PHYS 1443 – Section 001
Lecture #16

Wednesday, July 6, 2011

Dr. Jaehoon Yu

- Calculation of Moment of Inertia
- Torque and Angular Acceleration
- Rolling Motion & Rotational Kinetic Energy
- Work, Power and Energy in Rotation
- Angular Momentum & Its Conservation
- Equilibrium

The final homework is homework #9, due 10pm, Saturday, July 9!!

Announcements

- Quiz #3 results
  - Class average: 20.1/35
    - Equivalent to 59.7/100
    - Extremely consistent: 62.5 and 61.4
  - Top score: 33/35
- Quiz #4 tomorrow, Thursday, July 7
  - Beginning of the class
  - Covers CH10.1 through what we learn today (CH12.2?)
- Please to not forget the planetarium special credit sheet submission tomorrow
- Final Comprehensive Exam
  - 8 – 10am, Monday, July 11 in SH103
  - Covers CH1.1 through what we learn Thursday, July 7
  - Mixture of multiple choice and free response problems
- Bring your two special projects during the intermission

Moment of Inertia

Rotational Inertia: Measure of resistance of an object to changes in its rotational motion. Equivalent to mass in linear motion.

For a group of particles $I \equiv \sum_i m_i r_i^2$

For a rigid body $I \equiv \int r^2 dm$

What are the dimension and unit of Moment of Inertia? $\left[ML^2\right]$ $kg \cdot m^2$

Determining Moment of Inertia is extremely important for computing equilibrium of a rigid body, such as a building.

Dependent on the axis of rotation!!!

Calculation of Moments of Inertia

Moments of inertia for large objects can be computed, if we assume the object consists of small volume elements with mass, Δm_i.

The moment of inertia for the large rigid object is $I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i = \int r^2 dm$

It is sometimes easier to compute moments of inertia in terms of volume of the objects rather than their mass

How can we do this?

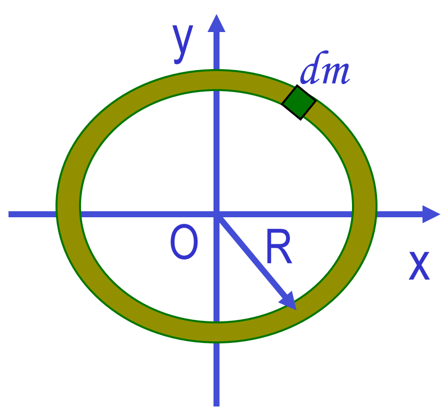
Using the volume density, ρ, replace dm in the above equation with dV.

$$\rho = \frac{dm}{dV} \Rightarrow dm = \rho dV$$

The moments of inertia becomes

$$I = \int \rho r^2 dV$$

Example: Find the moment of inertia of a uniform hoop of mass M and radius R about an axis perpendicular to the plane of the hoop and passing through its center.



The moment of inertia is

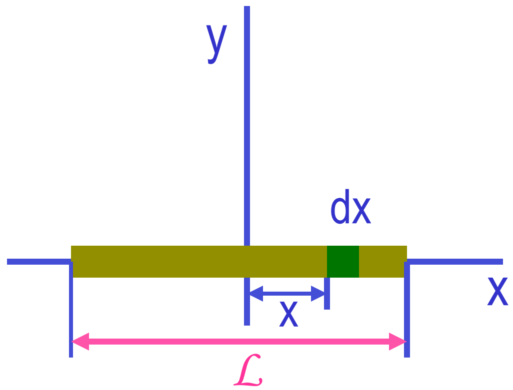
$$I = \int r^2 dm = R^2 \int dm = MR^2$$

What do you notice from this result?

The moment of inertia for this object is the same as that of a point of mass M at the distance R.

Ex.10 – 11 Rigid Body Moment of Inertia

Calculate the moment of inertia of a uniform rigid rod of length L and mass M about an axis perpendicular to the rod and passing through its center of mass.



The line density of the rod is $\lambda = \frac{M}{L}$

so the masslet is $dm = \lambda dx = \frac{M}{L} dx$

The moment of inertia is

$$I = \int r^2\, dm = \int_{-L/2}^{L/2} \frac{x^2 M}{L} dx = \frac{M}{L}\left[\frac{1}{3}x^3\right]_{-L/2}^{L/2}$$

$$= \frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right] = \frac{M}{3L}\left(\frac{L^3}{4}\right) = \frac{ML^2}{12}$$

What is the moment of inertia when the rotational axis is at one end of the rod.

