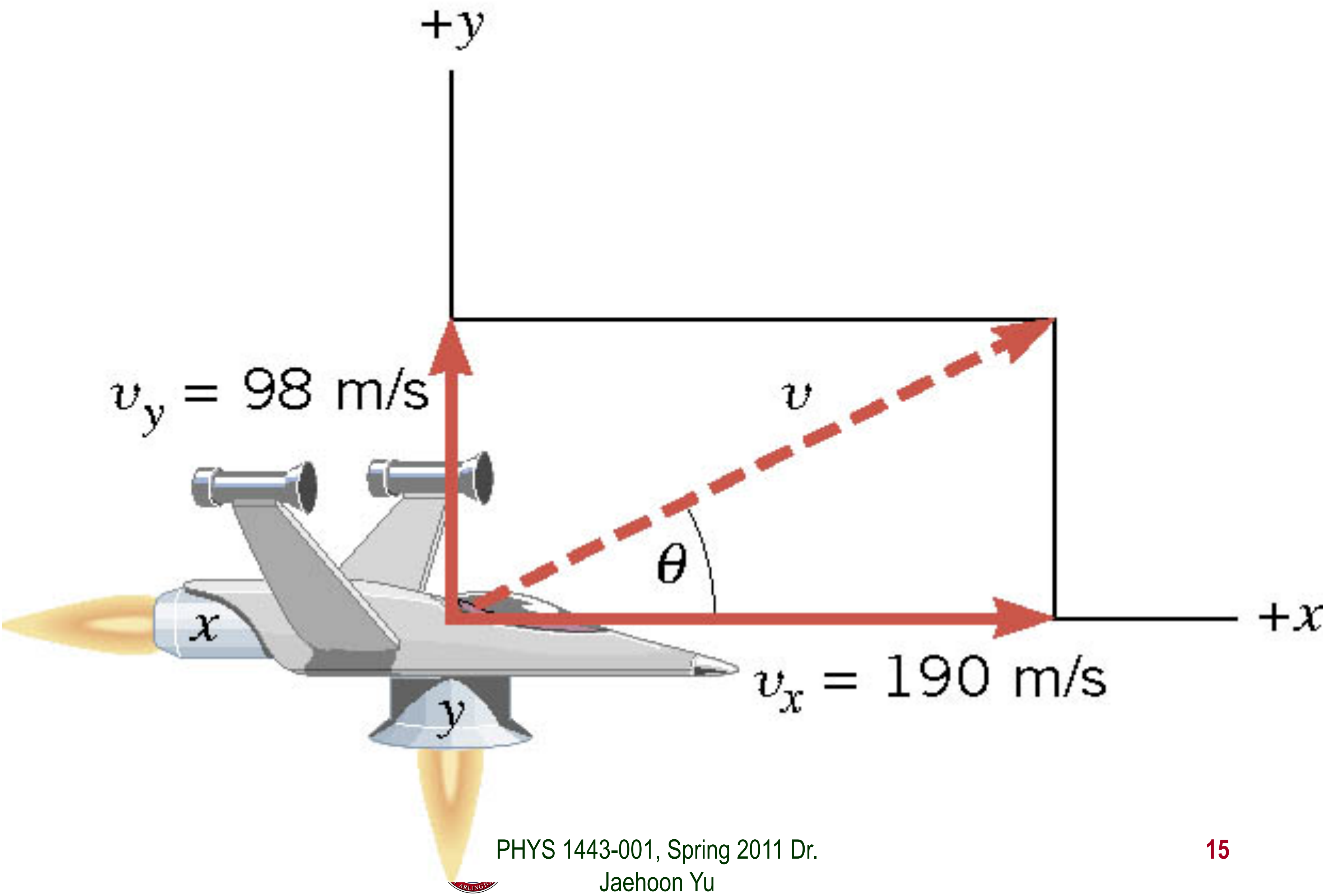
$$\theta = \tan^{-1}(98/190) = 27^\circ$$

A vector can be fully described when the magnitude and the direction are given. Any other way to describe it?

