PHYS 1443 – Section 001
Lecture #11

Thursday, June 23, 2011

Dr. Jaehoon Yu

- Energy Diagram
- General Energy Conservation & Mass Equivalence
- More on gravitational potential energy
  - Escape speed
- Power
- Linear Momentum and Forces
- Linear Momentum Conservation
- Collisions and Impulse

Today's homework is homework #6, due 10pm, Monday, June 27!!

Announcements

- Mid-term exam results
  - Class average: 65.3/99
    - Equivalent to 66/100
    - Incredibly consistent with quiz results!!
  - Class top score: 92/99
- Evaluation policy
  - Homework: 30%
  - Midterm and final comprehensive exam: 22.5% each
  - Lab: 15%
  - Quizzes: 10%
  - Extra credit: 10%

Reminder: Special Project

- Derive the formula for the gravitational acceleration (g_{in}) at the radius $R_{in}\left(< R_E\right)$ from the center, inside of the Earth. (10 points)
- Compute the fractional magnitude of the gravitational acceleration 1km and 500km inside the surface of the Earth with respect to that on the surface. (6 points, 3 points each)
- Due at the beginning of the class Monday, June 27

Energy Diagram and the Equilibrium of a System

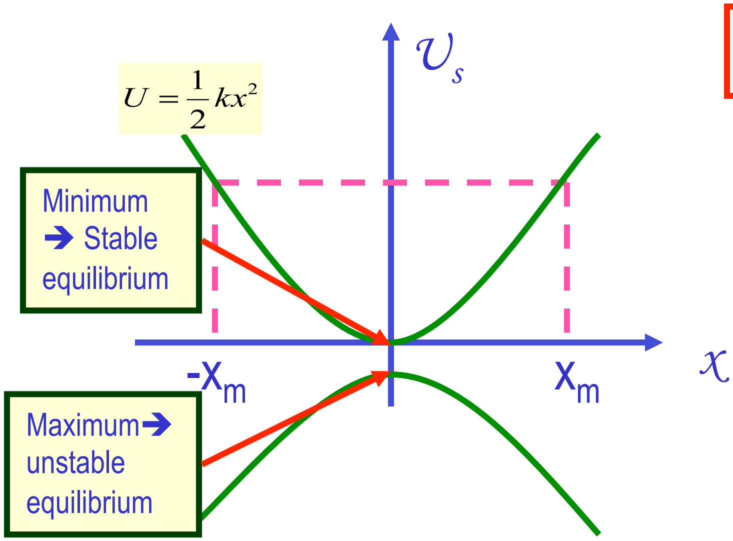
One can draw potential energy as a function of position ➔ Energy Diagram

Let's consider potential energy of a spring-ball system

$$U_s = \frac{1}{2}kx^2$$

What shape is this diagram?

A Parabola



What does this energy diagram tell you?

1. *Potential energy for this system is the same independent of the sign of the position.*
2. *The force is 0 when the slope of the potential energy curve is 0 at the position.*
3. *x=0 is the stable equilibrium position of this system where the potential energy is minimum.*

Position of a stable equilibrium corresponds to points where potential energy is at a minimum.

Position of an unstable equilibrium corresponds to points where potential energy is a maximum.

General Energy Conservation and Mass-Energy Equivalence

General Principle of Energy Conservation

The total energy of an isolated system is conserved as long as all forms of energy are taken into account.

What about friction?

Friction is a non-conservative force and causes mechanical energy to change to other forms of energy.

However, if you add the new forms of energy altogether, the system as a whole did not lose any energy, as long as it is self-contained or isolated.

In the grand scale of the universe, no energy can be destroyed or created but just transformed or transferred from one to another. ***The total energy of universe is constant as a function of time!!*** ***The total energy of the universe is conserved!***

Principle of Conservation of Mass

In any physical or chemical process, mass is neither created nor destroyed. Mass before a process is identical to the mass after the process.

Einstein's Mass-Energy equality.

$$E_R = mc^2$$

How many joules does your body correspond to?

The Gravitational Field

The gravitational force is a field force. The force exists everywhere in the universe.

If one were to place a test object of mass m at any point in the space in the existence of another object of mass M, the test object will feel the gravitational force exerted by M, $\vec{F}_g = m\vec{g}$.