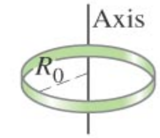
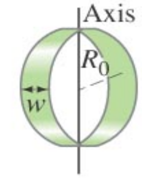
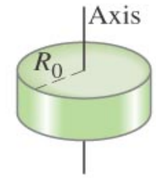
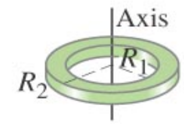
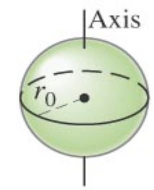
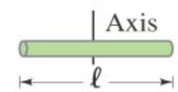
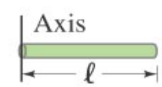
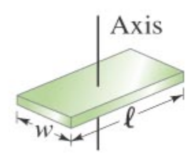
$$I = \int r^2\, dm = \int_0^L \frac{x^2 M}{L} dx = \frac{M}{L}\left[\frac{1}{3}x^3\right]_0^L$$

$$= \frac{M}{3L}\left[(L)^3 - 0\right] = \frac{M}{3L}\left(L^3\right) = \frac{ML^2}{3}$$

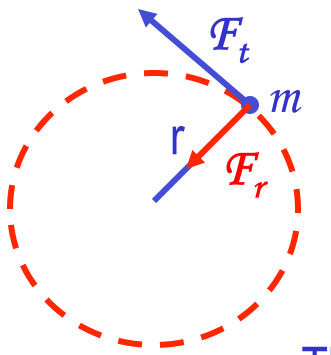
Will this be the same as the above. Why or why not?

Since the moment of inertia is resistance to motion, it makes perfect sense for it to be harder to move when it is rotating about the axis at one end.

Check out Figure 10 – 20 for moment of inertia for various shaped objects

	Object	Location of axis		Moment of inertia
(a)	**Thin hoop,** radius R_0	Through center		MR_0^2
(b)	**Thin hoop,** radius R_0 width w	Through central diameter		$\frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2$
(c)	**Solid cylinder,** radius R_0	Through center		$\frac{1}{2}MR_0^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius r_0	Through center		$\frac{2}{5}Mr_0^2$
(f)	**Long uniform rod,** length ℓ	Through center		$\frac{1}{12}M\ell^2$
(g)	**Long uniform rod,** length ℓ	Through end		$\frac{1}{3}M\ell^2$
(h)	**Rectangular thin plate,** length ℓ, width w	Through center		$\frac{1}{12}M(\ell^2 + w^2)$

Torque & Angular Acceleration



Let's consider a point object with mass m rotating on a circle.

What forces do you see in this motion?

The tangential force $\mathcal{F}_t$ and the radial force $\mathcal{F}_r$

The tangential force $\mathcal{F}_t$ is $\quad F_t = ma_t = mr\alpha$

The torque due to tangential force $\mathcal{F}_t$ is $\quad \tau = F_t r = ma_t r = mr^2\alpha = I\alpha$

What do you see from the above relationship?

$$\tau = I\alpha$$

What does this mean?

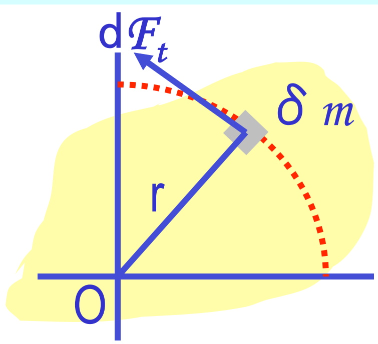
Torque acting on a particle is proportional to the angular acceleration.

What law do you see from this relationship?

Analogs to Newton's 2nd law of motion in rotation.

How about a rigid object?

The external tangential force $\delta\mathcal{F}_t$ is $\quad \delta F_t = \delta m a_t = \delta m r\alpha$



The torque due to tangential force $\mathcal{F}_t$ is $\quad \delta\tau = \delta F_t r = \left(r^2\delta m\right)\alpha$

The total torque is $\quad \tau = \lim_{\delta\tau \to 0} \sum \delta\tau = \int d\tau = \alpha \lim_{\delta m \to 0} \sum r^2 \delta m = \alpha \int r^2\, dm = I\alpha$

What is the contribution due to radial force and why?

Contribution from radial force is 0, because its line of action passes through the pivoting point, making the moment arm 0.

Rolling Motion of a Rigid Body

What is a rolling motion?

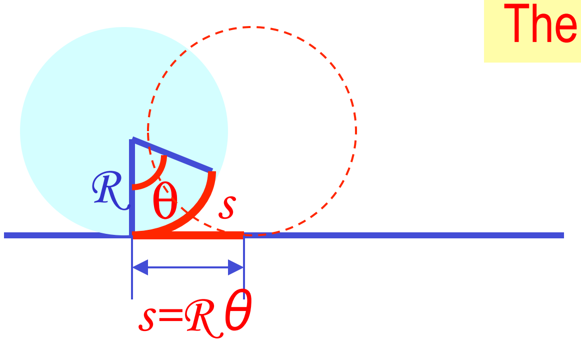
A more generalized case of a motion where the rotational axis moves together with an object

A rotational motion about a moving axis

To simplify the discussion, let's make a few assumptions

1. Limit our discussion on very symmetric objects, such as cylinders, spheres, etc
2. The object rolls on a flat surface

Let's consider a cylinder rolling on a flat surface, without slipping.