Yes, you are right! Using the components and unit vectors!!

$$\vec{v} = v_x \vec{i} + v_y \vec{j} = \left(190\vec{i} + 98\vec{j}\right) m/s$$

If we visualize the motion…



2-dim Motion Under Constant Acceleration

- Position vectors in x-y plane:

$$\vec{r}_i = x_i\vec{i} + y_i\vec{j} \qquad \vec{r}_f = x_f\vec{i} + y_f\vec{j}$$

- Velocity vectors in x-y plane:

$$\vec{v}_i = v_{xi}\vec{i} + v_{yi}\vec{j} \qquad \vec{v}_f = v_{xf}\vec{i} + v_{yf}\vec{j}$$

Velocity vectors in terms of the acceleration vector

X-comp $v_{xf} = v_{xi} + a_x t$ Y-comp $v_{yf} = v_{yi} + a_y t$

$$\vec{v}_f = \left(v_{xi} + a_x t\right)\vec{i} + \left(v_{yi} + a_y t\right)\vec{j} = \left(v_{xi}\vec{i} + v_{yi}\vec{j}\right) + \left(a_x\vec{i} + a_y\vec{j}\right)t =$$

$$= \vec{v}_i + \vec{a}t$$

2-dim Motion Under Constant Acceleration

- How are the 2D position vectors written in acceleration vectors?

Position vector components

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Putting them together in a vector form

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} =$$

$$= \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right)\vec{i} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2\right)\vec{j}$$

Regrouping the above

$$= \left(x_i\vec{i} + y_i\vec{j}\right) + \left(v_{xi}\vec{i} + v_{yi}\vec{j}\right)t + \frac{1}{2}\left(a_x\vec{i} + a_y\vec{j}\right)t^2$$

$$= \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

2D problems can be interpreted as two 1D problems in x and y

Example for 2-D Kinematic Equations

A particle starts at origin when t=0 with an initial velocity **v**=(20**i**-15**j**)m/s. The particle moves in the xy plane with $a_x=4.0m/s^2$. Determine the components of the velocity vector at any time t.

$$v_{xf}=v_{xi}+a_x t=20+4.0t\,(m/s)\quad v_{yf}=v_{yi}+a_y t=-15+0t=-15\,(m/s)$$

Velocity vector

$$\vec{v}(t)=v_x(t)\vec{i}+v_y(t)\vec{j}=(20+4.0t)\vec{i}-15\vec{j}\,(m/s)$$

Compute the velocity and the speed of the particle at t=5.0 s.

$$\vec{v}_{t=5}=v_{x,t=5}\vec{i}+v_{y,t=5}\vec{j}=(20+4.0\times5.0)\vec{i}-15\vec{j}=(40\vec{i}-15\vec{j})\;m/s$$

$$speed=\left|\vec{v}\right|=\sqrt{(v_x)^2+(v_y)^2}=\sqrt{(40)^2+(-15)^2}=43m/s$$

Example for 2-D Kinematic Eq. Cnt'd

Angle of the Velocity vector

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = \tan^{-1}\left(\frac{-3}{8}\right) = -21^\circ$$

Determine the x and y components of the particle at t=5.0 s.

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = 20\times5 + \frac{1}{2}\times4\times5^2 = 150(m)$$

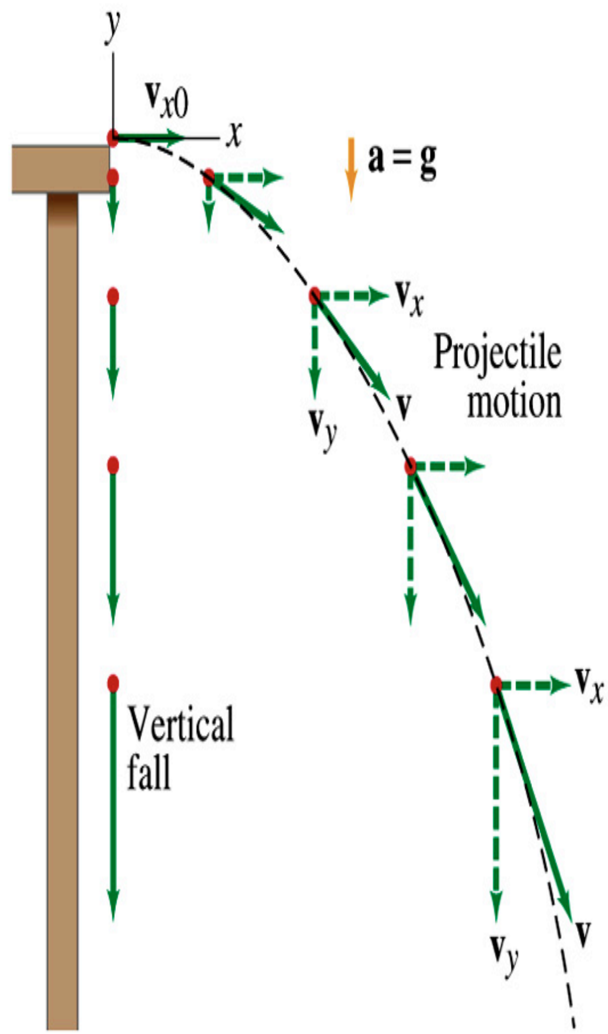
$$y_f = v_{yi}t = -15\times5 = -75\ (m)$$

Can you write down the position vector at t=5.0s?