Therefore the gravitational field *g* is defined as $\vec{g} \equiv \frac{\vec{F}_g}{m}$

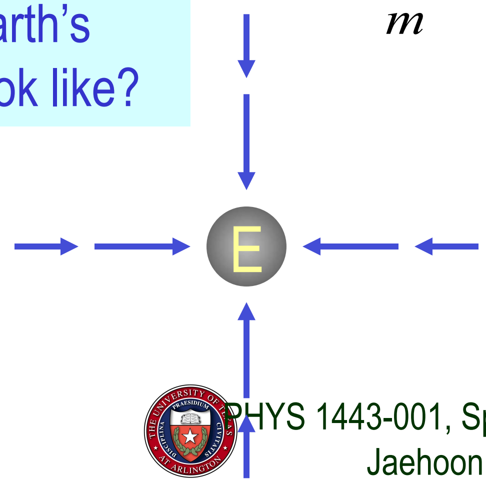
In other words, the gravitational field at a point in space is the gravitational force experienced by a test particle placed at the point divided by its mass.

$$\vec{g} = \frac{\vec{F}_g}{m} = -\frac{GM_E}{R_E^2}\hat{r}$$

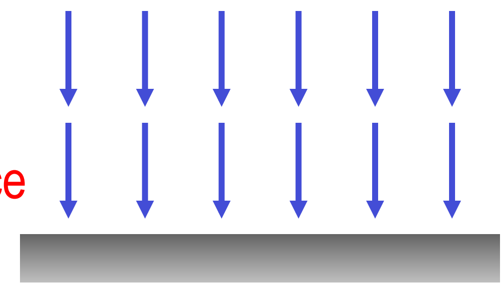
Where $\hat{r}$ is the unit vector pointing outward from the center of the Earth

So how does the Earth's gravitational field look like?

Far away from the Earth's surface



Close to the Earth's surface



The Gravitational Potential Energy

What is the potential energy of an object at the height y from the surface of the Earth?

$$U = mgy$$

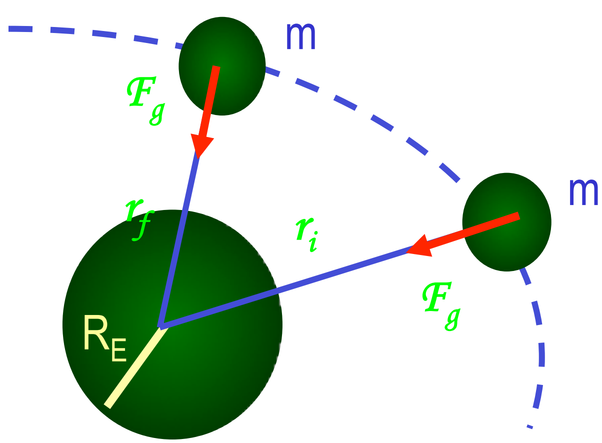
Do you think this would work in general cases?

No, it would not.

Why not?

Because this formula is only valid for the case where the gravitational force is constant, near the surface of the Earth, and the generalized gravitational force is inversely proportional to the square of the distance.

OK. Then how would we generalize the potential energy in the gravitational field?



Since the gravitational force is a central force, and a central force is a conservative force, the work done by the gravitational force is independent of the path.

The path can be considered as consisting of many tangential and radial motions.
Tangential motions do not contribute to work!!!

More on The Gravitational Potential Energy

Since the gravitational force is a radial force, it performs work only when the path has component in radial direction. Therefore, the work performed by the gravitational force that depends on the position becomes:

$$dW = \vec{F} \cdot d\vec{r} = F(r)\,dr$$

For the whole path

$$W = \int_{r_i}^{r_f} F(r)\,dr$$

Potential energy is the negative of the work done through the path

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)\,dr$$

Since the Earth's gravitational force is

$$F(r) = -\frac{GM_E m}{r^2}$$

Thus the potential energy function becomes

$$U_f - U_i = \int_{r_i}^{r_f} \frac{GM_E m}{r^2}\,dr = -GM_E m\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

Since only the difference of potential energy matters, by taking the infinite distance as the initial point of the potential energy, we obtain

$$U = -\frac{GM_E m}{r}$$

For any two particles?

$$U = -\frac{Gm_1 m_2}{r}$$

The energy needed to take the particles infinitely apart.

For many particles?

$$U = \sum_{i,j} U_{i,j}$$

Example of Gravitational Potential Energy

A particle of mass m is displaced through a small vertical distance Δy near the Earth's surface. Show that in this situation the general expression for the change in gravitational potential energy is reduced to the $\Delta U = -mg\Delta y$.

