PHYS 1443 – Section 001
Lecture #5

Monday, June 13, 2011

Dr. Jaehoon Yu

- Newton's Laws of Motion
  - Force
  - Newton's Law of Inertia
  - Mass and Newton's second law of motion
  - Newton's third law of motion
  - Categories of Forces
- Free Body Diagram

Today's homework is homework #3, due 10pm, Thursday, June 16!!

Announcements

- Quiz #2 tomorrow, Tuesday, June 14
  - Beginning of the class
  - Covers: CH 1.1 – what we finish today (CH4.5)

Reminder: Special Project for Extra Credit

- Show that the trajectory of a projectile motion is a parabola!!
  - 20 points
  - Due: tomorrow, Tuesday, June 14
  - You MUST show full details of your OWN computations to obtain any credit
    - Much beyond what was covered in page 21 of this lecture note!!

Example for a Projectile Motion

- A stone was thrown upward from the top of a cliff at an angle of 37° to horizontal with initial speed of 65.0m/s. If the height of the cliff is 125.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos\theta_i = 65.0\times\cos 37^\circ = 51.9 m/s$$

$$v_{yi} = v_i \sin\theta_i = 65.0\times\sin 37^\circ = 39.1 m/s$$

$$y_f = -125.0 = v_{yi}t - \frac{1}{2}gt^2$$

Becomes

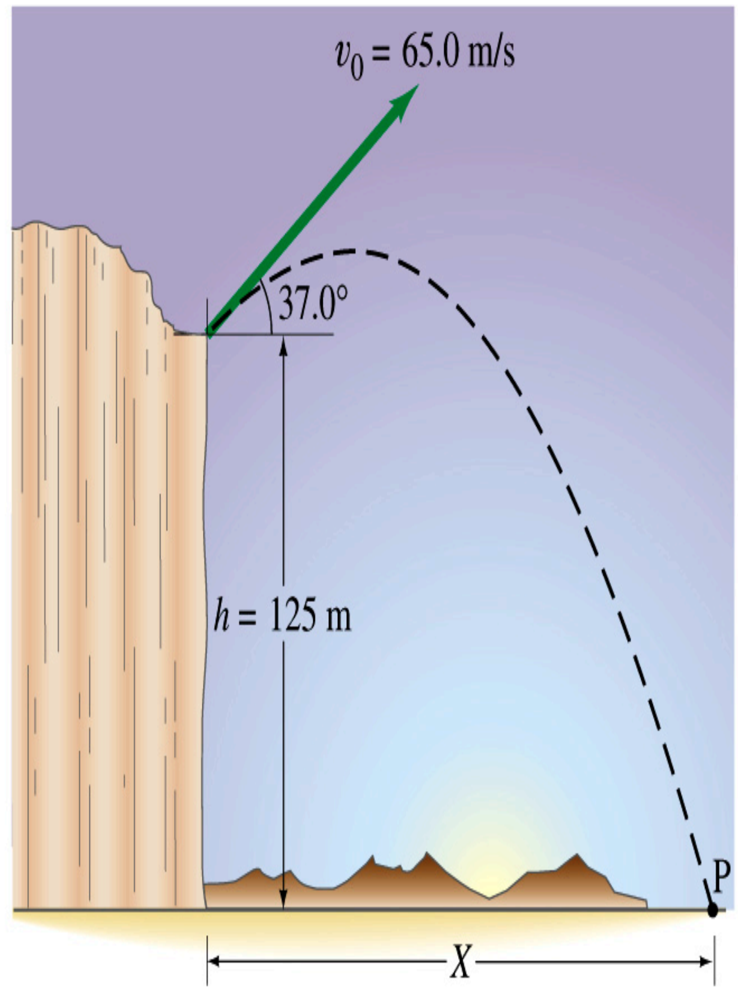
$$gt^2 - 78.2t - 250 = 9.80t^2 - 78.2t - 250 = 0$$

$$t = \frac{78.2 \pm \sqrt{(-78.2)^2 - 4\times 9.80\times(-250)}}{2\times 9.80}$$

$$t = -2.43s \text{ or } t = 10.4s$$

$$t = 10.4s$$

Since negative time represents the time with the stone on the ground if it were thrown from the ground.