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} = 150\vec{i} - 75\vec{j}\,(m)$$

Projectile Motion

- A 2-dim motion of an object under the gravitational acceleration with the following assumptions
  - Free fall acceleration, *g*, is constant over the range of the motion
    - $\vec{g} = -9.8\vec{j}\left(m/s^2\right)$
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity (no acceleration)
  - Vertical motion under constant acceleration (*g*)



Show that a projectile motion is a parabola!!!

x-component $v_{xi} = v_i \cos\theta_i$

y-component $v_{yi} = v_i \sin\theta_i$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = -g\vec{j}$$

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

a_x=0 $\Rightarrow$

$$x_f = v_{xi}t = v_i \cos\theta_i t$$

$$t = \frac{x_f}{v_i \cos\theta_i}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)\,t^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

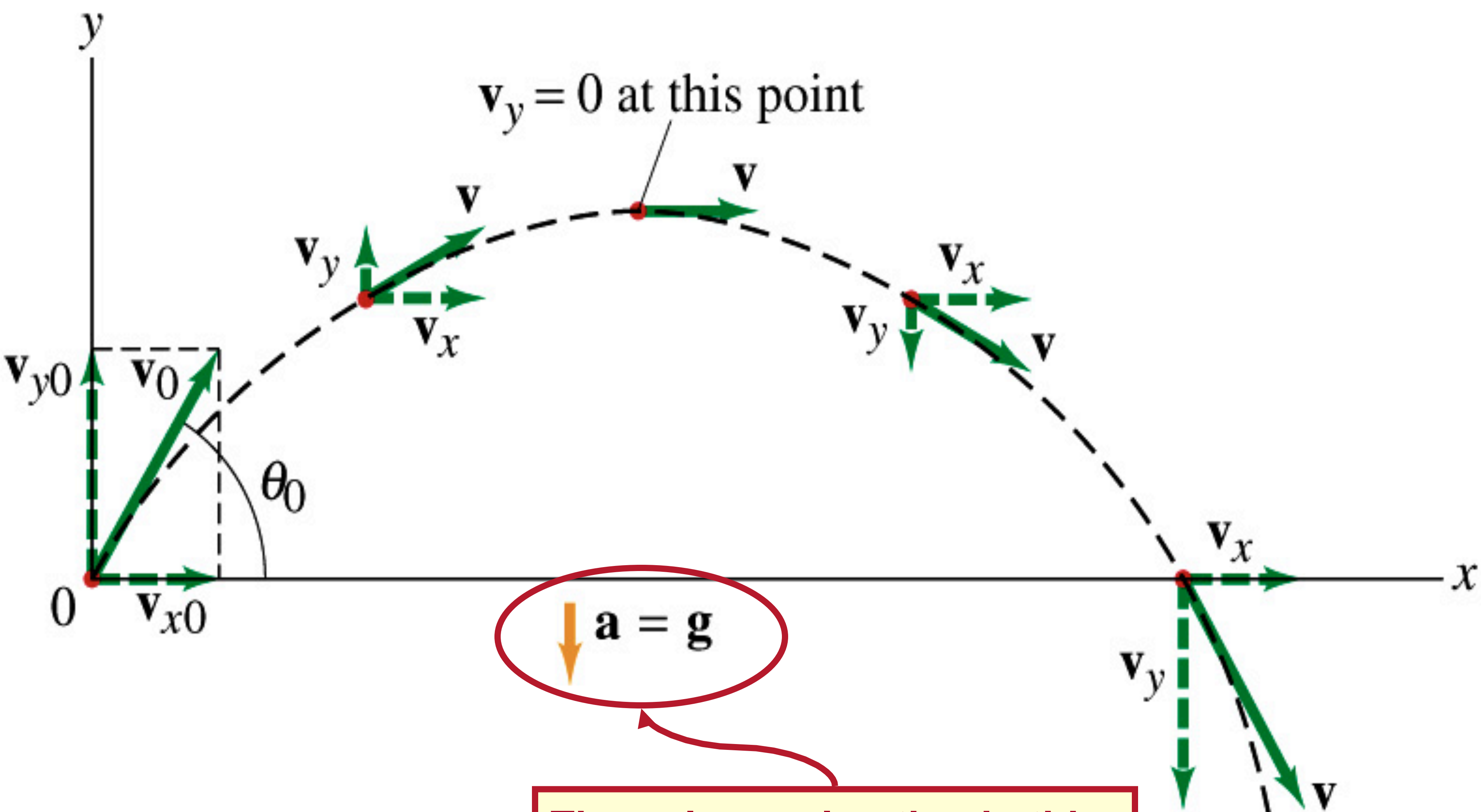
Plug t into the above $\Rightarrow$

$$y_f = v_i \sin\theta_i \left(\frac{x_f}{v_i \cos\theta_i}\right) - \frac{1}{2}g\left(\frac{x_f}{v_i \cos\theta_i}\right)^2$$