Taking the general expression of gravitational potential energy

$$\Delta U = -GM_E m\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

Reorganizing the terms w/ the common denominator

$$= -GM_E m\frac{(r_f - r_i)}{r_f r_i} = -GM_E m\frac{\Delta y}{r_f r_i}$$

Since the situation is close to the surface of the Earth

$r_i \approx R_E$ and $r_f \approx R_E$

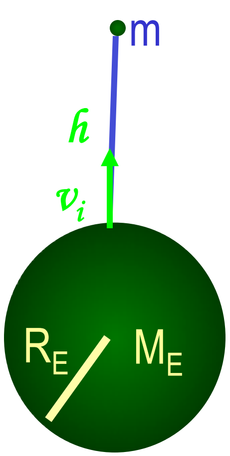
Therefore, ΔU becomes

$$\Delta U = -GM_E m\frac{\Delta y}{R_E^2}$$

Since on the surface of the Earth the gravitational field is $g = \frac{GM_E}{R_E^2}$

The potential energy becomes $\Delta U = -mg\Delta y$

The Escape Speed



Consider an object of mass m is projected vertically from the surface of the Earth with an initial speed v_i and eventually comes to stop v_f=0 at the distance r_{max}.

Since the total mechanical energy is conserved

$$ME = K + U = \frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{\max}}$$

Solving the above equation for v_i, one obtains

$$v_i = \sqrt{2GM_E\left(\frac{1}{R_E} - \frac{1}{r_{\max}}\right)}$$

Therefore if the initial speed v_i is known, one can use this formula to compute the final height h of the object.

$$h = r_{\max} - R_E = \frac{v_i^2 R_E^2}{2GM_E - v_i^2 R_E}$$

In order for an object to escape Earth's gravitational field completely, the initial speed needs to be

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2\times 6.67\times 10^{-11}\times 5.98\times 10^{24}}{6.37\times 10^{6}}}$$

$$= 1.12\times 10^{4}\, m/s = 11.2\, km/s$$

This is called the escape speed. This formula is valid for any planet or large mass objects.

How does this depend on the mass of the escaping object?

Independent of the mass of the escaping object

Power

- Rate at which the work is done or the energy is transferred
  - What is the difference for the same car with two different engines (4 cylinder and 8 cylinder) climbing the same hill?
  - ➔ The time… 8 cylinder car climbs up the hill faster!

Is the total amount of work done by the engines different? NO

Then what is different? The rate at which the same amount of work performed is higher for 8 cylinders than 4.

Average power $\overline{P} \equiv \frac{\Delta W}{\Delta t}$

Instantaneous power $P \equiv \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} = \lim_{\Delta t \to 0} \left(\sum \vec{F}\right) \cdot \frac{\Delta \vec{s}}{\Delta t} = \left(\sum \vec{F}\right) \cdot \vec{v} = \left|\sum \vec{F}\right| \left|\vec{v}\right| \cos\theta$

Unit? $J/s = Watts$ $1HP \equiv 746\,Watts$

What do power companies sell? $1kWH = 1000Watts \times 3600s = 3.6 \times 10^6 J$

Energy

Energy Loss in Automobile

Automobile uses only 13% of its fuel to propel the vehicle.

Why?

67% in the engine:
- Incomplete burning
- Heat
- Sound

16% in friction in mechanical parts

4% in operating other crucial parts such as oil and fuel pumps, etc

13% used for balancing energy loss related to moving vehicle, like air resistance and road friction to tire, etc

Two frictional forces involved in moving vehicles

$m_{car} = 1450kg \quad Weight = mg = 14200N$

Coefficient of Rolling Friction; m=0.016

$\mu n = \mu mg = 227N$

Air Drag

$f_a = \frac{1}{2} D\rho A v^2 = \frac{1}{2} \times 0.5 \times 1.293 \times 2v^2 = 0.647v^2$

Total Resistance

$f_t = f_r + f_a$

Total power to keep speed v=26.8m/s=60mi/h

$P = f_t v = (691N) \cdot 26.8 = 18.5kW$

Power to overcome each component of resistance

$P_r = f_r v = (227) \cdot 26.8 = 6.08kW$

$P_a = f_a v = (464.7) \cdot 26.8 = 12.5kW$

Yearly Solar Fluxes and Human Energy Consumption

Source	Energy Amount (J)
Solar	3.85×10^{21}
Wind	2.25×10^{18}
Biomass	2.0×10^{18}
Primary E use (2005)	4.87×10^{17}
Electricity (2005)	5.7×10^{16}