Example cont'd

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yf} = v_{yi} - gt = v_i \sin\theta_i - gt = 39.1 - 9.80 \times 10.4 = -62.8 m/s$$

$$|v| = \sqrt{v_{xf}^{\ 2} + v_{yf}^{\ 2}} = \sqrt{51.9^2 + (-62.8)^2} = 81.5 m/s$$

- What are the height and the range of the stone?

Do these yourselves!!!

Force

We've been learning kinematics; describing motion without understanding what the cause of the motion is. Now we are going to learn dynamics!!

Can someone tell me what FORCE is?

FORCE is what causes an object to move.

The above statement is not entirely correct. Why?

Because when an object is moving with a constant velocity no force is exerted on the object!!!

FORCE is what causes changes to the velocity of an object!!

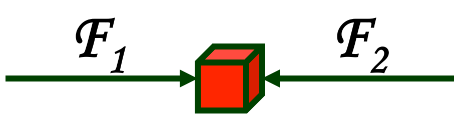
What does this statement mean?

When there is force, there is change of velocity!!

What does force cause? It causes an acceleration.!!

What happens if there are several forces exert on an object?

Forces are vector quantities, so vector sum of all forces, the NET FORCE, determines the direction of the resulting acceleration of the object.



NET FORCE, $F = F_1 + F_2$

When the net force on an object is **0***, it has constant velocity and is at its equilibrium!!*

More Forces

There are various classes of forces

Contact Forces: Forces exerted by physical contact of objects

Examples of Contact Forces: Baseball hit by a bat, Car collisions

Field Forces: Forces exerted without physical contact of objects

Examples of Field Forces: Gravitational Force, Electro-magnetic force

What are possible ways to measure the strength of a force?

A calibrated spring whose length changes linearly with the force exerted .

Forces are vector quantities, so the addition of multiple forces must be done following the rules of vector additions.

Newton's First Law and Inertial Frames

Aristotle (384-322BC): *The natural state of a body is rest. Thus force is required to move an object. To move faster, ones needs larger forces.*

Galileo's statement on natural states of matter: *Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed!!*

Galileo's statement is formulated by Newton into the **1st law of motion (Law of Inertia)**: *In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.*

What does this statement tell us?

- When no net force is exerted on an object, the acceleration of the object is 0.
- Any isolated object, the object that does not interact with its surroundings, is either at rest or moving at a constant velocity.
- Objects would like to keep its current state of motion, as long as there are no net force that interferes with the motion. This tendency is called the Inertia.

A frame of reference that is moving at a constant velocity is called the ***Inertial Frame***

Is a frame of reference with an acceleration an ***Inertial Frame?***

NO!

Mass

Mass: *A measure of the inertia of an object* or *the quantity of matter*

- Independent of the object's surroundings: The same no matter where you go.
- Independent of the method of measurement: The same no matter how you measure it.

The heavier the object, the bigger the inertia !!

It is harder to make changes of motion of a heavier object than a lighter one.

The same forces applied to two different masses result in different acceleration depending on the mass.

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1}$$

Note that the mass and the weight of an object are two different quantities!!

Weight of an object is the magnitude of the gravitational force exerted on the object.

Not an inherent property of an object!!!

Weight will change if you measure on the Earth or on the moon but the mass won't!!

Unit of mass? kg

Newton's Second Law of Motion

The acceleration of an object is directly proportional to the net force exerted on it and is inversely proportional to the object's mass.

How do we write the above statement in a mathematical expression?

$$\vec{a} = \frac{\sum_i \vec{F}_i}{m}$$

From this we obtain

$$\sum_i \vec{F}_i = m\vec{a}$$

Newton's 2nd Law of Motion

Since it's a vector expression, each component must also satisfy:

$$\sum_i F_{ix} = ma_x \qquad \sum_i F_{iy} = ma_y \qquad \sum_i F_{iz} = ma_z$$

Unit of the Force

From the vector expression in the previous page, what do you conclude the dimension and the unit of the force are?