Under what condition does this "Pure Rolling" happen?

The total linear distance the CM of the cylinder moved is $s = R\theta$

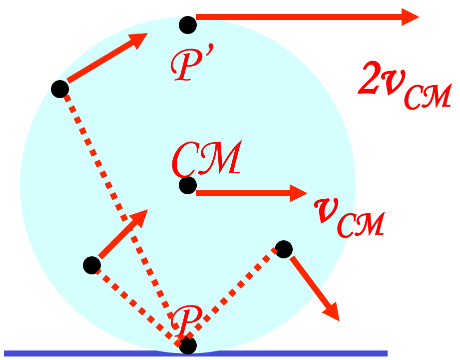
Thus the linear speed of the CM is $v_{CM} = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega$

The condition for a "Pure Rolling motion"

More Rolling Motion of a Rigid Body

The magnitude of the linear acceleration of the CM is

$$a_{CM} = \frac{dv_{CM}}{dt} = R\frac{d\omega}{dt} = R\alpha$$



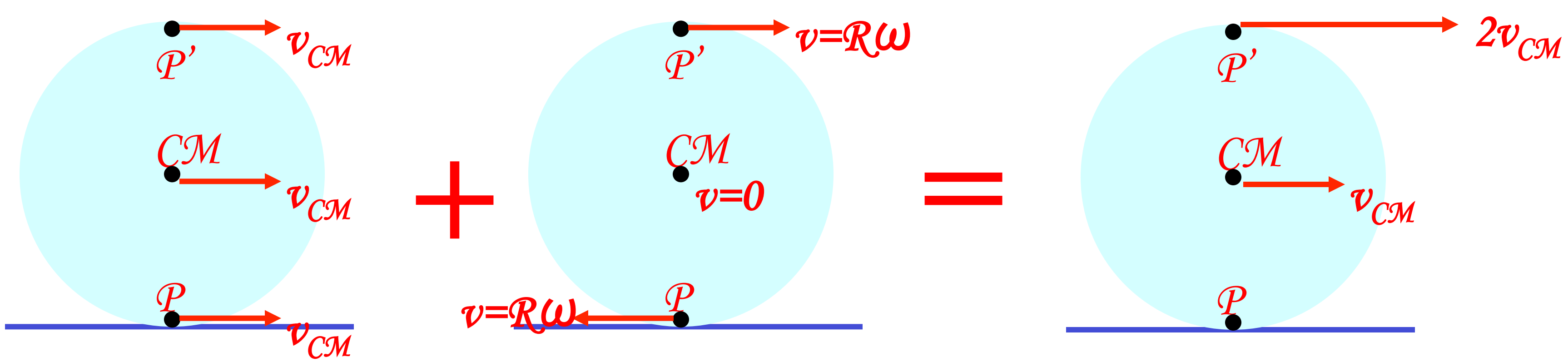
As we learned in rotational motion, all points in a rigid body moves at the same angular speed but at different linear speeds.

CM is moving at the same speed at all times.

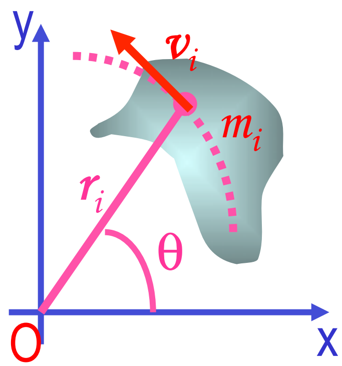
At any given time, the point that comes to P has 0 linear speed while the point at P' has twice the speed of CM

Why??

A rolling motion can be interpreted as the sum of Translation and Rotation



Rotational Kinetic Energy



What do you think the kinetic energy of a rigid object that is undergoing a circular motion is?

Kinetic energy of a masslet, m_i, moving at a tangential speed, v_i, is

$$K_i = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i r_i^2 \omega^2$$

Since a rigid body is a collection of masslets, the total kinetic energy of the rigid object is

$$K_R = \sum_i K_i = \frac{1}{2} \sum_i m_i r_i^2 \omega^2 = \frac{1}{2} \left(\sum_i m_i r_i^2 \right) \omega^2$$