$$y_f = x_f \tan\theta_i - \left(\frac{g}{2v_i^2 \cos^2\theta_i}\right)x_f^2$$

What kind of parabola is this?

Projectile Motion



The only acceleration in this motion. It is a constant!!

Example for Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance the ball is from the original position when landed.

Which component determines the flight time and the distance?

Flight time is determined by the y component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

So the possible solutions are…

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\,\text{sec}$$

$$\therefore\ t \approx 8\,\text{sec}$$

Why isn't 0 the solution?

Distance is determined by the x component in 2-dim, because the ball is at $y=0$ position when it completed it's flight.

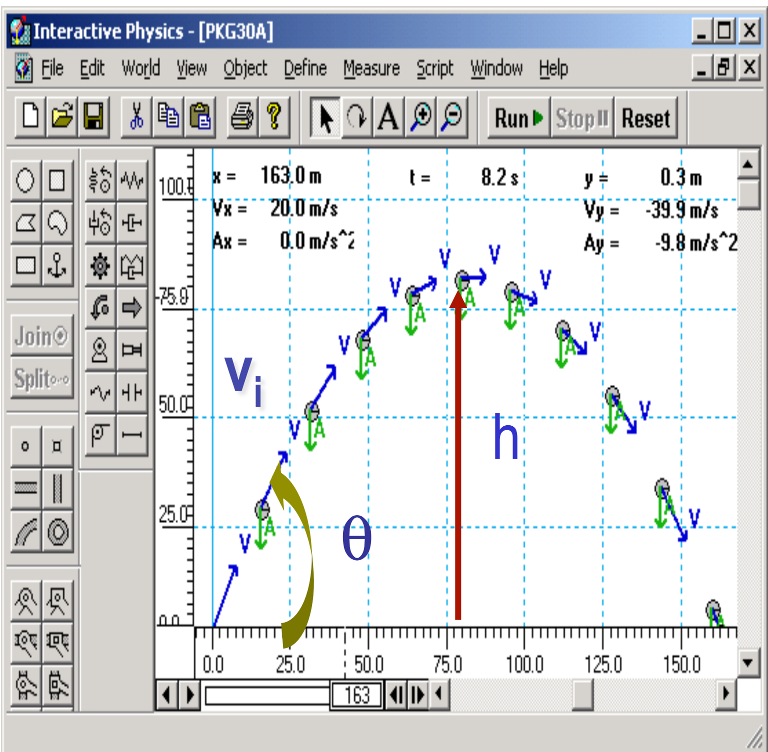
$$x_f = v_{xi}t = 20 \times 8 = 160\,(m)$$

Horizontal Range and Max Height

- Based on what we have learned, one can analyze a projectile motion in more detail
  - Maximum height an object can reach
  - Maximum range

What happens at the maximum height?