$$\sum_i \vec{F_i} = m\vec{a}$$

The dimension of force is $[m][a] = [M][LT^{-2}]$

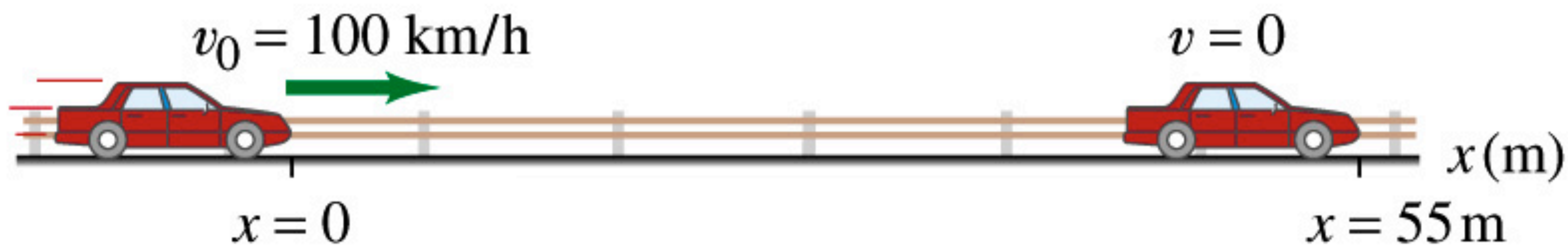
The unit of force in SI is $[Force] = [m][a] = [M][LT^{-2}] = (\mathrm{kg})\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right) = kg \cdot m/s^2$

For ease of use, we define a new derived unit called, Newton (N)

$$1N \equiv 1kg \cdot m/s^2 \approx \frac{1}{4} lbs$$

Example 4.3

What constant net force is required to bring a 1500kg car to rest from a speed of 100km/h within a distance of 55m?



What do we need to know to figure out the force? **Acceleration!!**

What are given? Initial speed: $v_{xi} = 100km/h = 28m/s$ Final speed: $v_{xf} = 0m/s$

Displacement: $\Delta x = x_f - x_i = 55m$

This is a one dimensional motion. Which kinetic formula do we use to find acceleration?

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Acceleration →

$$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{-(28m/s)^2}{2(55m)} = -7.1m/s^2$$

Thus, the force needed to stop the car is

$$F_x = ma_x = 1500kg \times (-7.1m/s^2) = -1.1 \times 10^4 N$$

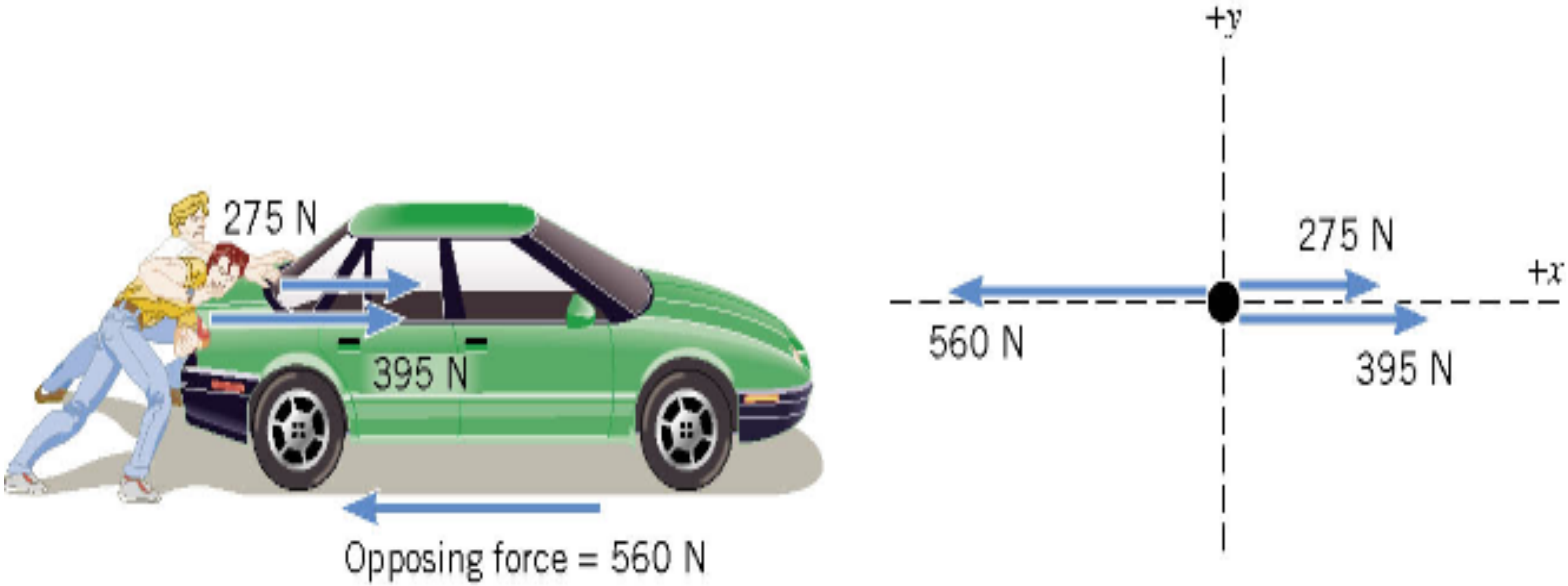
Given the force how far does the car move till it stops?