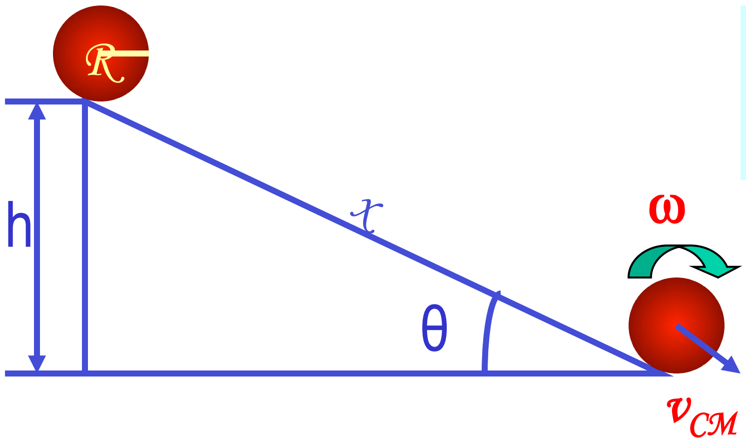
Since moment of Inertia, I, is defined as

$$I = \sum_i m_i r_i^2$$

The above expression is simplified as

$$K_R = \frac{1}{2} I \omega^2$$

Kinetic Energy of a Rolling Sphere



Let's consider a sphere with radius R rolling down the hill without slipping.

$$KE = KE_{Rotation} + KE_{Linear} = \frac{1}{2} I_{CM}\omega^2 + \frac{1}{2} M v_{CM}^2$$

$$= \frac{1}{2} I_{CM}\left(\frac{v_{CM}}{R}\right)^2 + \frac{1}{2} M v_{CM}^2$$

$$= \frac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right) v_{CM}^2$$

Since $v_{CM} = R\omega$

What is the speed of the CM in terms of known quantities and how do you find this out?

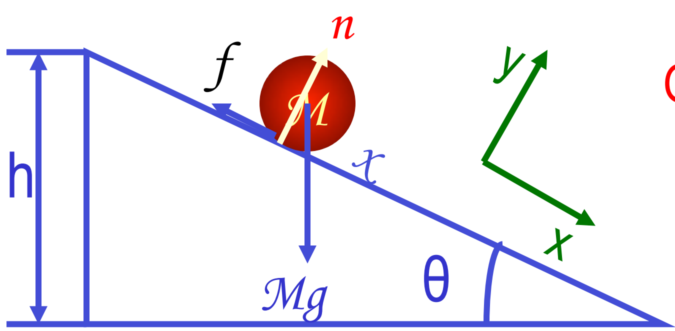
Since the kinetic energy at the bottom of the hill must be equal to the potential energy at the top of the hill

$$K = \frac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right) v_{CM}^2 = Mgh$$

$$v_{CM} = \sqrt{\frac{2gh}{1 + I_{CM}/MR^2}}$$

Ex. 10 – 16: Rolling Kinetic Energy

For solid sphere as shown in the figure, calculate the linear speed of the CM at the bottom of the hill and the magnitude of linear acceleration of the CM. Solve this problem using Newton's second law, the dynamic method.



What are the forces involved in this motion?

Gravitational Force, Frictional Force, Normal Force

Newton's second law applied to the CM gives

$$\sum F_x = Mg\sin\theta - f = Ma_{CM}$$
$$\sum F_y = n - Mg\cos\theta = 0$$

Since the forces $\mathcal{M}g$ and **n** go through the CM, their moment arm is 0 and do not contribute to torque, while the static friction f causes torque

$$\tau_{CM} = fR = I_{CM}\alpha$$

We know that

$$I_{CM} = \frac{2}{5}MR^2$$
$$a_{CM} = R\alpha$$

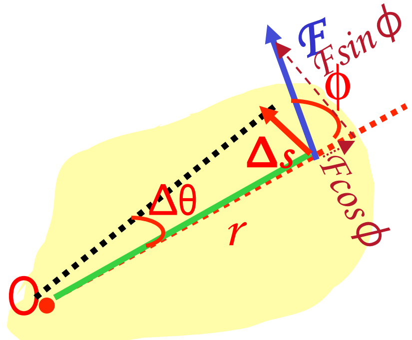
We obtain

$$f = \frac{I_{CM}\alpha}{R} = \frac{\frac{2}{5}MR^2}{R}\left(\frac{a_{CM}}{R}\right) = \frac{2}{5}Ma_{CM}$$

Substituting f in dynamic equations

$$Mg\sin\theta = \frac{7}{5}Ma_{CM} \qquad a_{CM} = \frac{5}{7}g\sin\theta$$

Work, Power, and Energy in Rotation



Let's consider the motion of a rigid body with a single external force $\mathcal{F}$ exerting on the point P, moving the object by Δs. The work done by the force $\mathcal{F}$ as the object rotates through the infinitesimal distance $\Delta s = r\Delta\theta$ is

$$\Delta W = \vec{F} \cdot \Delta\vec{s} = (F\sin\phi) r\Delta\theta$$

What is $\mathcal{F}\sin\phi$? The tangential component of the force $\mathcal{F}$.

What is the work done by radial component $\mathcal{F}\cos\phi$? Zero, because it is perpendicular to the displacement.

Since the magnitude of torque is $r\mathcal{F}\sin\phi$,

$$\Delta W = (rF\sin\phi)\Delta\theta = \tau\Delta\theta$$

The rate of work, or power, becomes

$$P = \frac{\Delta W}{\Delta t} = \frac{\tau\Delta\theta}{\Delta t} = \tau\omega$$

How was the power defined in linear motion?