At the maximum height the object's vertical motion stops to turn around!!



$$v_{yf} = v_{yi} + a_y t$$

$$= v_i \sin\theta_i - g t_A = 0$$

Solve for t_A

$$\therefore t_A = \frac{v_i \sin\theta_i}{g}$$

Horizontal Range and Max Height

Since no acceleration is in x direction, it still flies even if v_y=0.

$$R = v_{xi}t = v_{xi}(2t_A) = 2v_i \cos\theta_i \left(\frac{v_i \sin\theta_i}{g}\right)$$

Range

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g}\right)$$

$$y_f = h = v_{yi}t + \frac{1}{2}(-g)t^2 = v_i \sin\theta_i \left(\frac{v_i \sin\theta_i}{g}\right) - \frac{1}{2} g \left(\frac{v_i \sin\theta_i}{g}\right)^2$$

Height

$$y_f = h = \left(\frac{v_i^2 \sin^2\theta_i}{2g}\right)$$

Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left(\frac{v_i^2 \sin^2 \theta_i}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_i = 90^o$!!!

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i = 90^o$, i.e., $\theta_i = 45^o$!!!

Example for a Projectile Motion

- A stone was thrown upward from the top of a cliff at an angle of 37° to horizontal with initial speed of 65.0m/s. If the height of the cliff is 125.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yi} = v_i \sin\theta_i = 65.0 \times \sin 37^\circ = 39.1 m/s$$

$$y_f = -125.0 = v_{yi}t - \frac{1}{2}gt^2$$

Becomes

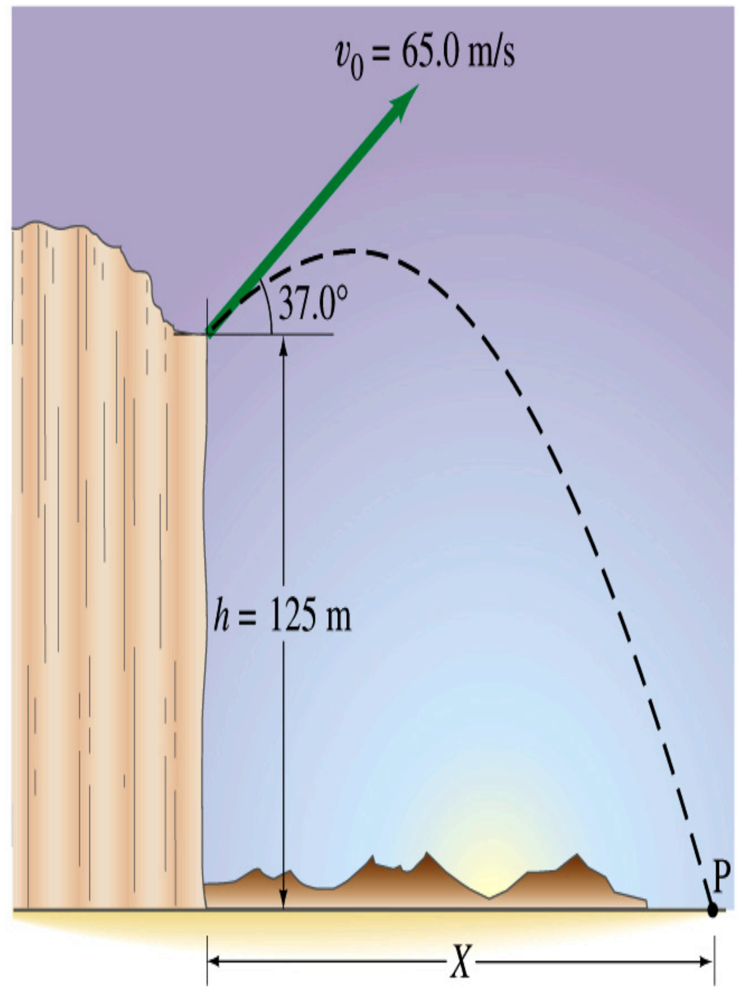
$$gt^2 - 78.2t - 250 = 9.80t^2 - 78.2t - 250 = 0$$

$$t = \frac{78.2 \pm \sqrt{(-78.2)^2 - 4 \times 9.80 \times (-250)}}{2 \times 9.80}$$

$$t = -2.43s \text{ or } t = 10.4s$$

$$t = 10.4s$$

Since negative time represents the time with the stone on the ground if it were thrown from the ground.



Example cont'd

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yf} = v_{yi} - gt = v_i \sin\theta_i - gt = 39.1 - 9.80 \times 10.4 = -62.8 m/s$$

$$|v| = \sqrt{v_{xf}^{\ 2} + v_{yf}^{\ 2}} = \sqrt{51.9^2 + (-62.8)^2} = 81.5 m/s$$

- What are the maximum height and the maximum range of the stone?

Do these yourselves at home for fun!!!