$$\Delta x = x_f - x_i = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{m(v_{xf}^2 - v_{xi}^2)}{2ma_x} = \frac{m(v_{xf}^2 - v_{xi}^2)}{2F_x}$$

- Linearly proportional to the mass of the car
- Squarely proportional to the speed of the car
- Inversely proportional to the force by the brake

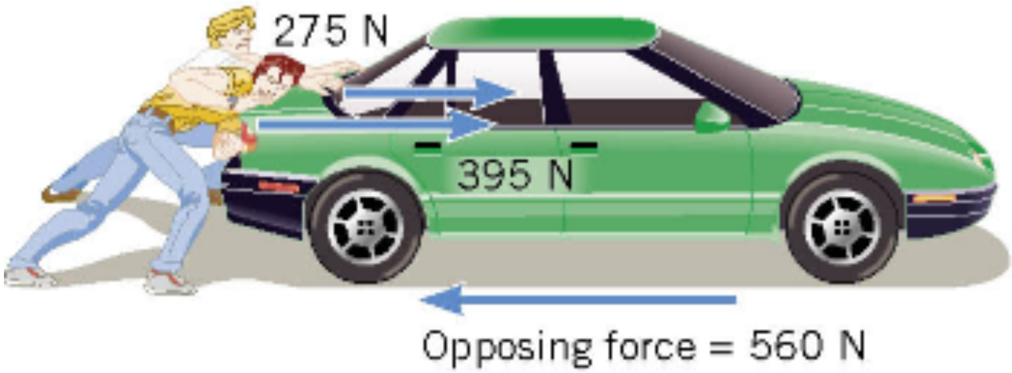
Free Body Diagram

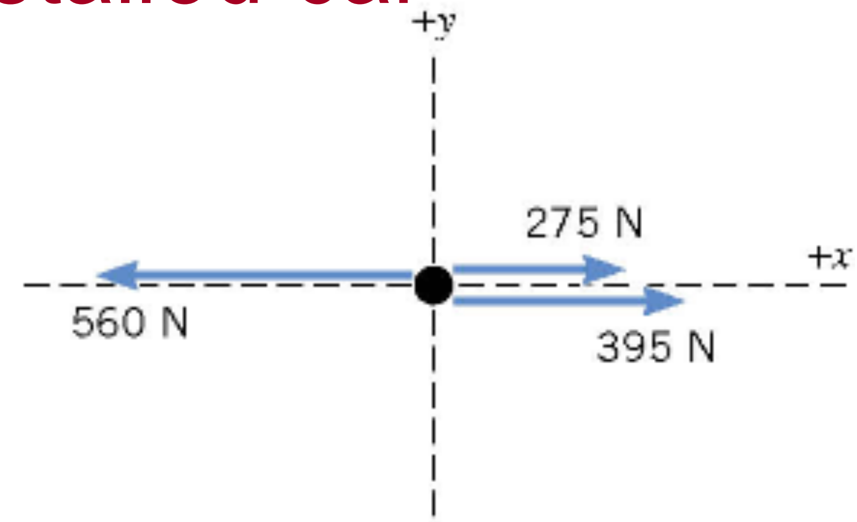
A ***free-body-diagram*** is a diagram that represents the object and the forces that act on it.



(a) (b) Free-body diagram of the car

Ex. Pushing a stalled car





What is the net force in this example?

F= 275 N + 395 N – 560 N = +110 N

Which direction? The + x axis of the coordinate system.