The rotational work done by an external force equals the change in rotational Kinetic energy.

$$\sum\tau = I\alpha = I\left(\frac{\Delta\omega}{\Delta t}\right) \Rightarrow \sum\tau\Delta\theta = I\omega\Delta\omega$$

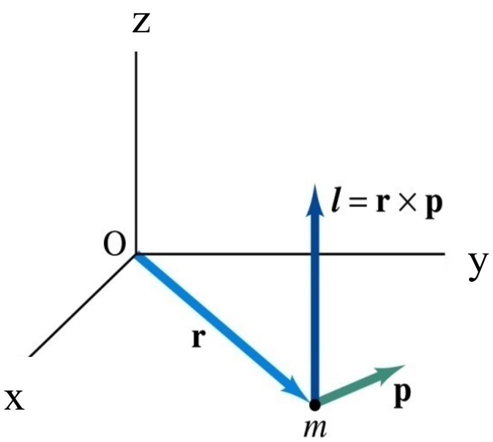
The work put in by the external force then

$$\Delta W = \int_{\omega_i}^{\omega_f} I\omega\, d\omega = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

Work-kinetic Energy Theorem

Angular Momentum of a Particle

If you grab onto a pole while running, your body will rotate about the pole, gaining angular momentum. We've used the linear momentum to solve physical problems with linear motions, the angular momentum will do the same for rotational motions.



Let's consider a point-like object (particle) with mass m located at the vector location $\mathbf{r}$ and moving with linear velocity $\mathbf{v}$

The angular momentum $\mathcal{L}$ of this particle relative to the origin O is

$$\vec{L} \equiv \vec{r} \times \vec{p}$$

What is the unit and dimension of angular momentum? $kg \cdot m^2/s$ $[ML^2T^{-1}]$

Note that $\mathcal{L}$ depends on origin O. Why? Because $\mathbf{r}$ changes

What else do you learn? The direction of $\mathcal{L}$ is +z.

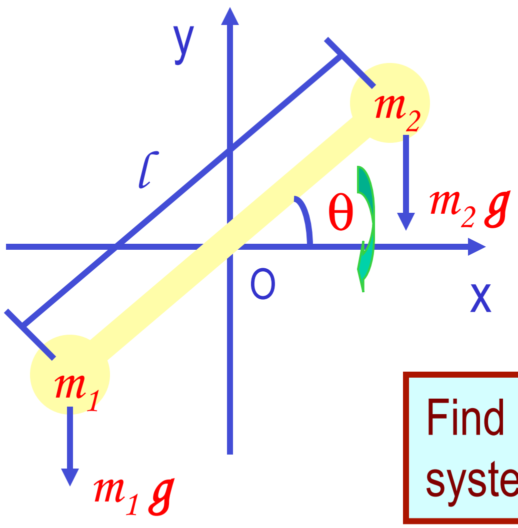
Since $\mathbf{p}$ is $m\mathbf{v}$, the magnitude of $\mathcal{L}$ becomes $L = mvr\sin\phi = mr^2\sin\phi v/r = I\omega$

What do you learn from this? If the direction of linear velocity points to the origin of rotation, the particle does not have any angular momentum.

If the linear velocity is perpendicular to position vector, the particle moves exactly the same way as a point on a rim.

Example for Rigid Body Angular Momentum

A rigid rod of mass $\mathcal{M}$ and length ℓ is pivoted without friction at its center. Two particles of mass m_1 and m_2 are attached to either end of the rod. The combination rotates on a vertical plane with an angular speed of ω. Find an expression for the magnitude of the angular momentum.



The moment of inertia of this system is

$$I = I_{rod} + I_{m_1} + I_{m_2} = \frac{1}{12}Ml^2 + \frac{1}{4}m_1 l^2 + \frac{1}{4}m_2 l^2$$

$$= \frac{l^2}{4}\left(\frac{1}{3}M + m_1 + m_2\right)$$

$$L = I\omega = \frac{\omega l^2}{4}\left(\frac{1}{3}M + m_1 + m_2\right)$$

Find an expression for the magnitude of the angular acceleration of the system when the rod makes an angle θ with the horizon.

If $m_1 = m_2$, no angular momentum because the net torque is 0.

If θ=+/−π/2, at equilibrium so no angular momentum.

First compute the net external torque

$$\tau_1 = m_1 g \frac{l}{2}\cos\theta \qquad \tau_2 = -m_2 g \frac{l}{2}\cos\theta$$

$$\tau_{ext} = \tau_1 + \tau_2 = \frac{gl\cos\theta\left(m_1 - m_2\right)}{2}$$

Thus α becomes

$$\alpha = \frac{\sum \tau_{ext}}{I} = \frac{\frac{1}{2}(m_1 - m_2)gl\cos\theta}{\frac{l^2}{4}\left(\frac{1}{3}M + m_1 + m_2\right)} = \frac{2(m_1 - m_2)\cos\theta}{\left(\frac{1}{3}M + m_1 + m_2\right)}\frac{g}{l}$$

Conservation of Angular Momentum

Remember under what condition the linear momentum is conserved?