What is the acceleration the car receives?

If the mass of the car is 1850 kg, then by Newton's second law, the acceleration is

$$\sum \vec{F} = m\vec{a}$$

Since the motion is in 1 dimension

$$\sum F = ma$$

Now we solve this equation for a

$$a = \frac{\sum F}{m} = \frac{+110\,\text{N}}{1850\,\text{kg}} = +0.059\,\text{m/s}^2$$

Vector Nature of the Force

The direction of the force and the acceleration vectors can be taken into account by using x and y components.

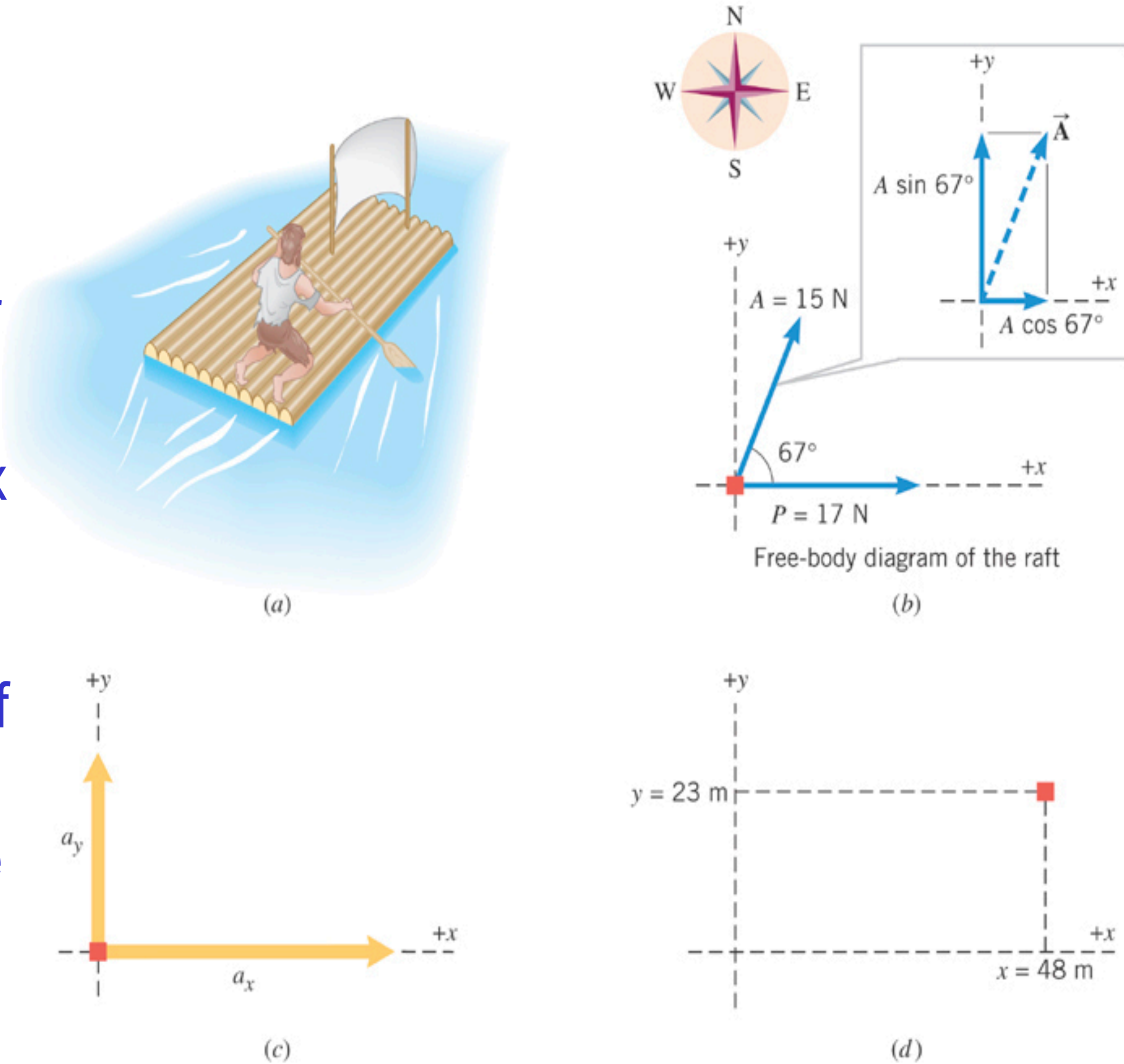
$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

is equivalent to

$$\sum F_y = ma_y \qquad \sum F_x = ma_x$$

Ex. Stranded man on a raft

A man is stranded on a raft (mass of man and raft = 1300kg)m as shown in the figure. By paddling, he causes an average force **P** of 17N to be applied to the raft in a direction due east (the +x direction). The wind also exerts a force **A** on the raft. This force has a magnitude of 15N and points 67° north of east. Ignoring any resistance from the water, find the x and y components of the rafts acceleration.



First, let's compute the net force on the raft as follows:

Force	*x* component	*y* component
$\vec{\mathbf{P}}$ $\vec{\mathbf{A}}$	+17 N $+(15N)\cos67^{o}$	0 N $+(15N)\sin67^{o}$
$\vec{F}=\vec{P}+\vec{A}$	$+17+15\cos67^{o}=$ +23(N)	$+15\sin67^{o}=$ +14(N)

Now compute the acceleration components in x and y directions!!

$$a_x = \frac{\sum F_x}{m} = \frac{+23\ \mathrm{N}}{1300\ \mathrm{kg}} = +0.018\ \mathrm{m/s^2}$$

$$a_y = \frac{\sum F_y}{m} = \frac{+14\ \mathrm{N}}{1300\ \mathrm{kg}} = +0.011\ \mathrm{m/s^2}$$

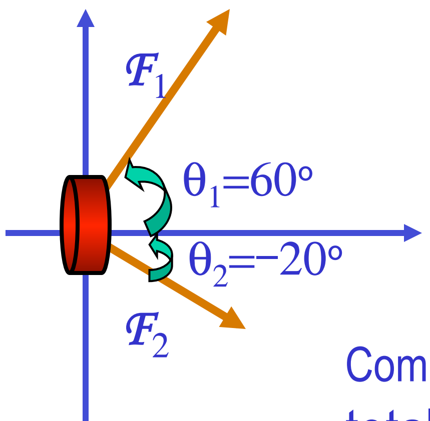
And put them all together for the overall acceleration:

$$\vec{a} = a_x\vec{i} + a_y\vec{j} =$$

$$\left(0.018\vec{i} + 0.011\vec{j}\right) m/s^2$$

Example for Newton's 2nd Law of Motion

Determine the magnitude and direction of the acceleration of the puck whose mass is 0.30kg and is being pulled by two forces, **F1** and **F2,** as shown in the picture, whose magnitudes of the forces are 8.0 N and 5.0 N, respectively.



Components of $\mathcal{F}_1$

$$F_{1x} = \left|\vec{F}_1\right| \cos\theta_1 = 8.0 \times \cos\left(60^\circ\right) = 4.0N$$

$$F_{1y} = \left|\vec{F}_1\right| \sin\theta_1 = 8.0 \times \sin\left(60^\circ\right) = 6.9N$$

Components of $\mathcal{F}_2$

$$F_{2x} = \left|\vec{F}_2\right| \cos\theta_2 = 5.0 \times \cos\left(-20^\circ\right) = 4.7N$$

$$F_{2y} = \left|\vec{F}_2\right| \sin\theta_2 = 5.0 \times \sin\left(-20^\circ\right) = -1.7N$$

Components of total force $\mathcal{F}$

$$F_x = F_{1x} + F_{2x} = 4.0 + 4.7 = 8.7N = ma_x$$

$$F_y = F_{1y} + F_{2y} = 6.9 - 1.7 = 5.2N = ma_y$$

Magnitude and direction of acceleration a

$$a_x = \frac{F_x}{m} = \frac{8.7}{0.3} = 29m/s^2 \quad a_y = \frac{F_y}{m} = \frac{5.2}{0.3} = 17m/s^2 \quad \left|\vec{a}\right| = \sqrt{(a_x)^2 + (a_y)^2} = \sqrt{(29)^2 + (17)^2} = 34m/s^2$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^\circ$$

Acceleration Vector a

$$\vec{a} = a_x \hat{i} + a_y \hat{j} = \left(29\hat{i} + 17\hat{j}\right) m/s^2$$