Linear momentum is conserved when the net external force is 0.

$$\sum \vec{F} = 0 = \frac{d\vec{p}}{dt}$$

$$\vec{p} = const$$

By the same token, the angular momentum of a system is constant in both magnitude and direction, if the resultant external torque acting on the system is 0.

$$\sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt} = 0$$

$$\boxed{\vec{L} = const}$$

What does this mean?

Angular momentum of the system before and after a certain change is the same.

$$\vec{L}_i = \vec{L}_f = \text{constant}$$

Three important conservation laws for isolated system that does not get affected by external forces

$$K_i + U_i = K_f + U_f$$ **Mechanical Energy**

$$\vec{p}_i = \vec{p}_f$$ **Linear Momentum**

$$\vec{L}_i = \vec{L}_f$$ **Angular Momentum**

Ex. 11 – 3 Neutron Star

A star rotates with a period of 30 days about an axis through its center. After the star undergoes a supernova explosion, the stellar core, which had a radius of 1.0×10^4km, collapses into a neutron star of radius 3.0km. Determine the period of rotation of the neutron star.

What is your guess about the answer?

The period will be significantly shorter, because its radius got smaller.

Let's make some assumptions:

1. There is no external torque acting on it
2. The shape remains spherical
3. Its mass remains constant

Using angular momentum conservation

$$L_i = L_f$$

$$I_i\omega_i = I_f\omega_f$$

The angular speed of the star with the period T is

$$\omega = \frac{2\pi}{T}$$

Thus

$$\omega_f = \frac{I_i\omega_i}{I_f} = \frac{mr_i^2}{mr_f^2}\frac{2\pi}{T_i}$$

$$T_f = \frac{2\pi}{\omega_f} = \left(\frac{r_f^2}{r_i^2}\right)T_i = \left(\frac{3.0}{1.0\times10^4}\right)^2 \times 30days = 2.7\times10^{-6}days = 0.23s$$

Similarity Between Linear and Rotational Motions

All physical quantities in linear and rotational motions show striking similarity.

Quantities	Linear	Rotational
Mass	Mass M	Moment of Inertia $I = mr^2$
Length of motion	Distance L	Angle θ (Radian)
Speed	$v = \frac{\Delta r}{\Delta t}$	$\omega = \frac{\Delta \theta}{\Delta t}$
Acceleration	$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta \omega}{\Delta t}$
Force	Force $\vec{F} = m\vec{a}$	Torque $\vec{\tau} = I\vec{\alpha}$
Work	Work $W = \vec{F} \cdot \vec{d}$	Work $W = \tau\theta$
Power	$P = \vec{F} \cdot \vec{v}$	$P = \tau\omega$
Momentum	$\vec{p} = m\bar{v}$	$\vec{L} = I\bar{\omega}$
Kinetic Energy	Kinetic $K = \frac{1}{2}mv^2$	Rotational $K_R = \frac{1}{2}I\omega^2$

Conditions for Equilibrium

What do you think the term "An object is at its equilibrium" means?

The object is either at rest (Static Equilibrium) or its center of mass is moving at a constant velocity (Dynamic Equilibrium).

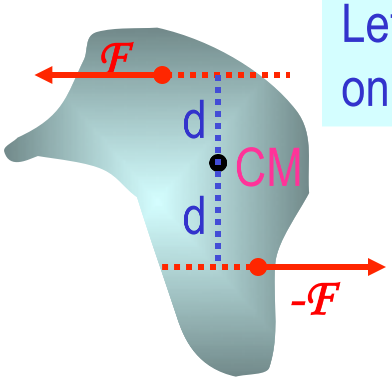
When do you think an object is at its equilibrium?

Translational Equilibrium: Equilibrium in linear motion

$$\sum \vec{F} = 0$$

Is this it?

The above condition is sufficient for a point-like object to be at its translational equilibrium. For an object with size, however, this is not sufficient. One more condition is needed. What is it?



Let's consider two forces equal in magnitude but in opposite direction acting on a rigid object as shown in the figure. What do you think will happen?

The object will rotate about the CM. The net torque acting on the object about any axis must be 0.

$$\sum \vec{\tau} = 0$$

For an object to be at its *static equilibrium*, the object should not have linear or angular speed. $v_{CM} = 0 \quad \omega = 0$

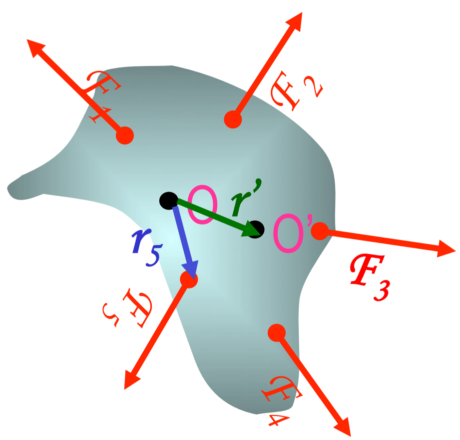
More on Conditions for Equilibrium

To simplify the problem, we will only deal with forces acting on x-y plane, giving torque only along z-axis. What do you think the conditions for equilibrium be in this case?

The six possible equations from the two vector equations turns to three equations.

$$\sum \vec{F} = 0 \Rightarrow \sum F_x = 0, \quad \sum F_y = 0 \quad \textbf{AND} \quad \sum \vec{\tau} = 0 \Rightarrow \sum \tau_z = 0$$

What happens if there are many forces exerting on an object?



If an object is at its translational static equilibrium, and if the net torque acting on the object is 0 about one axis, the net torque must be 0 about any arbitrary axis.

Why is this true?

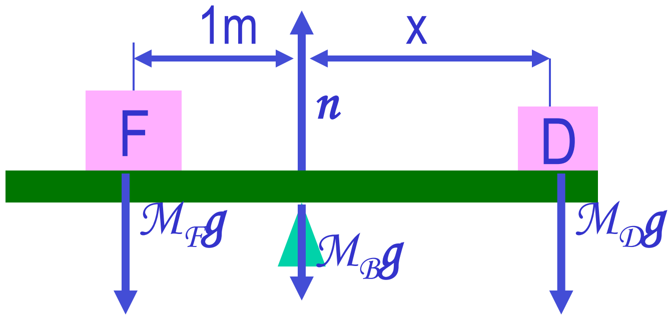
Because the object is *not moving*, no matter what the rotational axis is, there should not be any motion. It is simply a matter of mathematical manipulation.

How do we solve equilibrium problems?

1. Identify all the forces and their directions and locations
2. Draw a free-body diagram with forces indicated on it with their directions and locations properly noted
3. Write down force equation for each x and y component with proper signs
4. Select a rotational axis for torque calculations ➔ Selecting the axis such that the torque of one of the unknown forces become 0 makes the problem easier to solve
5. Write down the torque equation with proper signs
6. Solve the equations for unknown quantities

Ex. 12 – 3: Seesaw Balancing

A uniform 40.0 N board supports the father and the daughter each weighing 800 N and 350 N, respectively, and is not moving. If the support (or fulcrum) is under the center of gravity of the board, and the father is 1.00 m from CoG, what is the magnitude of the normal force n exerted on the board by the support?



Since there is no linear motion, this system is in its translational equilibrium

$$\sum F_x = 0$$

$$\sum F_y = n - M_B g - M_F g - M_D g = 0$$

Therefore the magnitude of the normal force $n = 40.0 + 800 + 350 = 1190\,N$

Determine where the child should sit to balance the system.

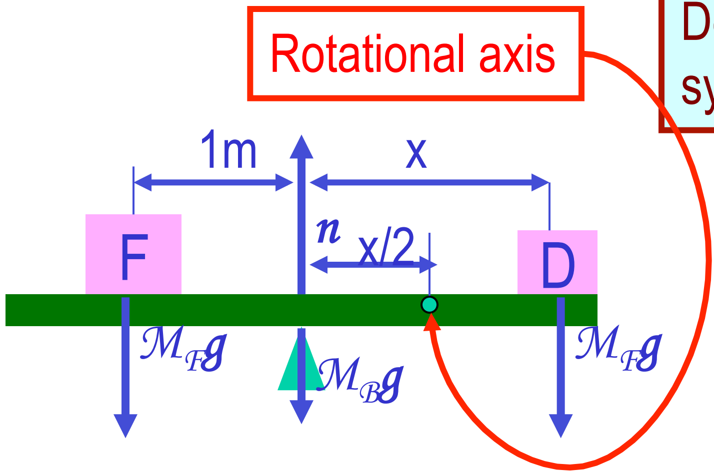
The net torque about the fulcrum by the three forces are

$$\tau = M_B g \cdot 0 + n \cdot 0 + M_F g \cdot 1.00 - M_D g \cdot x = 0$$

Therefore to balance the system the daughter must sit

$$x = \frac{M_F g}{M_D g} \cdot 1.00m = \frac{800}{350} \cdot 1.00m = 2.29m$$

Seesaw Example Cont'd



Determine the position of the child to balance the system for different position of axis of rotation.

The net torque about the axis of rotation by all the forces are

$$\tau = M_B g \cdot x/2 + M_F g \cdot (1.00 + x/2) - n \cdot x/2 - M_D g \cdot x/2 = 0$$

Since the normal force is $n = M_B g + M_F g + M_D g$

The net torque can be rewritten

$$\tau = M_B g \cdot x/2 + M_F g \cdot (1.00 + x/2) - (M_B g + M_F g + M_D g) \cdot x/2 - M_D g \cdot x/2$$
$$= M_F g \cdot 1.00 - M_D g \cdot x = 0$$

Therefore

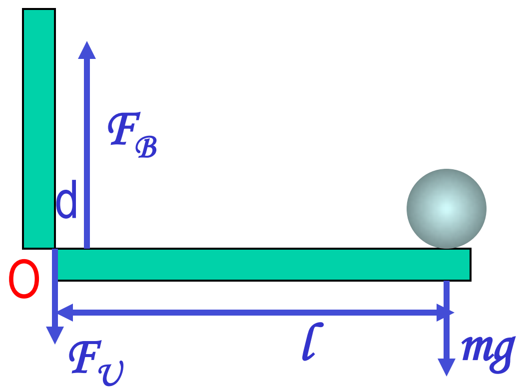
$$x = \frac{M_F g}{M_D g} \cdot 1.00m = \frac{800}{350} \cdot 1.00m = 2.29m$$

What do we learn?

No matter where the rotation axis is, net effect of the torque is identical.

Ex. 12.4 for Mechanical Equilibrium

A person holds a 50.0N sphere in his hand. The forearm is horizontal. The biceps muscle is attached 3.00 cm from the joint, and the sphere is 35.0cm from the joint. Find the upward force exerted by the biceps on the forearm and the downward force exerted by the upper arm on the forearm and acting at the joint. Neglect the weight of forearm.



Since the system is in equilibrium, from the translational equilibrium condition

$$\sum F_x = 0$$

$$\sum F_y = F_B - F_U - mg = 0$$

From the rotational equilibrium condition $\sum \tau = F_U \cdot 0 + F_B \cdot d - mg \cdot l = 0$

Thus, the force exerted by the biceps muscle is

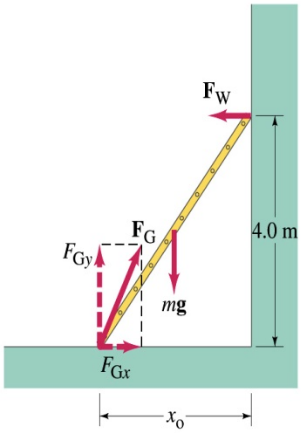
$$F_B \cdot d = mg \cdot l$$

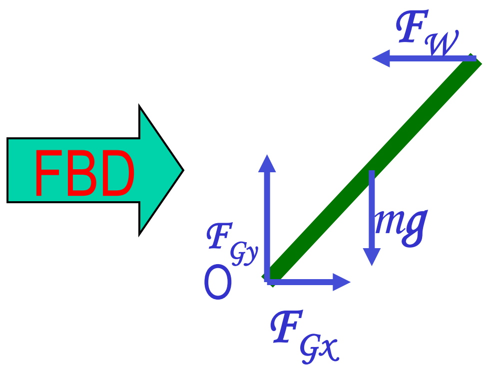
$$F_B = \frac{mg \cdot l}{d} = \frac{50.0 \times 35.0}{3.00} = 583N$$

Force exerted by the upper arm is $F_U = F_B - mg = 583 - 50.0 = 533N$

Example 12 – 6

A 5.0 m long ladder leans against a wall at a point 4.0m above the ground. The ladder is uniform and has mass 12.0kg. Assuming the wall is frictionless (but ground is not), determine the forces exerted on the ladder by the ground and the wall.





First the translational equilibrium, using components

$$\sum F_x = F_{Gx} - F_W = 0$$

$$\sum F_y = -mg + F_{Gy} = 0$$

Thus, the y component of the force by the ground is

$$F_{Gy} = mg = 12.0 \times 9.8N = 118N$$

The length x_0 is, from Pythagorian theorem

$$x_0 = \sqrt{5.0^2 - 4.0^2} = 3.0m$$

Example 12 – 6 cont'd

From the rotational equilibrium $\sum \tau_O = -mg\, x_0/2 + F_W\, 4.0 \;\; = 0$

Thus the force exerted on the ladder by the wall is

$$F_W = \frac{mg\, x_0/2}{4.0} = \frac{118 \cdot 1.5}{4.0} = 44N$$

The x component of the force by the ground is

$$\sum F_x = F_{Gx} - F_W = 0$$

Solve for F_{Gx}

$$F_{Gx} = F_W = 44N$$

Thus the force exerted on the ladder by the ground is

$$F_G = \sqrt{F_{Gx}^2 + F_{Gy}^2} = \sqrt{44^2 + 118^2} \approx 130N$$

The angle between the ground force to the floor

$$\theta = \tan^{-1}\left(\frac{F_{Gy}}{F_{Gx}}\right) = \tan^{-1}\left(\frac{118}{44}\right) = 70^\circ$$